Part 4

Modeling Teacher Learning Using Design Research
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11 Developing Design Studies in Mathematics Education
Professional Development
Studying Teachers’ Interpretive Systems

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Introduction

Design research has been used to investigate students’ mathematical development and to design more effective learning environments, with the twin purposes of designing and studying the impact of educational innovations (e.g., Brown, 1992; Design-Based Research Collective, 2003; Verschaffel et al., 1999). For example, Brown (1992) proposed using design experiments in classes in order to investigate the design of learning communities and to advance methods for carrying out research about learning within the complexity of the actual classroom situation. This research resulted both in the development of theory about learning and in improved innovations for helping students learn. Recent calls for more rigorous scientific research in education and more connections between research and classroom practice (e.g., National Research Council, 2002) extend to research on mathematics teaching practice. We propose the extension of design research to teachers’ learning and development in order to understand both how teachers develop in their practice and how to design environments and situations to encourage the development of that practice.

The purpose of this chapter is to propose how design study may be used for conducting research on teachers’ growth in the context of long-term, professional development, with two assumptions being made: that teachers will grow in ways that cannot be predicted always and that teachers’ growth will take place in complex situations. The first assumption is in contrast to professional development organized around a particular model of teaching to which teachers aspire, and in which predetermined characteristics and behaviors can be identified and measured. Research methodologies are needed for professional development experiences where participants are not expected to converge toward a particular standard, yet teachers grow and improve as a result of participating in the professional development experience. Teachers’ growth is often
unpredictable because they use their own interpretive systems to function in daily practice. In doing so, they draw on multiple theories and models: cognitive learning theory, economic models, political models, etc. Even so, Doerr and Lesh (2003) suggested that the knowledge base and expertise of teachers can develop in ways that can be seen as continually better, even without a particular end point in mind. The challenge is to design studies of professional development that characterize the ways in which teachers develop and how their growth can be documented.

The second assumption—that teachers’ growth takes place in complex situations—is based on the notion that professional development experiences should address the problems that teachers want to solve. The situations that teachers face are complex and involve trade-offs among many factors, such as the learning goals for the students, the costs of implementing desired programs and methods of instruction, and negotiations between the teachers’ goals and the goals of the various interested parties (e.g., parents, the school district’s statement of standards). Experts in the field, such as Lieberman (1996) and Putnam and Borko (1997), support the notion that professional development should be relevant to local needs. Ball and Cohen (1999), Darling-Hammond and Ball (1998), the National Council of Teachers of Mathematics (2001), and Putnam and Borko (2000) recommend that the planned experiences should engage teachers in work that is related directly to their classroom practice; others (Darling-Hammond & McLaughlin, 1995; Guskey, 1995; Hawley & Valli, 1999; Loucks-Horsley et al., 1987) suggest that the professional development experience needs to provide reasons for teachers to share their ways of thinking about their own practice with peers and to participate in the development of a community of practice. This collection of recommendations paints a picture of professional development as existing in complex dynamic systems where the system influences the nature of the professional development and the professional development influences the nature of the system. Consider, at the classroom level, that as a teacher interprets students’ thinking, the teacher’s actions toward the students change; the students change in response to the teacher’s altered expressions and expectations; and the cycle repeats as the teacher again interprets students’ thinking and modifies his or her thinking about the students. Consider also, at the school level, that when a teacher explains new insights about students to the principal, the principal changes his or her ways of conversing with parents; the principal’s expressed perceptions prompt changes in how parents interact with their children; students’ classroom behavior and performance change as a result of parental influence; and the cycle repeats as the teacher again modifies his or her thinking about the students. Speck and Knipe (2001: 217) say that professional development facilitators must “view professional development practices in the context of the larger system with the interconnectedness of all the parts and the continuous flow of change.” The notion of design study provides a means to think about research methods that can embrace and capitalize on dynamic and complex school situations, rather than thinking about how to control those same factors.

In 1992, Brown described the need to develop research methodologies that acknowledge that change in one part of the system may perturb the whole system. In addition to Brown, Collins (1992) introduced the concept of design experiment to the field of education, and other researchers, such as Cobb et al. (2003), the Design-Based Research Collective (2003), and Hawkins and Collins (1992), have carried on Brown’s recommendations, explaining that design experiments provide an avenue for studying learning within the complexity of interacting educational systems. In these lines of research, the term design experiment is used to indicate that a tangible product (e.g., for Cobb et al. [2003], a trajectory of student learning) is being designed by the research
team and developed over a series of trials, through systematic data collection and analysis.

The notion of design experiment draws on the scholarly work of engineers in which their design process involves expressing, testing, and revising functional objects, procedures, and theory. Unlike engineering research, educational research in professional development must deal with multiple interacting layers of designers: teachers design educational tools (e.g., lesson plans, questioning techniques, rubrics for interpreting students’ work) for their own practice; facilitators design professional development sessions based on what they have learned from previous sessions; researchers design theories for professional development that can guide similar experiences in different contexts with different teachers, which in turn impact the facilitator’s immediate professional development plans. Thus, aspects of the multitiered teaching experiments described by Lesh and Kelly (1997) are important for capturing the realities of the multiple constituencies involved in professional development in educational settings, the problems identified by teachers, and the inability to predict always the ways in which teachers grow.

This chapter focuses on and fleshes out the teacher development tier of the Lesh and Kelly (1997, 2000) multitiered teaching experiment model. The first section describes the characteristics of multitiered professional development experiences that provide opportunities to conduct design study research for investigating changes in teachers’ interpretive systems. Then, two illustrative cases of professional development (lesson study and students’ thinking sheets) are described and serve as examples for points we make in subsequent sections. The section after them characterizes teachers’ interpretive systems for teaching and learning and how professional development is designed to prompt changes in their interpretive systems. The final section addresses practical aspects of constructing a multitiered professional development design study.

The Nature of Multitiered Professional Development Design Studies

Professional development experiences that are planned explicitly as design studies have distinctive characteristics: teachers are engaged in the development of artifacts that reveal aspects of their own thinking; teachers are engaged in testing and revising the artifact; and teachers are asked to describe and document the guiding principles they have used while revising the artifact. When teachers produce educational objects for use in their own practice, they have the opportunity to examine their own thinking because their artifacts are an external representation of their own interpretive systems. Testing and revision take place when small groups of teachers negotiate the development of the artifact, when the educational object is field-tested in the classroom, and when classroom data are reported to and discussed among peers. Peer interaction provides important opportunities for teachers to express, test, and revise their artifact and their ways of thinking about teaching and learning as related to the artifact. When teachers are asked to be explicit about the principles that guide their decisions about artifact revisions, they externalize general aspects of their interpretive systems. The principles and guidelines that teachers express are revealed to themselves, their peers, the facilitator, and the researcher of the professional development experience. The facilitator and the researcher proceed through a similar process of testing and revising their plans for, and theories about, professional development. Hence, the design study is multitiered as teachers examine classroom situations and their own interpretive systems, and researchers and facilitators examine teachers’ interpretive systems as well as their own
and derive principles for professional development that may be transportable to other
contexts. In order to ensure transportability to other situations, Middleton, Gorard,
Taylor, and Bannan-Ritland and Hjalmarson and Lesh in their chapters in this volume
describe a model for design research that moves between the proposed or intended
function of the design and its actual implementation in the classroom. The two profes-
sional development cases described in this section—lesson study and students’ thinking
sheets—illustrate the assumptions underlying design study, the methodological con-
siderations for this type of research, and the characteristics of professional development
situations planned as design studies.

Lesh and Kelly (2000) describe multitiered teaching experiments that involve three
interacting tiers (researchers, teachers, and students) represented hierarchically in a dia-
gram—Figure 11.1. The design study perspective for teachers’ professional develop-
ment described in this chapter draws on their multitiered model but adds a facilitator
tier that is responsible for designing the professional development experience for
teachers and that focuses more on the collaboration between the tiers. In particular, the
diagram in Figure 11.1(a) shows the iterative design cycles involved in the design pro-
cess and the diagram in Figure 11.1 shows how the teachers, researchers, and facilita-
tors are collaborators in the design of educational objects. Figure 11.1(b) suggests a
more interactive and collaborative process than the hierarchical process (hence, their
use of the term tier) described by Lesh and Kelly. Students are put in the center because
all of the constituents are concerned with students’ thinking in some way (even if it is
through a different constituent—e.g., a researcher through teachers’ interpretations)
and all of the tiers are trying to cause change and improvement to students’ thinking
(even if it is through a different tier—e.g., improving students’ learning by improving
teaching). Each tier receives feedback from and influences the development of the other
two constituents.

Aspects of design study, as defined above, happen naturally in the classroom on a
daily basis. Teachers are designing constantly products for classroom instruction, such
as a curricular unit, a plan for implementing small group work, a daily lesson plan, or a
rubric to communicate to students what is valued in their work. Teachers engage in the
experimentation process as they implement the product they have designed, gather
information about its effectiveness, and revise the product on the spot or revise it for
future use. The difference in professional development design studies is that teachers

![Design Study Situated in Context](image-url)
externalize their thinking explicitly through the educational objects they design, then
they make their theories explicit by describing and explaining their principles (e.g., how
the educational object worked, under what conditions, and why). The challenge to
facilitators of professional development is to design experiences for participants that
address the problems important to teachers and also prompt cycles of teachers express-
ing, testing, and revising their educational objects. The researcher works hand in hand
with the facilitator in the design of the professional development experience while
expressing, testing, and revising his or her theory of professional development. In this
chapter, this type of professional development research is termed design study, which
conveys a sense of designing and studying at the same time and in multiple dimensions.

Examples of Multitiered Design Studies for
Professional Development

Next, we describe two examples of studies that can be characterized as multitiered
design studies. Both the lesson study and the students’ thinking sheet examples were
carried out in the complex setting of classroom practice and set about investigating
teachers’ thinking through the designing of an educational object. In the case of the
lesson study, the object under design was an algebra lesson; in the students’ thinking
sheet example, the object was a tool for representing students’ mathematical thinking
on model-eliciting activities. The two studies are used in subsequent sections to illus-
trate some of the considerations and decisions that need to be made when conducting a
multitiered design study for teachers’ professional development.

Lesson Study

Lesson study has long been the major form of professional development chosen by
Japanese teachers and is credited with the transformation of Japanese elementary math-
ematics and science instruction over the past several decades (Lewis, 2002a, 2002b;
has been initiated by educators at a number of sites in the United States (Lewis, 2002a,
2002b). The brief lesson study case described here is drawn from a school district in the
western United States where teachers founded and led a lesson study effort based on
English-language descriptions of Japanese models (Lewis, 2002b; Stigler & Hiebert,
1999; Yoshida, 1999), as well as collaboration with Japanese practitioners. The case
focuses on the six teacher-members of a lesson study group who worked together
during a summer workshop. The workshop was designed to immerse interested teachers
in the study of both algebra and lesson study and was part of a larger, teacher-led effort
in the school district to improve mathematics instruction and to build lesson study as an
effective means of professional development. In the context of a recent state mandate
for all grade-eight students to take algebra, the workshop engaged teachers in identify-
ing and building the elementary mathematics experiences that would lay a foundation
for success in algebra. The summer experience generally followed Japanese protocols
for lesson study that were introduced by local teacher-leaders and by Japanese educators
who had participated in local research lessons during several, prior, multiday workshops.

Lesson study begins by having teachers consider their long-term goals for students
and then design collaboratively one or more “research lessons” to bring these goals to
life (Lewis, 2002b). The goals serve as an “end in view” to keep in mind as the teachers
engage in design cycles of teaching and revising the lesson. They observe and collect
carefully data on student learning and development during implementation of the lessons and use these data to refine their instructional approach. In some cases, participants reteach a revised version of the research lesson (Lewis, 2002a, 2002b; Yoshida, 1999). In this lesson study case, after studying the state mathematics standards, the adopted textbook, other mathematics curriculum resources, and solving and discussing algebra problems, the small group of teachers decided to focus the lesson study on students’ capacity to recognize and extend patterns—a key, algebra-related theme in the state mathematics standards. The teachers planned a research lesson designed to promote students’ capacity to recognize, extend, and express patterns mathematically, using the pattern-extension activity in Appendix A. This began an iterative design cycle in which members of the lesson study group taught the lesson a total of three times (twice during the summer workshop and once during the following fall). On each occasion one member taught the lesson and the other group members observed and collected the data that would be used to reflect on and revise the lesson.

The iterative process of expressing, testing, and revising an educational object (the research lesson) was the focus of the lesson study sessions. For example, after the first teaching (and observing) of the lesson, the teachers revised the lesson by (a) eliminating a worksheet that (teachers hypothesized) had spoon-fed the pattern to students, (b) asking students to organize the data from the problem and write about it, and (c) asking students to share their counting methods with the class (based on evidence from the first lesson that these counting methods revealed ways of thinking about the problem). The teachers teaching the research lesson a third time made additional modifications (e.g., adding a requirement that the groups come to consensus about the pattern).

**Students’ Thinking Sheets**

The students’ thinking sheets workshop series was a professional development experience initiated in a midsized midwestern town in the United States in response to a strategic plan developed by the school community. The school’s long-term plan included a commitment to increase emphasis on problem-solving in both the teachers’ professional development and in the school curriculum. Teachers working collaboratively with a professional development facilitator and researcher developed a program for an initial group of volunteer teachers. The plan was for the teachers to implement a series of five problems with students and to design for each problem a students’ thinking sheet that would illustrate different mathematical approaches taken typically by students. The goal was to use the teacher-developed students’ thinking sheets in the development of a teacher’s handbook for their colleagues to use the following year as they began to incorporate problem solving into their classes. The particular problems used are called *model-eliciting activities* (Lesh et al., 2000) because the problems were designed to have students create and articulate a mathematical procedure, explanation, or description (i.e., a model) as a solution to the problem. (See Appendix B for a sample problem.) Students prepared their final product so that it could be presented to the client described in the problem statement and to meet the client’s well-specified need. Therefore, the students’ solutions revealed their thinking to the teachers, providing opportunities for the teachers to analyze their students’ mathematical approaches. The book edited by Lesh and Doerr (2003) provided the theoretical foundation for the design of the professional development study.

Seven teachers volunteered to attend the workshops, positioning them to be facilitators for their peers in the subsequent academic year. There were five cycles of professional development, each of which involved the teachers in: (a) completing the problem,
(b) implementing the problem with students, (c) examining the students’ work to identify mathematical approaches, (d) presenting an individually produced draft of a students’ thinking sheet to colleagues, and (e) developing a group consensus, students’ thinking sheet for the problem. This group of teachers decided to include in each student thinking sheet excerpts from students’ actual work, details about mathematical understandings and abilities associated with each way of thinking, and an evaluation of how effective each approach could be in meeting the needs of the client. Appendix C includes a sample students’ thinking sheet for the Departing-On-Time task illustrated in Appendix B.

**Teachers’ Interpretive Systems**

The lesson study and the students’ thinking sheet professional development experiences each have an implicit assumption that engaging teachers in the collaborative development of educational artifacts and in cycles of testing and revising the artifact will lead them to grow professionally. What does it mean to say that teachers grow professionally? In both illustrations, teachers used their own conceptual systems to make instructional decisions, interpret students’ thinking, design curriculum for students (and peers), analyze their own experiences and data, etc. The conceptual systems that teachers use for their educational practice can be called *interpretive systems* for teaching and learning mathematics. These systems are complex and cannot be defined precisely, but they are to a large extent mathematical (Koellner-Clark & Lesh, 2003). Teachers need to interpret students’ mathematical thinking even when students do not use conventional terminology or symbolism, which requires that the teachers’ understanding of the mathematics be profound (Ma, 1999); that is, teachers have to understand mathematics not only as a discipline but also through students’ perspectives and through the perspective of systems of teaching and learning. Teachers’ interpretive systems cannot be “seen,” are difficult to describe, are multidimensional and not labeled readily, yet they can be considered the most important target for professional growth. An important goal for the researcher in professional development is to find ways to reveal aspects of teachers’ interpretive systems and how those systems are changing. Lesh and Clarke (2000) have described educational researchers’ search for evidence of teachers’ interpretive systems as similar to physicists searching for evidence of neutrinos, which are particles of matter that cannot be seen. Although, with current technology, the particles themselves cannot be found, physicists are able to provide evidence of their existence by using a theoretical framework and setting up experiments that leave a trail of documentation of these particles. Both the lesson study and the students’ thinking sheet cases produced trails of documentation of changes in teachers’ interpretive systems. Then, the documentation could be analyzed by the researcher and the facilitator for the generation of theory related to teachers’ development, as well as for planning future sessions.

The lesson study case illustrates how change in teachers’ interpretive systems can be detected, in particular when examining the iterations of the written lesson plans created by the group of teachers and the conversations captured on video-tape that the teachers had concerning revisions to the lesson. In this case, at the first teaching of the lesson, students were given a worksheet that used the pattern problem in Appendix A and included a two-column table, with one column for the number of tables and one column for the number of seats, to be filled in by the students. All 22 students completed the sheet correctly, showing that the number of seats was always two more than the number of tables, but when asked to write about patterns in the problem, only five students mentioned the plus-two pattern (i.e., that there were always two more seats than the
number of tables). The plus-two rule was verbalized several times in the classwide discussion. However, when asked at the end of the lesson to represent this rule as an equation, few students could explain the connection between the plus-two pattern and the problem. In the postlesson symposium, the teacher recalled:

At the very end, when I was trying to get them to say the number of tables plus two equals the number of seats, there was a lot of confusion. It’s easy for them to just go plus two, plus two, and they sort of lose the whole picture of what the plus two is representing.

Teacher 5 noticed the same problem:

I could see that students were able to fill out the worksheets quickly but never really saw any indication of what does that mean they know ... they could add plus two to the numbers, but that work didn’t necessarily show the kids understand the pattern.

As they replanned the lesson later that day, the teachers revisited what was revealed in their lesson about students’ understanding:

Teacher 3: You tried the equation thing and ...
Teacher 1: It flopped.
Teacher 3: It was too much for them.
Teacher 1: But actually I’m really glad I tried it because I think it’s really clear to all of us that we were not where we thought we were. If I hadn’t done that, I might have thought they’d gotten it.

Building on Teacher 1’s analysis that “Our worksheet set it up for them, spoon-fed them,” the group decided to eliminate the worksheet, give each student a particular number of tables to calculate, and have him or her organize the data and find a pattern that would help to solve the problem. Although some members felt discomfort initially about eliminating the worksheet (because the lesson would be taught again just two days later), they agreed eventually that, in the messy work of organizing the data, students might see better the connection between the problem and the plus-two pattern. This lesson redesign suggests that the teachers were thinking in new ways about what it means for students to “understand” a pattern (i.e., more than merely filling out a worksheet correctly) and about how the challenge of organizing data, rather than merely filling in a pre-organized worksheet, might build such understanding.

The lesson study case also provides evidence that interactions with fellow teachers, as well as data from students, led to changes in thinking, which also influenced how the lesson was revised. Teacher Five reported during the postlesson colloquium that by watching how the students counted the seats, she had learned something about how they thought about the problem:

I noticed kids counting the seats different ways, and this was a kind of a big aha for me ... When I’ve done the problem myself, I’ve always counted [shows counting around the edge], and it didn’t occur to me that there was another way of counting it ... But [student name] had laid out 20 triangles ... and she was counting [demonstrates counting top and bottom alternately, followed by the ends], and then it looked totally different to me; I could see there are 10 triangles on top, 10 on
bottom, and a seat on either end. Now, I was seeing the pattern a different way. Up until then, I had always seen it as you’re taking away a seat and adding these two, taking away a seat and adding these two (shows adding a triangle and subtracting the side that is joined). I was seeing a pattern from somebody else’s perspective. That’s why I thought it might be helpful to have kids talking about how they’re counting it. How are you seeing the seats, and the numbers, and the increases, and where does that come from? So, I think definitely having the kids use the manipulatives is important, and watching how they use them is going to tell us a lot about how did they see the pattern.

(Teacher 5)

As a result of this observation, the group decided to redesign the lesson in order to have students share their counting methods. When reflecting at the end of the workshop on what they had learned from revising and reteaching the lesson, Teacher 1 recalled that when the suggestion was made to have students share their counting methods, she could not understand why this would be helpful but decided to go along with it. However, when students shared their counting methods during the second teaching of the lesson, she could see the geometric reason for the plus-two pattern (that each non-end triangular table contributes one seat and the two end triangular tables each contribute two seats). In this situation, not only were the teachers producing a trail of documentation in their research lessons, but also they were creating opportunities for their students to produce a trail of documentation about how they were counting. Incorporating opportunities for teachers to reflect on the changes they have made helps them to identify and document their own growth. The resulting documentation can be used to form a compelling case that can be reported to external audiences describing changes in teachers’ own interpretive systems.

Aspects of teachers’ interpretive systems for teaching and learning can be documented by planning professional development experiences that require the externalization of teachers’ thinking through the design and creation of educational objects that are subjected to testing and revision. When such documentation is available, the professional development facilitator and the researcher can use the information gleaned from the artifacts and observations of teachers’ interactions to gain some understanding of teachers’ evolving interpretive systems and to plan subsequent experiences based on what has been learned about the teachers. In addition, at the end of the professional development experience, the teachers have high-quality educational resources for their classes. In the case of the lesson study, the teachers had well-constructed lessons. In the students’ thinking sheets example, the teachers had a handbook that could be used the next year and shared with other teachers. Most important is the teachers’ opportunity to examine and reflect on the evidence of their own interpretive systems, which pushes the professional development system further. Professional development experiences that are designed to include teachers’ documentation of the principles underlying their decisions for revisions can provide an even more complete picture of teachers’ interpretive systems and the changes in those systems. If the goal is to study teachers’ interpretive systems as teachers develop, then professional development experiences need to be designed to make teachers’ interpretive systems grow and to trace those changes. Developing a program of professional development goes hand in hand with creating a design study of that professional development experience.
Creating a Multitiered Design Study

Creating a multitiered design study requires that the researcher and the facilitator work together to plan activities that will provide simultaneously opportunities for teachers to grow and for them to document their growth. The research questions need to be complementary to a clearly articulated end in view, which keeps the orientation of the professional development on track, addressing teachers’ problems while providing flexibility in the activities planned. Data, or documentation, may be obtained from multiple sources and is elicited as part of the professional development process. Each participant in the professional development experience analyzes data from his or her perspective, leading to multiple conclusions and implications of the research. Different aspects of formulating a design study are addressed in the following four sections:

• What are the research questions?
• What is the nature of the data?
• How are the data collected?
• How are the data analyzed?

What Are the Research Questions?

The researcher is interested in studying the teachers’ data for patterns to inform theory, principles, or frameworks that can be shared with mathematics educators in the field. By focusing on the mechanisms that prompt the development of teachers’ interpretive systems, the information gathered for the study has the potential to inform others in professional development about ways to devise cycles of teachers expressing, testing, reflecting, and revising. Cobb et al. (2003) emphasize the importance of clarifying the theoretical intent before each cycle of experimentation. In the lesson study and the students’ thinking sheet examples the design principle they held in common was: mechanisms that prompt teachers to express, test, and revise their interpretive systems should be incorporated into the teachers’ experience. The assumption underlying this principle is that as teachers express, test, and revise their educational object, their interpretive systems for learning and teaching will grow and change. Further, aspects of the changes are often evident in the iterative designs of the objects and available for the teachers’ self-assessment and for the researcher to trace aspects of the teachers’ development. The mechanism used to enact this principle in the lesson study and the students’ thinking sheet examples was to have teachers design an educational object relevant to their practice (i.e., the algebra lesson, the students’ thinking sheet). The research questions were: How do these mechanisms work? Why? Understanding how the mechanisms work helps to reduce the amount of design necessary for transporting the theory to new contexts.

What Is the Nature of the Data?

Because one cannot presume to know what is in another’s mind, investigating the interpretive systems of others is based necessarily on external representations: the spoken word, the written word, diagrams, actions, etc. Many professional development studies involve the use of surveys and self-reporting by individuals in which they testify about their own growth. However, these types of data have long been suspect in the social sciences research methodology literature (Cohen, 1990; Spillane & Zeuli, 1999). Other professional development studies use pre- and post-tests of teachers’ knowledge.
The use of tests as evidence of teachers’ growth presumes that what is tested represents and captures the most important aspects of teachers’ knowledge development, that teachers’ knowledge growth will converge on the content that is tested, and that what is tested will translate into classroom practice. A different kind of data is performance-based data or documentation that comes from what teachers actually do as they engage in practical work. These types of data have a high degree of face validity and have the potential to provide compelling evidence about how different teachers grow in different ways.

Performance-based data that have the potential to reveal teachers’ thinking can be gathered from activities designed to reveal teachers’ thinking. Chamberlin (2004) has proposed principles for developing effective, thought-revealing activities for teachers. She characterizes the desired activities as the design of classroom tools or resources that reveal teachers’ thinking as they engage in an iterative process of expressing, testing, and revising their creation. This design process results in thought-revealing products created by teachers that can be assessed by themselves, their peers, the facilitator, and the researcher. The lesson study case involved three points of written, performance-based data obtained from the series of revised lessons written by the teachers. Although the problem to be solved by the students remained the same over the three lesson revisions, the tasks given to the students were redesigned repeatedly in response to the data collected during the lessons. The final lesson produced by the teachers showed how they designed the activity to reveal the students’ thinking at various points during the lesson. The teachers obtained an initial round of data when the class discussed the solution to a problem for a small number of tables. Then, working with partners and using pattern blocks, the students were asked to solve the problem for some larger numbers, resulting in written records of the students’ data on the handout. Finally, the class was to discuss the patterns that the partners saw, providing another round of information to the teachers. Another task was given at the end of the lesson in which the students were asked, as a class, to write an equation that would express a mathematical rule relating the number of triangles to the number of seats. The equation that students chose revealed how they mathematized the relationship between the number of triangles and the number of seats. These data sources provided the primary information that the teachers used to revise the lesson, and the facilitator and the researcher gained performance-based data from the teachers.

How Are the Data Collected?

Similar to the process of data collection described by the Design-Based Research Collective (2003), the collection of performance-based data in professional development design studies faces issues of complexity. Lesh and Kelly (2000) focused on the interactions among various tiers involved in teaching studies. In particular, they emphasized the role of the facilitator/researcher in planning for ways to document the thinking of all the constituencies: the students, the teachers, and the facilitator/researcher. The data collected in a design study can be considered a trail of documentation, rather than a series of static data points over time. Figure 11.2 illustrates the products designed by different constituencies in the students’ thinking sheet case. For example, in the students’ thinking sheet workshops, the activities given to students were model-eliciting, meaning that the activities were crafted carefully, using the design principles described by Lesh et al. (2000) to require students to reveal their thinking in their final products. The products created by students serve as the primary sources of information about the groups’ mathematical thinking, which typically prompt teachers to reflect on the
relationship between how their students are using mathematical knowledge and how the teacher is planning for instruction (Chamberlin, 2002). In the students’ thinking sheet case, the thought-revealing activity for teachers was to develop the students’ thinking sheets. The teachers needed to interpret the students’ responses and categorize common approaches that students took in their final products. A trail of documentation was produced by planning for cycles of teachers expressing their individual ideas about students’ approaches, testing their ideas with peers during the workshop session, and revising their ideas as the group of teachers converged on a consensus students’ thinking sheet. In Chamberlin’s study, teachers’ reflections were captured by video-taping, transcribing, and analyzing the conversations among teachers that took place during the session.

The construction of a design study should plan not only for the teachers’ documentation, but also for the documentation of teachers’ analysis of their own reflections. In the lesson study case, each research lesson was preceded by a presentation by the teachers of the principles that governed their lesson design and redesign. For example, before the research lesson, group members made a presentation to outside observers in order to explain the changes in the lesson, the reason for each change, and what they had learned from the experience of revising and reteaching the lesson. The teachers justified their decision to eliminate the worksheet by citing evidence from the first lesson that students did not understand the pattern fully despite filling out the worksheet correctly. For the students’ thinking sheet, the teachers might have been asked to write a final chapter in the handbook that would provide principles or frameworks for their peers to use in identifying students’ approaches to model-eliciting activities. Planning for cycles of expressing, testing, and revising those principles would provide not only a trail of documentation of the teachers’ metacognitive development, but also the teachers themselves would have been engaged in the analysis process.

**How Are the Data Analyzed?**

Data analysis methods need to be aligned with the nature of the research questions posed (Shavelson et al., 2003). Although the two illustrations in this chapter involve the
qualitative analysis of data, different types of research questions may draw on various
types of qualitative and quantitative analyses, in part depending on the phase of the
research in the complete design cycle described in Middleton, Gorard, Taylor, and
Bannan-Ritland (this volume). Data analysis in multitiered design studies at these
particular phases involve the:

- Teachers analyzing their students’ data for the purpose of designing an educational
tool or resource.
- Teachers analyzing data about their own interpretive systems for the purpose of
developing principles or frameworks for designing the educational object.
- Facilitator interpreting the teachers’ analyses for the purpose of planning further
professional development experiences.
- Researcher analyzing data about his or her own interpretive systems concerning
the teachers’ teaching and learning for the purpose of revising and refining their
theories, frameworks, and mechanisms for professional development.

Just as complexity is assumed in professional development research, so it is assumed
to exist in the analysis of the data. Figure 11.3 illustrates the process of analysis em-
bedded in the process of designing an educational object, a professional develop-
ment session, or a theory. It should be kept in mind that this is happening for each
constituent in the design study, from the teachers through the researcher. Teachers an-
alyze the students’ data because it not only facilitates their professional development,
but also it prompts them to express aspects of their own interpretive systems. The educational objects produced by the teachers (e.g., the lessons) provide primary evidence of changes in their thinking over the course of the professional development experience.

The facilitator/researcher in the lesson study analyzed performance-based data (i.e., video-tapes of the teachers’ planning meetings, lessons, and postlesson symposia). The goals were to document the teachers’ thinking over time for use in planning subsequent sessions and to understand the teachers’ developing of interpretive systems. For example, during the symposium following the first lesson, one lesson study team member commented:

Our worksheet set it up for them, spoon-fed them. . . . One of the things we’re going to be talking about later is: Was the worksheet helpful in focusing their thinking, or did it close off that aspect and not give us the feedback about where the students were starting from?

Another team member added:

I could see that students were able to fill out the worksheets quickly but never really saw any indication of what does that mean they know . . . they could add plus two to the numbers, but that work didn’t necessarily show the kids understand the pattern. Since we’ll have an opportunity to reteach it, we’re thinking about . . . how to revise it so we can see more of the students’ thinking in the lesson.

These data from the teachers’ conversations about the first teaching suggest that the mechanism of the lesson design prompted them to think critically about how their lesson was prompting the students to think deeply about the problem. The process of lesson design also prompted them to modify the lesson to reveal the students’ thinking better.

After discussing what the students might learn from organizing the data on their own, rather than being presented with a two-column table to fill out, the team decided to eliminate the two-column table from the next teaching of the lesson. Initially, the teachers disagreed about the wisdom of having the students organize the data without a two-column table. In the course of the discussion, the teachers agreed that the students might learn something about the problem from organizing the data themselves, or at least that the teachers could learn something from trying it this new way. One teacher said, “It’s exciting to see what will happen when we make this change. I guess that’s what you call lesson study.” Thus, the video transcripts revealed further confirmation that, over the course of their planning, the teachers crafted progressively a lesson that they perceived was more adequate for building the students’ understanding of patterns than the one in the textbook. Furthermore, they had developed the students’ materials to go with the lesson and had obtained information about the mathematical issues related to the lesson (e.g., the difference between a pattern, a rule, an equation, and a formula).

In the students’ thinking sheets case, the researcher also analyzed video-taped data. She video-taped each session to capture the interactions among the teachers in the professional development session in order to obtain evidence of their processes of posing and testing interpretations of the students’ thinking. The method of analyzing the transcripts produced from these video-tapes and the teachers’ artifacts was guided by the grounded theory approach of Strauss and Corbin (1998). Two different trails of
documentation (using both written and video-taped data) could be examined: the intermediate and final products over the course of a problem and the form of the students’ thinking sheets over the course of the five problem-based cycles.

The researcher found that the production of the students’ thinking sheets led the teachers to engage in mini-inquiries, occasions during which the teachers inquired into why their students thought about the associated, model-eliciting activities as they did or the teachers inquired into the underlying mathematical complexities associated with the model-eliciting activities. During these mini-inquiries, the teachers met some of the challenges of attending to students’ thinking that are described in the reform documents, including looking for sense in their students’ thinking (Chamberlin, 2005). For example, during the Departing On-Time (Appendix B) discussion, the teachers saw an evolution in the students’ thinking. They reported that students often initially computed values for total minutes late or average minutes late, decided that the results were too close to use to rank the airlines, and therefore moved on to another strategy either as a tiebreaker or as a new way to approach the problem (e.g., the frequency of on-time departures).

At other times, the teachers’ discussions did not result in such inquiries, leading to the need to revise for a subsequent study the locally developed professional development mechanism (i.e., the students’ thinking sheets developed by the teachers). Specifically, through further analysis, the researcher recognized three adaptations to the production of the students’ thinking sheets that may have led the teachers to express, test, and revise their interpretive systems more frequently. First, the facilitator should strive to establish more specific norms for the development of the students’ thinking sheets. According to Ball and Cohen (1999), to take on a stance of inquiry, teachers need to develop collective social norms, such as avoiding leaps to definitive conclusions, presenting interpretations as conjectures, and relying upon evidence and critical methods of reasoning for making conclusions. Second, in order to enhance the teachers’ ability to examine the students’ ways of thinking, the facilitator should ensure that every teacher has a copy of the students’ thinking sheets that each teacher designed individually. This would enable the teachers to examine more critically the other teachers’ descriptions of their students’ thinking (i.e., by being able to refer directly to the other teachers’ interpretations of the students’ work). Finally, as mentioned previously, the facilitator and the researcher should ask the teachers to produce a final chapter for the teachers’ handbook that would help the teachers express, test, and revise their principles for implementing model-eliciting activities and for developing students’ thinking sheets.

**Summary**

Guided by the professional development design principle that teachers should be engaged in experiences that result in expressing, testing, and revising their interpretive systems, the researcher and the facilitator in each professional development case designed mechanisms to enact this principle. As a result of testing these mechanisms, trails of documentation were revealed about a teacher’s development and were used to revise the mechanisms further. The lesson study experience designed for this particular context was built upon the established literature in the field and, as such, was being field-tested in a new context, similar to Phase Five of the complete design cycle described in the chapter by Middleton, Gorard, Taylor, and Bannan-Ritland in this volume. The students’ thinking sheet example, on the other hand, was in the initial stages of having a mechanism designed for it, which was expected to lead to further
rounds of revision. Thus, in the latter example, one sees how the express-test-and-revise process applies to the development of mechanisms for local contexts and hints at how design experiments move from prototype and trial (Middleton et al.) to a field study.

Some Practical Considerations

During a design experiment, in some ways, the teacher and the researcher function more as co-researchers than as researcher and participant (Design-Based Research Connective, 2003; Lesh, 2002); that is also the case for the teachers, facilitators, and researchers in design studies for professional development. Each is involved in the research and development of an educational object, each is testing and analyzing data in order to refine the object under design, and each is reflecting on his or her own interpretive systems. Clearly, the students and their teachers are two different constituencies, but although the facilitator and the researcher have different purposes, much of their work is collaborative and closely related. Thus, it is possible to design a study in which the researcher and the facilitator are one and the same person. In fact, in the students’ thinking sheets case, the facilitator and the researcher were the same person. The facilitator/researcher sought outside support to keep track of her different roles, to keep the purposes distinct yet in harmony, to discuss the possibilities for future sessions based on information gathered, and to keep the end-in-view in mind. Although it is challenging to take on both roles simultaneously, it is possible, especially in a small-scale study. Further, it is possible for an individual to conduct a series of studies over time, keeping both roles and building a program of research that can lead to transportable theories and frameworks for designing the particular type of professional development. Another way to approach creating and implementing a design study for professional development would be to use a team-based model, in which the facilitator and the researcher are two individuals (or teams). In the lesson study case, the teacher-leaders researched the process actively (e.g., documenting each iteration of the lesson and asking the group members to report on what they learned from lesson changes), but outside researchers assisted with additional elements of the process (e.g., videotaping, transcribing discussions, and developing a flowchart to describe changes in the lesson). The advantage is the opportunity to have multiple perspectives involved in the design of the professional development and of the theory, principles, or frameworks being developed for the field. Continual opportunities for the facilitator and the researcher to express, test, and revise their ideas are important for all the tiers in the design study.

Design studies in professional development require that multiple goals and purposes be kept in mind simultaneously. The teachers’ problem to be solved is always in the forefront, as the end-in-view, and the product to be designed at each level needs to be thought revealing in order to prompt growth to take place (through cycles of express, test, and revise) and to provide documentation of the conceptual or interpretive systems of the constituency. Given that the professional development process is flexible and responsive to the changing conditions of the school environment, organizational tools are needed to facilitate the planning, implementation, documentation, and analysis processes. Table 11.1 is a chart intended to help a facilitator and a researcher begin planning a design study and represents an initial externalization of the authors’ theory of how to create a design study for professional development.
Conclusions

The purpose of this chapter has been to propose a way of approaching research in professional development in situations where the nature of teachers’ growth cannot be predicted always and where the professional development takes place in the complex context of schools. The proposed form of the design study for professional development is based on gathering information from what teachers do in preparing for instruction rather than from what they say about instruction. Designing the professional development experience around teachers’ needs and around teachers’ designs of educational objects or resources can work to enhance the professional development experience and, simultaneously, to provide documentation of the teachers’ interpretive systems for learning and teaching. Planning the experience in a way that motivates teachers to go through cycles of expressing, testing, and revising the educational object they are designing leads to trails of documentation that can be used as evidence of change in teachers’ interpretive systems. Engaging teachers in the process of reflecting on and analyzing evidence of their own interpretive systems not only enhances their professional development experience, but also provides the facilitator and the researcher with

Table 11.1 An Initial Planning Chart for a Design Study

<table>
<thead>
<tr>
<th>Tier (participant level)</th>
<th>Product to be designed</th>
<th>Nature of the thought-revealing activity</th>
<th>Research and design process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td>What product is the student being asked to design?</td>
<td>How does the student’s product reveal his or her thinking?</td>
<td>What is the purpose, or need, that the student is addressing by designing the product? How will the motivation to assess and revise the design be built into the classroom experience?</td>
</tr>
<tr>
<td>Teacher</td>
<td>What educational object will the teachers design? How will it help them advance toward the end in view?</td>
<td>How will the educational object reveal and document the teachers’ thinking?</td>
<td>How will the students’ data motivate the teachers’ assessment and revision of the educational object under design?</td>
</tr>
<tr>
<td>Facilitator</td>
<td>What is the professional development experience being designed by the facilitator?</td>
<td>How will the facilitator document the changes and rationales for the changes in the planned professional development experience?</td>
<td>How will the facilitator use information gathered at the professional development sessions to reflect on and revise plans for the subsequent professional development sessions?</td>
</tr>
<tr>
<td>Researcher</td>
<td>What theory, principles, or framework is the researcher designing?</td>
<td>How will the researcher document the changes and rationales for the changes in the theory, principles, or framework being designed?</td>
<td>How will the researcher use information gathered at the professional development sessions to reflect on and revise the theory, principles, or framework under design?</td>
</tr>
</tbody>
</table>
better insights into teachers’ ways of thinking and their interpretive systems. The facilitator and the researcher, like the teacher, also are designing educational theories about which they reflect and produce a trail of documentation. The facilitator is designing the professional development experience and the researcher is designing the theory, principles, or frameworks that can be shared with and used by other researchers or educators interested in professional development in mathematics.

Rather than trying to control the factors of complexity associated with school-based, professional development, the proposed approach to design study is intended to embrace the complexity and the dynamic nature of the system in which teachers learn and grow. In addition, the professional development should feel more collaborative in nature. Fishman et al. (2004) describe a model for technology innovation that encourages explicitly collaboration between partners in an educational innovation. For professional development, we have described a model that encompasses the teachers’ perspective in an end in view and the needs of the local school situation. However, the model also provides for the development of principles that are transportable to other situations in the sense that Middleton et al. (in this volume) describe. Thus, design experiments for professional development as we have conceptualized them are one way to think about what might be next for investigating teachers’ interpretive systems and the development of those systems over time.

Acknowledgments

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Notes

1 The studies described here seem to fall into the design experiment phases (three, four, and five) of the complete design cycle for a program of research described by Middleton, Gorard, Taylor, and Bannan-Ritland in another chapter in this book. It is conceivable that each study could become part of a larger program of research that would complete the seven phases described by them.
2 See the on-line database of lesson study groups at: www.tc.columbia.edu/lessonstudy/lgroups.html
3 They received an honorarium for their attendance.
4 A group of 26 teachers initiated a lesson study in the autumn of 2000. By mid-2002, 87 teachers in the school district had joined lesson study groups voluntarily. For the protocols, see Lewis, 2002b; http://www.ger.com; http://www.tc.columbia.edu/lessonstudy.
5 English and Lesh (2003) used the term end in view to describe the final goal of a task in which the solution process is likely to be characterized by cycles of formulating the problem, modeling the problem, and revising the problem’s interpretation and the model. In other words, the solution process mirrors a complex system that changes over time as the problem is being solved.
6 The illustrative, model-eliciting activity in Appendix B is typical, in that it begins with an article or a story that provides background in the contextual information and then uses “readiness questions” to review the context and address prerequisite skills. Finally, the model-eliciting product (i.e., model) should serve.
7 When the students completed this activity, they had access to computers. Thus, they performed their calculations by hand on paper and then used the word-processing capabilities of the computers to type their letters. The excerpts here are from their typed letters.
References


Verschaffel, L., DeCorte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H. & Ratinckx, E.
Appendix A

Lesson Study Table Task

We have a long skinny room and triangle tables that we need to arrange in a row with their edges touching, as shown. Assuming each side can hold one seat, how many seats will 1 table, 2 tables, 3 tables hold? Is there a pattern that helps you figure out how many seats 10 tables will hold?

![Figure 11.A1](image-url)
Appendix B

A Model-Eliciting Activity

Newspaper Article: The Challenges of Flying

Chicago, Illinois—With 180,000 people flying in and out of O’Hare International Airport in Chicago each day, nearly 70 million people per year, O’Hare is one of the busiest airports in the world. Being this busy has advantages for passengers. For instance, if one’s flight is canceled, one has a very good chance of finding another flight. Also, O’Hare has flights to virtually every other airport in the world.

However, along with these advantages come some disadvantages for passengers. It can be difficult to get to one’s gate, to park one’s car, to pick up one’s baggage, and to check in when you have to compete with thousands of other people each day. Despite these disadvantages, people keep coming back to the airport and passengers have even rated the airport as their favorite airport in the world (on an internet survey). On the survey, passengers provided numerous reasons for their like of O’Hare airport. A popular reason was that all of the airlines at O’Hare try to stay on schedule. Staying on schedule is very important because one or two little disturbances can offset the entire airport schedule.

Travelers typically have three main concerns when flying to their destination. First and foremost, they are concerned with safety. When asked, most passengers say that they would not mind being a few minutes late to ensure that they arrive at their destination safely and without incident. After safety, the passengers’ second most common concern is whether the flight takes off and arrives on time. Third, they want their baggage to be shipped to the correct destination and to arrive on time also.

O’Hare does a fantastic job of making sure the planes arrive and leave on time; however, many things can impact this timing. Those who travel regularly can make a calculated guess about whether their flights will arrive on time. This timing is contingent on several factors.

First, the origin of the flight impacts the plane’s chance of arriving on time. For example, planes rarely leave late from San Diego, California due to San Diego’s great weather, but they frequently leave San Francisco late due to weather conditions such as fog. Veteran travelers often try to avoid flights that leave San Francisco to come to O’Hare. Second, the on-time arrival is based on the flight’s destination. For example, sometimes, a destination takes a plane into a very busy airport that may be too small for the amount of daily air traffic. In this case, a gate may not be ready always for the plane to pull up to and unload the passengers. Thus, the plane will have to wait. Similarly, an understaffed maintenance department may impact the company’s ability to fix planes on a timely basis. Third, the on-time arrival may be dependent on the company. Some airlines are known for being on time consistently, whereas other airlines are known for not being on time.

For some travelers, arriving on time is not an
issue because they are not in a hurry. For example, a family flying from Pittsburgh to Orlando in order to visit Disney World may not be too concerned if they arrive 15 to 30 minutes late. However, business travelers may miss important meetings if their flights arrive late.

**Readiness Questions**

1. Where is one of the busiest airports in the world?
2. What do you believe might be another busy airport?
3. Why would arriving on time be important to some travelers and not as important to other travelers?
4. List one thing cited in the article that may cause a plane to be late.
5. Can you think of other reasons for a plane to be late that are not mentioned in the article?

**Problem Statement**

**Information**

In June, Ridgewood High School’s Spanish club is going on a study abroad trip to Venezuela, and they have hired your class to help them select which airline to fly. Last year the Spanish club had a miserable experience when traveling to Barcelona. Their connecting flight to Reykjavik, Iceland was late, so they missed their next flight to Barcelona. The entire class had to stay overnight in the airport.

This year the class has decided to take a more systematic approach to choosing an airline. So far, the class has identified five airlines with economical fares that fly from O’Hare Airport to Venezuela, but they are still in the process of identifying more airlines that fly to Venezuela. Most of the flights have a connecting flight in Mexico City. They are hoping to find the airline that has the smallest chance of departing late from O’Hare so that they are less likely to arrive late in Mexico City. They don’t want to miss their one connecting flight to Venezuela this year!

In Table 11.B1, you will find information about departure times for flights on the five airlines that the Spanish Club has identified thus far. The departure times are for flights leaving from O’Hare Airport and scheduled to arrive in Mexico City. Rank the five airlines in terms of most likely to be on time to least likely to be on time for departing from O’Hare Airport. As you rank the airlines, keep track of your process. Describe your process in a letter to the Spanish Club so that they may use a similar process to rank the additional airlines they may identify at a later time.
**Table 11.B1** Number of Minutes Late for Flights Departing from O’Hare Airport

<table>
<thead>
<tr>
<th>Sky Voyage Airlines</th>
<th>Central American Airlines</th>
<th>Mexico Express</th>
<th>Sudamerica Internacional</th>
<th>Southeast Airlines</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>5</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>5</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>125</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>10</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
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<td>10</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>0</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>9</td>
<td>55</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>25</td>
<td>7</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>12</td>
<td>0</td>
<td>5</td>
</tr>
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<td>5</td>
<td>0</td>
<td>0</td>
</tr>
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<td>7</td>
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<td>5</td>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>10</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
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</tr>
<tr>
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<td>4</td>
<td>0</td>
<td>0</td>
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<td>15</td>
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<td>5</td>
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</tr>
<tr>
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<td>25</td>
<td>7</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

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Appendix C

Students’ Thinking Sheet for Departing on Time Task

<table>
<thead>
<tr>
<th>Description of the Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy 1</strong> Total or Average Number of Minutes Late:</td>
</tr>
<tr>
<td>Students may find the total number of minutes late for each airline. Then, some students will continue to find the average number of minutes late per day for each airline. They then order the airlines from lowest to highest to rank the airlines from most likely to depart on time to least likely to depart on time.</td>
</tr>
<tr>
<td><strong>Total Number of Minutes Late:</strong></td>
</tr>
<tr>
<td>We found this information out by adding all the minutes for every airline in the month of June. In this case the airline with the smallest sum was the best.</td>
</tr>
<tr>
<td><strong>Average Number of Minutes Late:</strong></td>
</tr>
<tr>
<td>We have been looking over various airlines’ flight times from the month of June in 1999. We added up the total amount of minutes they were late and divided it by 30 to find the average amount of minutes a particular airline was late per day.</td>
</tr>
<tr>
<td><strong>Strategy 2</strong> Counting the Number of On-Time Flights:</td>
</tr>
<tr>
<td>Students may count the number of times that each airline is on time; that is, count the number of zeros for each airline. Then, they may find either the ratio of flights on time per the 30 flights for each airline or they may find the percentage of on-time flights for each airline. They then order the airlines from highest to lowest to rank the airlines from most likely to depart on time to least likely to depart on time.</td>
</tr>
<tr>
<td><strong>Counting the Number of On-Time Flights:</strong></td>
</tr>
<tr>
<td>We found our answer by counting how many times the airlines were on time. We picked one that was on time the most.</td>
</tr>
<tr>
<td><strong>Strategy 3</strong> Redefining Late:</td>
</tr>
<tr>
<td>We found this information out by adding all the minutes for every airline in the month of June. In this case the airline with the smallest sum was the best.</td>
</tr>
</tbody>
</table>
Students may recognize that departing anywhere between one and twenty minutes late will still allow a passenger to catch a connecting flight. Therefore, they may redefine late as occurring when a flight departs more than 20 minutes late. Then, they may incorporate this new definition for lateness into Strategy 1 or Strategy 2. For example, similar to Strategy 1, they may find the average number of minutes that flights are late when they are late. Similar to Strategy 2, they may count the number of times that each airline departs later than 20 minutes.

Counting the Number of Flights Departing Less than 5 Minutes Late:

If you are wanting to know the probability of the airlines being late, you must find the ratio. We did this by not counting 5 minutes and under as being late. Sky Voyage Airline’s ratio of being late, for instance, is 22/30. 22 being the times the airlines is not late and 30 being the amount of days. We found that Sky Voyage and Sudamerica Internacional are tied for first and Southeast Airlines somewhat behind at 15/30, so half the time it is late. Central American Airlines is late 2/5 of the time, coming in last place. We hope this helps you and thanks for asking us for the job.

<table>
<thead>
<tr>
<th>Mathematics in the Strategy</th>
<th>Meeting the Needs of the Client</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy 1</strong></td>
<td>Adding the total number of minutes for each airline.</td>
</tr>
<tr>
<td>Finding the average by dividing the total number of minutes for each airline by 30.</td>
<td></td>
</tr>
<tr>
<td>Comparing and ordering the averages or total minutes to rank the airlines.</td>
<td></td>
</tr>
<tr>
<td>Students often rounded their averages to the nearest tenth of a decimal point.</td>
<td></td>
</tr>
<tr>
<td>This strategy would be easy for the Spanish Club to implement. However, it is not very effective for the Spanish Club. More specifically, one or two very late departures on the part of an airline will be equal to a large number of minimally late departures on the part of another airline. Also, the differences between the averages are only a matter of seconds, not a good measure for ranking the airlines.</td>
<td></td>
</tr>
<tr>
<td><strong>Strategy 2</strong></td>
<td>Adding the number of times that an airline left on time.</td>
</tr>
<tr>
<td>Finding the ratio of on-time departures to total departures (30).</td>
<td></td>
</tr>
<tr>
<td>Dividing the number of times on time by 30 for each airline, then multiplying by 100 to find the percentage of on-time departures for each airline.</td>
<td></td>
</tr>
<tr>
<td>Comparing and ordering the ratios or percentages to rank the airlines.</td>
<td></td>
</tr>
<tr>
<td>This strategy is also fairly easy for the Spanish Club to implement. It is more effective than Strategy 1, but the strategy does not take into account that departing up to 20 minutes late may still be acceptable for catching a connecting flight.</td>
<td></td>
</tr>
<tr>
<td><strong>Strategy 3</strong></td>
<td>Determining an acceptable range for departure times.</td>
</tr>
<tr>
<td>The same mathematics as Strategy 1 or 2 depending on which approach is used.</td>
<td></td>
</tr>
<tr>
<td>This strategy is a little bit more difficult for the Spanish Club to implement. It depends on identifying an acceptable range for departures and requires associated computations. However, this strategy proves more effective than Strategy 1 or 2. It takes into account that departing up to 20 minutes late may still be acceptable for catching a connecting flight.</td>
<td></td>
</tr>
</tbody>
</table>

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