Although dynamical systems have been used by cognitive scientists for more than a decade already (e.g. Kugler, Kelso, and Turvey, 1980), dynamical systems first gained widespread attention in the mid-1990s (e.g. Kelso, 1995; Port and van Gelder, 1995; Thelen and Smith, 1994). Dynamical systems theory was then, and continues to be, a crucial tool for embodied cognitive science. The word *dynamical* simply means “changing over time” and thus a *dynamical system* is simply a system whose behavior evolves or changes over time. The scientific study of dynamical systems is concerned with understanding, modeling, and predicting the ways in which the behavior of a system changes over time. In the last few decades, thanks to increasing computational power, researchers have begun to investigate and understand the dynamic behavior of complex biological, cognitive, and social systems, using the concepts and tools of non-linear dynamical systems. In the next section, we will describe the key concepts of modern dynamical systems theory (complexity, self-organization, soft assembly, interaction dominance, and non-linearity). In the second section, we briefly discuss some dynamical analysis techniques used in the cognitive sciences. In the third, we give some examples of the application of complex dynamical systems theory and analysis in cognitive science. In the last, we sketch some consequences of the widespread applicability of dynamical approaches to understanding neural, cognitive, and social systems.

**Complex dynamical systems**

Complex dynamical systems exhibit three key characteristics (Gallagher and Appenzeller, 1999). First, they consist of a number of interacting components or agents. These components can be homogeneous or heterogeneous. A collection of cortical areas or simple artificial agents can comprise a homogeneous, complex dynamical system; a brain in a body in an environment can comprise a heterogeneous, complex dynamical system. A second property is that these systems exhibit *emergent* behavior in that their collective behavior exhibits a coherent pattern that could not be predicted from the behavior of the components separately. Third, and most importantly, this emergent behavior is *self-organized* in that it does not result from a controlling component agent. These three characteristics can be seen clearly in phenomena such as bird flocking. Starlings, for example, gather in flocks of hundreds to thousands known as “murmurations.” Starling murmurations exhibit striking, globally unified behavior, in which large numbers of starlings move as a single, dark blob that changes shape as it moves across the sky. Murmurations are a
coordination phenomenon in which interactions between individuals produce collective, large-scale patterns. Starling murmurations, and bird flocks more generally, exhibit all the key features of complex dynamical systems and have been modeled as such (Cavagna et al., 2010).

**Self-organization**

The term “self-organization” is used to refer to behavioral patterns that emerge from the interactions that bind the components of a system into a collective system, without a centralized controller. A murmuration’s behavior is emergent and self-organized: murmurations form when sufficient numbers of starlings gather, without a head starling leading the way. In fact, in order to model the velocity of individual birds in a murmuration, Cavagna et al. (2010) start with the velocity of the murmuration as a whole, and work inward from there to model the velocities of individual birds. Starling flocking highlights how coordinated social behavior can result spontaneously from the interactions of agents. Coordinated, collective behavior among herds of mammals and schools of fish is self-organized in the same way, as is the nest-building behavior of ants, bees, and termites. In each of these cases, no individual animal or subset of the whole controls the behavior of the group (Camazine et al., 2001).

**Soft-assembly**

Murmurations are temporary coalitions of starlings that are put together in a fluid and flexible manner. It doesn’t matter which particular bird ends up in which position in the flock, and each bird will take up many different positions as the flock moves and takes shape. The behavior of the birds that are the components in the flock is context dependent. Dynamical systems that exhibit this kind of emergent, context-dependent behavior are often referred to as softly assembled systems, in that the behavioral system reflects a temporary coalition of coordinated entities, components, or factors. The term synergy is sometimes used to refer to softly assembled systems—a functional grouping of structural elements that are temporarily constrained to act as a single coherent unit (Kelso, 2009). In contrast, most non-biological systems or machines are hard-molded systems. A laptop computer, for example, is a hard-molded system, in that it is composed of a series of components, each of which plays a specific, predetermined role in the laptop’s behavior. Coordinated behavior in social animals, including humans, is often softly assembled.

**Interaction-dominant dynamics**

Softly assembled systems exhibit interaction-dominant dynamics, as opposed to component-dominant dynamics. For component-dominant dynamical systems, system behavior is the product of a rigidly delineated architecture of system modules, component elements or agents, each with predetermined functions. For softly assembled, interaction-dominant dynamical systems, system behavior is the result of interactions between system components, agents, and situational factors, with these intercomponent or interagent interactions altering the dynamics of the component elements, situational factors and agents themselves (Anderson, Richardson, and Chemero, 2012; Van Orden, Kloos and Wallot, 2011). As noted above, to model the behavior of individual starlings in a murmuration, Cavagna et al. (2010) began with the behavior of the flock as a whole. Within the murmuration, the behavior of any bird is primarily determined by the behavior of the whole murmuration, even though the murmuration is nothing other than the collection of individual birds. If one were to examine the relationship between any two levels of an interaction-dominant dynamical system, one would observe that elements or agents at the
lower level of the system modulate the macroscopic order of the higher level, and at the same
time are structured by the macroscopic order of the system. For interaction-dominant systems, it
is difficult, and often impossible, to assign precise causal roles to particular components. It is also
difficult, and often impossible, to predict the behavior of components within interaction-dominant
systems from their behavior in isolation.

**Non-linearity**

A non-linear system is one in which the system’s output is not directly proportional to the input,
as opposed to a linear system in which the output can be simply represented as a weighted sum of
input components. Complex dynamical systems are non-linear in this sense, so their behavior is
never merely the sum of the behavior of the components (Van Orden, Holden, and Turvey,
2003). Non-linearity cuts two ways. On one hand, the non-linearity of complex dynamical
systems makes them much more difficult to understand. In fact, non-linear systems are non-
decomposable, in that you cannot isolate components of the system and predict their behavior.
On the other hand, it is only because complex dynamical systems are non-linear that they can
exhibit complex behavior.

**Dynamical analysis**

Central to identifying the causal structures or processes that underlie and shape the physical and
cognitive behavior of complex biological agents is time-series analysis. Substantial advances in
the types of non-linear analysis techniques that have occurred in recent years, combined with the
increasing availability of these techniques (i.e. via open source software packages and code
sharing), has further compounded their importance. In fact, it is becoming increasingly clear that
non-linear time-series analysis is essential for understanding how the ordered regularity of human
behavior and cognition can emerge and be maintained. The advantage of these methods over the
traditional statistical techniques commonly employed in cognitive psychology is that they can
handle the time dependence of behavior and are not restricted to making linear assumptions
about behavioral organization. Indeed, contemporary methods of non-linear dynamics embrace
the complexity of self-organized behavior and, accordingly, can provide deep insights about the
behavior of real-world time-evolving processes. Here we discuss two methods of non-linear
time-series analysis that have had a transformative impact on our ability to classify and understand
a wide range of embodied cognition, namely recurrence analysis and fractal analysis.

**Recurrence analysis**

Recurrence analysis is a phase-space method that allows one to determine the dynamical structure
of a recorded time series, no matter how complex the time series is, nor the number of state
dimensions needed to capture the time series within its corresponding state space. The beauty of
recurrence analysis, in comparison to other time-series methods, is that it does not require one
to make any assumptions about the structure of the time series being investigated: it can be
stationary, non-stationary periodic, stochastic, discrete, or categorical.

Essentially, recurrence analysis identifies the dynamics of a system by discerning (a) whether
the states of system behavior recur over time and, if states do recur, (b) the deterministic reg-
ularity of the patterning of recurrences. Conceptually, performing recurrence analysis on behavioral
data is relatively easy to understand; one simply plots whether recorded points, states, or events
in a time series are revisited or reoccur over time on a two-dimensional plot, called a recurrence
plot. This plot provides a visualization of the patterns of revisitations in a system’s behavioral state space and can be quantified in various ways in order to identify the structure of the dynamics that exist (see Marwan, 2008 for details). The plots in Figure 4.1 are examples of what recurrence plots look like for a categorical (left plot) and continuous (right plot) behavioral time series.

Recurrence analysis can also be extended to uncover the dynamic similarity and coordinated structure that exists between two different behavioral time series. This latter form of recurrence analysis is termed cross-recurrence analysis and is performed in much the same way as standard (auto-)recurrence analysis. The key difference is that recurrent points in cross-recurrence correspond to states or events in two time series that are recurrent with each other. Cross-recurrence analysis can therefore be employed to quantify the co-occurring dynamics of two behavioral time series.

**Fractal analysis**

Researchers in cognitive and behavioral psychology commonly collapse repeated measurements into summary variables, such as the mean and standard deviation, under the assumption that the
measured data contain uncorrelated variance that is normally distributed. Real-time behavior and cognition, however, are rarely static and thus summary statistics often reveal little about how a system evolves over time. Indeed, time-series recordings of human performance and cognition typically contain various levels of correlated variance or non-random fluctuations that are not normally distributed (Stephen and Mirman, 2010) and, moreover, are structured in a \textit{fractal or self-similar} manner (Gilden, 2001, 2009; Van Orden \textit{et al.}, 2003; Van Orden \textit{et al.}, 2011). Indexing the correlated and self-similar variance within a behavioral time series requires the use of fractal methods of analysis, sometimes called \textit{fractal statistics}.

A fractal or self-similar pattern is simply a pattern that is composed of nested copies of itself and looks similar at different scales of observation. A fractal time series is therefore a time series that contains nested patterns of variability (Figure 4.2). That is, the patterns of fluctuation over time look similar at different scales of magnification. The time series displayed in Figure 4.2 is a good example, with the self-similarity of its temporal fluctuations revealed by zooming in on smaller and smaller sections. At each level of magnification the temporal pattern looks similar (Holden, 2005).

A fractal time series is characterized by an inverse proportional relationship between the power ($P$) and frequency ($f$) of observed variation. That is, for a fractal time series there exists a proportional relationship between the size of a change and how frequently changes of that size occur, with this relationship remaining stable across changes in scale. It is in this sense that the pattern of variability in a repeatedly measured behavior is self-similar; large-scale changes occur

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fractal_patterns.png}
\caption{Example geometric and temporal fractal patterns (i.e. contain self-similar structure at different magnitudes of observation). (Left) Koch Snowflake at three levels of magnification. (Right) Fractal time series at three levels of magnification. (Adapted from Holden, 2005.)}
\end{figure}
with the same relative frequency as small-scale changes. The degree to which a data set approximates this ideal relationship between power and frequency, \( P = 1/f^a \), is summarized in the scaling exponent, \( a \). If one plots the power of the different spectral frequencies that make up a time series on double-logarithmic axes, \( a \) is equivalent to the slope of the line that best fits the data (Figure 4.3). That is, \( a \) captures the relationship between size and frequency of fluctuations in the time series of behavior. Random fluctuations (i.e. white noise) produce a flat line in a log-log spectral plot with a slope close to 0, which indicates that changes of all different sizes occur with approximately the same frequency. Alternatively, fractal fluctuations, often referred to as pink or \( 1/f \) noise, produce a line in a log-log spectral plot that has a slope closer to \(-1\), which indicates the scale-invariant scaling relationship characteristic of fractal patterns.

The import of determining whether a behavioral time series contains fractal or \( 1/f \) variability is highlighted by a growing body of research demonstrating that the most human behaviors exhibit fractal structure. For example, numerous studies have demonstrated how the fluctuations in time series of ongoing stimulus-response activity, time estimation, cognitive performance, postural control, and eye movements exhibited fractal structure (see Delignières et al., 2006; Gilden, 2009; Holden, 2005). Even the flow of social interaction and behavior has a fractal structure (e.g. Delignières, Fortes, and Ninot, 2004; Newton, 1994). Of particular relevance for the current discussion, however, is that this research has also demonstrated that the degree to which fluctuations within a behavioral time series are fractal (i.e. pink) or not (i.e. white), can provide evidence about whether a behavior is non-linear and the result of interaction-dominant dynamics (Van Orden et al., 2003).

**Complex, dynamical cognitive systems**

The above analysis techniques have been applied widely at all spatial scales relevant to cognitive science, from brain areas, to embodied behavior, to agent-environment systems, and to social interaction. Although recurrence analysis is still relatively new, there is now substantial evidence
to suggest that it is potentially one of the most generally applicable methods for assessing the
dynamics of biological and human behavior (e.g. Marwan and Meinke, 2002; Zbilut, Thomasson,
and Webber, 2002). This is due to the fact that recurrence analysis provides researchers with a
way of determining whether the nested fluctuations and complex time-evolving patterns within
almost any type of behavioral time series are deterministic and interrelated or stochastic and
disconnected (i.e. the degree a behavioral structure is the result of interaction-dominant
dynamics). For instance, auto- and cross-recurrence analysis has already been employed to
uncover the non-obvious changes that goal constraints produce on the synergistic dynamics of
postural movements (Riley, Balasubramaniam, and Turvey, 1999), the noise structure of limb
movements (e.g. Pellecchia, Shockley, and Turvey, 2005; Richardson, Schmidt, and Kay, 2007)
the intermitted perceptual-motor synchrony that occurs between people interacting (Richardson
and Dale, 2005; Richardson, Marsh, and Schmidt, 2005; Richardson, Marsh, Isenhower,
Goodman, and Schmidt, 2007; Shockley, Santana, and Fowler, 2003; Shockley, Baker,
Richardson, and Fowler, 2007), the deterministic structure inherent in eye movements and
stimulus-response reaction-time data (e.g. Cherubini, Nüssli, and Dillenbourg, 2010; Pannasch,
Helmert, Müller, and Velichkovsky, 2012), even semantic similarity during conversation (Angus,
Smith, and Wiles, 2011) and the vocal dynamics of children during development (Warlaumont
et al., 2010). In each case, recurrence analysis was able to reveal whether the observed dynamics
were the result of nested physical, neural, and informational couplings that bound cognition
and action to each other and to the relevant objects (individuals) and events within the task
environment.

As noted above, the presence of 1/f scaling and complex patterns of recurrent structure in a
cognitive and behavioral phenomenon is evidence that the softly assembled system is interaction
dominant. Complex patterns of recurrent behavior and 1/f scaling has been observed in the
brain, and in a wide variety of cognitive and behavioral tasks, from tapping, to key pressing, to
word naming, and many others (Van Orden et al., 2011). This indicates that softly assembled
coalitions of components encompassing portions of the participants’ brain and body were
responsible for the performance of the experimental task. That the portions of the cognitive sys-
tem that engage in tasks such as these are not fully encapsulated in the brain is perhaps not
surprising, since each has a strong motor component. But we also see time-evolving recurrent
structures and 1/f scaling in “purely cognitive” phenomena. In one example, Stephen, Dixon,
and Isenhower (2009) have shown that problem-solving inference is accomplished by an
interaction-dominant system. Using fractal statistics and recurrence analysis, they found that
learning a new strategy for solving a problem coincides with changes in the complexity and
amount of recurrent activity in an individual’s eye movements. This indicates that even leaps of
insight do not occur in the brain alone—the eye movements are part of the interaction-domi-
nant system that realizes the cognitive act. Findings such as this impact not only the extent of
the biological resources required for cognitive faculties, but also the separation of cognitive
faculties from one another. Finding that moving eyes are components of the interaction-dominant
system that has the problem-solving insight makes it more difficult to separate cognition from
motor control.

There is reason to think that this expansion of the cognitive system does not stop at the
boundaries of the biological body. For example, Dotov, Nie, and Chemero (2010) describe
experiments designed to induce and then temporarily disrupt an extended cognitive system.
Participants in these experiments play a simple video game, controlling an object on a monitor
using a mouse. At some point during the 1-minute trial, the connection between the mouse
and the object it controls is disrupted temporarily before returning to normal. Dotov et al.
found 1/f scaling at the hand-mouse interface while the mouse was operating normally, but not
during the disruption. As discussed above, this indicates that, during normal operation, the computer mouse is part of the smoothly functioning interaction-dominant system engaged in the task; during the mouse perturbation, however, the $1/f$ scaling at the hand-mouse interface disappears temporarily, indicating that the mouse is no longer part of the extended interaction-dominant system. These experiments were designed to detect, and did in fact detect, the presence of an extended cognitive system, an interaction-dominant system that included both biological and non-biological parts. The fact that such a mundane experimental set-up (using a computer mouse to control an object on a monitor) generated an extended cognitive system suggests that extended cognitive systems are quite common. These, of course, are not the only examples of interaction dominance in cognition (for a review, see Van Orden et al., 2011).

The phenomena of $1/f$ scaling and recurrent dynamics are ubiquitous in the brain as well. Heterogeneous coupling and multiscale recurrent dynamics are widespread features of the brain. Brain connectivity is organized on a hierarchy of scales ranging from local circuits of neurons to functional topological networks. At each scale the relevant neural dynamics are determined not just by processes at that scale, but by processes at other smaller and larger scales as well. Such multilevel clustered architectures promote varied and stable dynamic patterns via criticality and other dynamical and topological features. There is therefore also growing evidence that neural circuits are interaction dominant. Several recent studies have found evidence of $1/f$ scaling in human neural activity (e.g. Freeman, Rogers, Holmes, and Silbergeld, 2000; Bullmore et al., 2001; Freeman, 2009). Research on the dynamics of brain activity using recurrence analysis has also produced evidence that the dynamic behavior of the brain is characteristic of an interaction-dominant system. For example, Acharya and colleagues have employed recurrence analysis to uncover the non-linear and interaction-dominant dynamics of EEG singles during various sleep cycles and for individuals with epilepsy (e.g. Acharya, Faust, Kannathal, Chua, and Laxminarayan, 2005).

Finally, the dynamics of many social behaviors are interaction dominant and characterized by complex recurrent patterns and $1/f$ scaling. For instance, Shockley et al. (2003) employed cross-recurrence analysis to examine the postural dynamics of two co-present participants completing a conversational task together. The experiment included two key manipulations. The first manipulation was whether the two participants were performing the task together, or whether the participants were co-present but performed the task with a confederate. The second manipulation was whether the participants were positioned facing each other or back to back. The analysis revealed that the postural activity of the two participants was more similar when performing the puzzle task together (i.e. conversing with each other) compared to when performing the task with the confederate. Surprisingly, the interpersonal postural dynamics was not influenced by vision, in that the same magnitude of recurrent activity was observed irrespective of whether the participants could see each other or not. Thus, the findings not only demonstrated how an individual’s postural dynamics are spontaneously influenced by interactions with other conspecifics, but also how conversation alone can couple the behavioral dynamics of interacting individuals.

The fact that the physical and informational interactions that characterize social interaction operate to shape behavior (often spontaneously and without awareness) means that the behavioral dynamics of social activity is inherently interaction dominant. In addition to the postural work of Shockley et al. (2003), other studies investigating various forms of social movement coordination have produced findings that demonstrated that the dynamics of social behavior is interaction dominant (see Riley, Richardson, Shockley, and Ramenzoni, 2011; Richardson, Marsh, and Schmidt, 2010; Schmidt and Richardson, 2008). The implication is that co-acting individuals form a synergy, whereby the behavioral order of the individuals involved is enslaved by the functional order of the group or team as a whole. Accordingly, the behavioral
performance of interacting individuals is not simply an additive function of each individual’s cognitive or behavioral capabilities and, moreover, cannot be understood by studying the individuals in isolation from each other or the social setting. Ramenzoni (2008) highlighted this point, using cross-recurrence analysis to demonstrate how the informational interaction that occurs during joint action results in the dimensional compression of each individual’s behavioral degrees of freedom and the formation of a low-dimensional reciprocally compensating synergy. Similar findings have been made by Richardson, Dale, and colleagues in studies investigating social eye coordination and language comprehension (Richardson, Dale, and Tomlinson, 2009). Using categorical cross-recurrence analysis, they have demonstrated across several studies that a shared task context results in synergistic eye movements and that the coordinated stability of such eye movements reflects how well two people comprehend each other (Richardson and Dale, 2005), the strength of their shared knowledge and how much two people converge in language use (Richardson, Dale, and Spivey, 2007).

With respect to 1/f scaling and the fractal nature of social behavior, Delignières et al. (2004) have demonstrated that fractal processes underlie the dynamics of self-esteem and physical self. Twice a day, for 512 consecutive days, they collected data about the global self-esteem of four individuals. Consistent with a conception of self-perception as an emergent product of an interaction-dominant dynamical system, an analysis of the resulting time series found converging evidence of 1/f scaling in the behavioral series. At a more local level, Malone and colleagues (Malone, Castillo, Holden, Kloos, and Richardson, 2013) recently employed a social Simon stimulus-response compatibility task to demonstrate how the mere presence of another actor constrains the fractal variability of an individual’s response behavior. The results revealed how the presence of another actor alters a task setting and, as such, the ongoing dynamics of individual behavior (even if the co-present individual is engaged in an independent task). Eiler, Kallen, Harrison, and Richardson (2013) have uncovered preliminary evidence that social stereotypes and gender salience can influence the fractal structure of an individual’s cognitive and behavioral performance. Perhaps most compelling is the work by Correll (2008), which has shown that participants who are trying to avoid racial bias show decreased fractal signature in their response latencies in a video game. In light of characterizing social perception and other processes as a system of many intertwined dependencies—as processes of an interaction-dominant dynamical system—these findings suggest that the behavioral fluctuations of socially situated performance reflects the distributed influence of positive and negative perceptions and judgments, and the cultural regulations that define them.

**Consequences**

The complexity and unpredictability of human behavior has led many cognitive scientists to attempt to understand cognitive systems as complex dynamical systems, and to approach them using complex dynamical analysis. The result of this has been the widespread recognition of interaction-dominant dynamics in the brain and in individual and social cognition. This recognition has consequences both for the nature of cognition and for the practice of cognitive science. Here we focus on consequences concerning modularity and extended cognition. (See Chemero, in press.)

**Modularity**

An interaction-dominant system is a highly interconnected system, each of whose components alters the dynamics of many of the others to such an extent that the effects of the interactions are more
powerful than the intrinsic dynamics of the components. In an interaction-dominant system, inherent variability (i.e. fluctuations or noise) of any individual component propagates through the system as a whole, altering the dynamics of the other components. In interaction-dominant systems one cannot treat the components of the system in isolation: because of the widespread feedback in interaction-dominant systems, one cannot isolate components to determine exactly what their contribution is to particular behavior. And because the effects of interactions are more powerful than the intrinsic dynamics of the components, the behavior of the components in any particular interaction-dominant system is not predictable from their behavior in isolation or from their behavior in some other interaction-dominant system. Interaction-dominant systems, in other words, are not modular. They are in a deep way unified in that the responsibility for system behavior is distributed across all of the components. Given the rapid pace at which cognitive systems have been shown to be interaction dominant in the twenty-first century, there is good reason to think that cognitive systems are not, in general, modular (Anderson et al., 2012; Chemero, in press).

**Extended cognition**

We have seen that not just neural and brain-body systems are interaction dominant; so too are human-tool cognitive systems and social cognitive systems. Because interaction-dominant systems are unified, we should identify the cognitive systems in these cases with human-tool and social systems as a whole. That is, the cognitive system in question is not encapsulated within an individual brain or even an individual body. This supports the hypothesis of extended cognition. According to the hypothesis of extended cognition, cognitive systems sometimes include portions of the non-bodily environment (Clark and Chalmers, 1998; Chemero 2009). When human-tool or social cognitive systems are complex dynamical systems with interaction-dominant dynamics, they are extended cognitive systems. Moreover, these studies support extended cognition empirically, and not with a priori philosophical argumentation.

**References**


