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A Methodology for Documenting Collective Activity

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10 A Methodology for Documenting Collective Activity

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Introduction

Mathematics and science education has witnessed an increase in the number of design research studies in which researchers spend extended amounts of time in classrooms implementing and investigating interventions to support students’ conceptual growth (Kelly & Lesh, 2000; Suter & Frechtling, 2000). One issue of theoretical and pragmatic concern that has emerged from design research is documentation of the normative or collective ways of reasoning that develop as learners engage in mathematical or scientific activity. The purpose of this chapter is to explicate a methodology for documenting learners’ collective activity. Although methodologies for documenting the learning of individuals are well established in the mathematics and science education fields, methodologies for detailing the intellectual activities of classroom cultures are less so. Therefore, we maintain a focus in this chapter on a methodology for documenting collective activity, an important endeavor in its own right.

We define collective activity as the normative ways of reasoning of a classroom community. We stress that collective activity is a social phenomenon in which mathematical or scientific ideas become established in a classroom community through patterns of interaction. To give an analogy, consider a couple, let us call them Sam and Pat, that, as a pair or dyad, are characterized aptly as argumentative. All of us know couples whom we would describe as having a particular characteristic: “Oh, the Smiths, they’re a fun couple, or the Robinsons are argumentative.” In other words, being argumentative is a quality of the pair. However, we might say that, as individuals, neither Pat nor Sam is particularly argumentative. Being argumentative is a characteristic of the collective activity of the couple (or, in this case, dyadic activity) and is not necessarily a characteristic of each individual. Similarly, the collective activity of a mathematics class refers to the normative ways of reasoning that develop as learners solve problems, explain their thinking, represent their ideas, etc. These normative ways of reasoning that can be used to describe the mathematical or scientific activity of, say, Mr. Jackson’s class may or may not be appropriate descriptions of the characteristics of each individual student in the class. This last point is critical to our notion of collective activity because it offers a view of the social context of the classroom that affords students opportunities for conceptual growth without necessarily being deterministic.

The issue of collective activity is one that resonates with a variety of social theories, among them activity theory (Davydov & Radzikhovskii, 1985; Leont’ev, 1981), sociocultural theory (Lerman, 1996; Moschovich, 2004; van Oers, 1996), and socioconstructivist theory (Bauersfeld et al., 1988; Cobb, 2000; Simon, 2000; Yackel, 1997). These social theories of learning offer analytical lenses for viewing and explaining the
complexities of learning in classrooms; however, detailed methods for documenting collective activity under the guidance of these theoretical orientations are underdeveloped. Rigorous qualitative methodologies are needed to document the collective activity of a class over extended periods of time. We address this need by first detailing the theoretical orientation that can serve as the basis for such an analysis. Then, we describe Toulmin’s (1969) model of argumentation, which serves as an analytic tool in the methodology. Next, we outline a three-phase, methodological approach for documenting the collective activity of a classroom community and highlight two criteria using Toulmin’s scheme for determining when ideas begin to function “as if shared.” Our use of the phrase “as-if-shared” is intended to emphasize that normative ways of reasoning in a community function as if everyone shares this way of reasoning. However, there is individual variation within collective activity.¹ The usefulness of a methodology that examines collective activity, rather than individual variation, within a group’s normative ways of reasoning is two-fold. First, it offers an empirically grounded basis for design researchers to revise instructional environments and curricular interventions (cf., Brown & Campione, 1994). Second, it is a mechanism for comparing the quality of students’ learning opportunities across different enactments of the same intervention. Then, we illustrate the two criteria using Toulmin’s scheme with an example from a first-grade class learning to measure and from a university course in differential equations. These two content areas are chosen in order to increase the likelihood that the methodology we describe is accessible to a wide range of readers and applicable across content domains. We conclude by discussing issues of the generalizability and trustworthiness of the methodology.

Theoretical Orientation

A research methodology expresses both the methods used in analysis and the theoretical orientation that underpins such methods because the particular methods one chooses are influenced by the theoretical lens one is using to view the results of an investigation (Moschkovich & Brenner, 2000; Skemp, 1982). For example, if a researcher or teacher desires to create a model of how individuals construct rate of change concepts, then a cognitive theory that views learning as primarily a mental construction of accommodations and assimilations might be most useful. On the other hand, if a researcher or teacher is attempting to analyze the activity of a community of learners, social theories can be more useful.

One particular lens that has been helpful for analyzing students’ learning in social contexts is that of symbolic interactionism (Blumer, 1969), which treats interaction among people as central to the creation of meaning. From this perspective, learning refers to the conceptual shifts that occur as a person participates in and contributes to the meaning that is negotiated in a series of interactions with other individuals. Such a view is consistent with theories of argumentation that define an argument as a social event in which individuals engage in genuine argumentation (Krummheuer, 1995). Genuine argumentation occurs when the persons involved engage in a back-and-forth flow of contributions (often in verbal, gestural, and symbolic forms), in which each person interprets actively the meaning of another’s statement and adjusts his or her response based upon the meaning they infer. Often, these adjustments and interpretive actions are at the implicit level of students’ awareness. Moreover, genuine argumentation requires a social situation in which participants explain and justify their thinking routinely. Such a social situation does not happen by accident; it requires the proactive role of a knowledgeable teacher.

¹ Chris Rasmussen and Michelle Stephan
Social theories, such as symbolic interactionism, that treat meaning as created in interactions among individuals serve as the theoretical underpinnings of the methodology we describe in this chapter. Also at the foundation of our methodology are those social theories that place prominence on the roles that tools and gestures play in mathematical development. Our methodology takes as primary the notion that learning is created in argumentations when individuals engage language, tools, symbols, and gestures (cf. Meira, 1998; McNeill, 1992; Nemirovsky & Monk, 2000; Rasmussen et al., 2004). This view is consistent with distributed theories of intelligence in which learning is said to be distributed across tools and symbols (Pea, 1993).

With this theoretical basis, we now turn explicitly to our central analytic tool, Toulmin’s argumentation scheme. Because collective activity refers to the negotiated normative ways of reasoning that evolve as learners engage in genuine argumentation, we use argumentations that occur in public discourse (i.e., discussions that can be heard by all participants, usually whole-class discussions) as our unit of analysis. Next, we explain Toulmin’s (1969) model of argumentation and how it can be used to document when particular mathematical ideas begin to function as if they were shared.

**Toulmin’s Argumentation Scheme**

In his seminal work, Toulmin (1969) created a model to describe the structure and function of certain parts of an individual’s argument. Figure 10.1 illustrates that, for Toulmin, the core of an argument consists of three parts: the data, the claim, and the warrant.

In any argumentation, the speaker makes a claim and presents evidence or data to support that claim. Typically, the data consist of facts or procedures that lead to the conclusion that is made. For example, imagine a fourth-grade class that has been asked to find the area of a $4 \times 7$ rectangle. During a discussion, Jason makes a claim that the answer is 28. When pushed by the teacher to say more about how he got his answer, Jason says, “I just multiplied the length times the width.” In terms of Toulmin’s scheme, Jason has made a claim of 28 and given evidence (data) in the form of his method for obtaining his answer. Although his explanation is clear to us, other students may not understand why the warrant has authority.
understand what Jason’s statement “multiply length times width” has to do with obtaining 28 as the answer. In fact, a student (or the teacher) may challenge Jason to clarify how his evidence relates to his conclusion, so Jason must present some kind of bridge between the data and the conclusion. When this type of challenge is made and a presenter provides more clarification that connects the data to the conclusion, the presenter is providing a warrant, or a connector between the two.

Often, the content of a warrant is algorithmic (Forman et al., 1998) in that the presenter states more precisely the procedures that led to the claim. In our fourth-grade example, Jason might provide the following warrant: “Because you multiply length times width to find area, I just said 4 × 7 and that’s 28.” A student may see now how Jason went from data to claim with such an explanation but not understand or agree with the content of the warrant used: “I see where you get 28 now, but why do you multiply 4 by 7 to get 28?” The mathematical authority (or validity) of the argument can be challenged, and the presenter must provide a backing to justify why the warrant, and therefore the core of the argument, is valid. Then, Jason might provide a backing by drawing unit squares inside the rectangle and saying, “You see, you have 4 rows of 7 squares. That’s why you can multiply 4 by 7 to get the answer.”

**Documenting Argumentation**

In general, documenting the structure and function of students’ argumentations is facilitated by the following rules of thumb. Claims are the easiest type of contribution to identify in an argumentation and consist of either an answer to a problem or a mathematical statement for which the student may need to provide further clarification. Data are less easy to document but usually involve the method or mathematical relationships that lead to the conclusion. Most times, warrants remain implied by the speaker and are elaborations that connect or show the implications of the data to the conclusion. Finally, a backing is identified typically by answering the question: “Why should I accept your argument (the core) as being sound mathematically?” Backings, therefore, function to give validity to the argumentation.

Documenting ongoing collective argumentation is much more difficult than we have portrayed in the example above. Normally, classroom conversations do not occur in the clean crisp manner we have used to illustrate Toulmin’s (1969) model. Often, many claims are made simultaneously, such as when students get different answers for the same problem. In such cases, the researchers must record the data, warrants, and backings, if any, that students give as each claim is being justified and which claims are rejected in the classroom. Sometimes, the class seems to agree on one answer (claim) but may offer different warrants and backings as support. This situation occurs frequently in classes in which open-ended problem-solving is used because teachers encourage a variety of solution processes. To use our 4” × 7” rectangle example again, the class may agree that the conclusion is 28 square inches and the students may use the same data (e.g., a drawing of the rectangle cut into 4 rows of 7” unit squares), yet several different warrants might emerge in their arguments. For example, a student may say that he or she used the picture to count all the squares one by one and got 28 (W1). A different warrant that ties the data or inscription to the conclusion may be that a student added 4 seven times (W2), added 7 four times (W3), or merely multiplied 4 by 7 (W4). We have seen at least six different warrants (and two backings) in support of the same conclusion. Analyzing the collective argumentations that emerge in classrooms and shift in quality over time can shed sight on the mathematical ideas and learning that gain currency within that community. In the next section, we describe more specifically the
general, three-phase approach we have developed for documenting collective activity in classrooms—an approach that capitalizes on analyzing the collective argumentations that evolve over time.

The Three-Phase Approach to Documenting Collective Activity

To document the collective activity of a classroom community we developed a three-phase approach. Classroom teaching experiments are a form of design research in which the goals include investigating and developing effective means to support student learning of particular content (Cobb et al., 2003). Previous reports (Rasmussen et al., 2004; Stephan & Rasmussen, 2002; Stephan et al., 2003) placed the collective mathematical learning in the foreground, whereas the method itself remained in the background. The goal of this chapter is to offer a broader theoretical account by foregrounding the method itself and backgrounding all the content-specific details that appear in earlier reports. We encourage the reader to refer to these earlier reports for more information on the collective activity of the various classroom communities that we have studied. This approach evolved in the course of our efforts to document the collective activity in different classroom teaching experiments.

Phase One

In our classroom teaching experiments, we gather data from multiple sources: video-recordings of each classroom session, video-taped interviews with students, field notes from multiple researchers, reflective journals (from the teacher and/or researchers), and students’ work. With such a vast amount of data to organize, we start Phase One by creating transcripts of every whole-class discussion from all the class periods under consideration. Then, we watch video-recordings of every whole-class discussion and note each time a claim is made by a student or the teacher. Next, we use Toulmin’s (1969) model to create an argumentation scheme for each claim that was made. This is a time-intensive process that yields an “argumentation log” across whole-class discussions for several weeks. In our differential equations experiment, we found anywhere from two to seven different conclusions being made and/or debated in any one class period. The argumentation log orders all of the argumentation schemes sequentially.

For reliability purposes, a team of at least two researchers is needed to draw up an argumentation scheme (or a sample thereof) and then to verify and/or refute the argumentation scheme for each instance of a claim or a conclusion. We come to agreement on the argumentation scheme by presenting and defending our identification of the argumentation elements (i.e., the data, conclusion, warrant, and backing) to each other. In general, paying particular attention to the function that various contributions make is critical for identifying properly the elements of each argumentation. For example, is the student’s contribution functioning to provide a bridge between the data and the conclusion? If so, we would label it as a warrant. In the fourth-grade example, we pointed to several rules of thumb for identifying these elements.

Phase Two

The second phase of the analysis involves taking the argumentation log as data itself and looking across all the class sessions to see what mathematical ideas expressed in the
arguments become part of the group’s normative ways of reasoning. We developed these two criteria for when mathematical ideas function as if shared:

1. When the backings and/or warrants for an argumentation no longer appear in students’ explanations (i.e., they become implied rather than stated or called for explicitly, no member of the community challenges the argumentation, and/or if the argumentation is contested and the student’s challenge is rejected), we consider that the mathematical idea expressed in the core of the argument stands as self-evident.

2. When any of the four parts of an argument (the data, warrant, claim, or backing) shifts position (i.e., function) within subsequent arguments and is unchallenged (or, if contested, challenges are rejected), the mathematical idea functions as if it were shared. For example, when students use a previously justified claim as unchallenged justification (the data, warrant, or backing) for future arguments, we would conclude that the mathematical idea expressed in the claim has become a part of the group’s normative ways of reasoning.

We then make a “mathematical ideas” chart for each day that includes three columns: (a) a column for the ideas that now function as if shared, (b) a column of the mathematical ideas that were discussed and that we want to keep an eye on to see if they function subsequently as if they were shared, and (c) a third column of additional comments, both practical and theoretical, or connections to related strands of literature. For example, a page out of our charts from the differential equations teaching experiment for this second phase of analysis looked like Table 10.1.

Table 10.1 is only one page out of a series of daily charts that took the argumentation logs for each day as data and summarized the days on which we observed certain mathematical ideas moving from the “keep-an-eye-on” column to the “as-if-shared” column. As we create these charts, we look at previous days’ charts to see which ideas in the second and third columns move to the first or second column in the current day’s argumentation schemes (from right to left). This is consistent with Glaser and Strauss’ (1967) constant comparison method in which we look for regularities in students’ argumentations one class period at a time. We make conjectures about argumentations in which ideas function as if they were shared and look for further evidence (or refutations) of these conjectures in subsequent class periods.

Table 10.1 A Page of the Mathematical Ideas Charts

<table>
<thead>
<tr>
<th>Ideas that function as-if-shared</th>
<th>Ideas to keep-an-eye-on</th>
<th>Additional comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>The slopes are the same horizontally for autonomous differential equations (shifts from conclusion to data)</td>
<td>The graphs are merely horizontal shifts of one another for autonomous differential equations</td>
<td>The rate of change is based on an equation, not on a real-world setting; relate basis of claims to RME heuristics</td>
</tr>
<tr>
<td>No real-world backing is given</td>
<td>Using the rate of change equation as data to show that slopes are invariant</td>
<td>Making predictions about solutions functions as if it is a shared goal (sociomathematical norm?) Isomorphism between graphic and analytic techniques</td>
</tr>
</tbody>
</table>

Note
RME = Realistic Mathematics Education.
Phase Three

In the third phase of the analysis, we take the pages of charts from Phase Two, list the ideas from the “as-if-shared” column, and organize them around common mathematical activities. For example, in the differential equations teaching experiment, we took our list of mathematical ideas and organized several of them under the general activity of predicting individual solution functions. Hence, the third phase of the analysis involved taking the list of as-if-shared mathematical ideas and organizing them according to the general mathematical activity in which the students were engaged when these ideas emerged and became established.

We define this level of general mathematical activity as a classroom mathematical practice. This definition of a classroom mathematical practice is different from earlier definitions put forth by Cobb and Yackel (1996), which are restricted to one mathematical idea. In our methodological approach, a classroom mathematical practice is a collection of as-if-shared ideas that are integral to the development of a more general mathematical activity. For example, in the differential equations teaching experiment, the fifth classroom mathematical practice, which we called “Creating and organizing collections of solution functions,” entailed the following four normative ways of reasoning:

1. The graphs of solution functions do not touch or cross each other (at least for the equations studied thus far).
2. Two graphs of solution functions are horizontal shifts of each other for autonomous differential equations.
3. Solution functions can be organized with different inscriptions.
4. The phase line signifies the result of structuring a space of solution functions.

Moreover, the collection of classroom mathematical practices is different from the typical scope and sequence for a course in two ways. First, classroom mathematical practices, unlike a typical scope and sequence, can be established in a non-sequential time fashion. In a previous analysis that documented the constitution of classroom mathematical practices in a first-grade mathematics class (Stephan et al., 2003), the various practices proceeded in a more or less sequential fashion: the initiation and constitution of the first practice preceded the initiation and constitution of the second practice in time, etc. In our experience, a linear temporal development of classroom mathematical practices is not always the case. The second way in which classroom mathematical practices differ from a typical scope and sequence is that classroom mathematical practices can emerge in a non-sequential structure. On some occasions, we characterized some as-if-shared ideas as parts of more than one practice. This suggests that the practices themselves can have structural overlap, rather than a timing overlap of when the practices are initiated and constituted (Stephan & Rasmussen, 2002).

Now that we have described the general, three-phase approach to documenting the evolution of classroom mathematical practices, we will use examples from two different classroom teaching experiments to illustrate the two criteria that we developed for documenting the researcher’s belief that an idea is taken as-if-shared.

Criterion One: The Dropping Off of Warrants and Backings

Consider an example from a first-grade classroom discussion on linear measurement to help illustrate the first criterion for determining when an idea becomes part of a group’s...
normative ways of reasoning. This example is significantly more difficult to interpret than the fourth-grade example because there will be a number of individuals contributing to the emerging argumentation rather than only one student. During small group work, students had been asked to measure the length of items with their feet. The teacher noticed that the students had two different ways of counting their paces, as pictured in Figure 10.2; some students counted “one” as they began stepping (see Figure 10.2(a)), and others counted “two” as they began stepping (see Figure 10.2(b)). The teacher’s mathematical goal was to support the students’ interpreting the purpose of their measuring activity as one of covering space. Therefore, she organized a follow-up, whole-class discussion in which the students could contrast these two methods of measuring.

In the transcript below, Sandra explains her method of measuring and Alice contrasts Sandra’s method with her own.

*Sandra:* Well, I started right here and went 1 [she starts counting as in Figure 10.2(a)], 2, 3, 4, 5, 6, 7, 8 [the teacher stops her].

*Teacher:* Were people looking at how she did it? Did you see how she started? Who thinks they started a different way? Or did everybody start like Sandra did? Alice, did you start a different way or the way she did it?

*Alice:* Well, when I started, I counted right here [she counts as in Figure 10.2(b)], 1, 2, 3.

*Teacher:* Why is that different . . . [from what Sandra did]?

*Alice:* She put her foot right here [she places it next to the rug] and went 1 [she counts like Sandra], 2, 3, 4, 5.

*Teacher:* How many people understand that Alice says that what she did and what Sandra did was different?

In analyzing the preceding episode using Toulmin’s (1969) model of argumentation, we notice that Sandra first presented the data that led to her conclusion (of eight paces). The data consisted of her method of starting at a particular spot on the rug and counting 1 with the next step, 2, 3, 4, and so on until the teacher stopped her. In this portion of Sandra’s argumentation, her warrant was not explicitly present. Later, the teacher will ask questions and provide visual supports that lead the students to articulate the warrant for Sandra’s argumentation more explicitly. The teacher asked if other students started differently from how Sandra began. Alice presented different data and a conclusion: “Well, when I started, I counted right here. 1, 2, 3.” Her conclusion was three paces, and her evidence for her conclusion consisted of displaying her method of measuring in front of the class.

When the teacher asked whether other students in the class understood the difference in the methods, many students expressed confusion. The problem in understanding the
difference between the methods lies in the difficulty of distinguishing which paces were or were not being counted. As soon as the students lift his or her foot, the record of their pace disappears. Therefore, after the students had paced three or four paces, it was difficult for: (a) the demonstrating student to communicate what “1” referred to and, (b) the other students to see what the demonstrator meant by the first pace. The teacher then asked questions to support a discussion where the warrants (i.e., more information connecting a student’s method to his or her answer) would become more explicit. The teacher asked a student, Melanie, to measure the rug while she placed a piece of masking tape at the beginning and end of each pace. In the dialogue below, we will see that the record of pacing preserved by the masking tape contributed to the teacher’s agenda of making the difference between the two methods explicit. In Toulmin’s terms, the warrants were articulated.

Melanie: Sandra didn’t count this one [she puts her foot in the first taped space]; she just put it down and then she started counting 1, 2. She didn’t count this one, though [she points to the space between the first two pieces of tape].

Teacher: So she would count 1, 2 [she refers to the first three spaces because the first space is not being counted by Sandra]. How would Alice count those?

Melanie: Alice counted them 1, 2, 3.

Teacher: So, for Alice, there’s 1, 2, 3 there, and, for Sandra, there’s 1, 2.

Melanie: Because Alice counted this one [she points to the first taped space] and Sandra didn’t, but if Sandra . . . [had] counted it, Alice would have counted three, and Sandra would have too. But Sandra didn’t count this one, so Sandra has one less than her.

In this portion of the dialogue, Melanie used the record of paces to communicate clearly the difference between the methods, why Sandra counted the first three spaces as two, and why Alice counted them as three. This instance is an example of how a student’s explanation serves the function of making explicit how each student’s method (way of starting to count) related to the two conclusions they were drawing (i.e., three paces or two); in Toulmin’s (1969) terms, the warrants for each claim were made explicit. The record of paces played a crucial role in supporting the emergence of warrants that could be interpreted readily by the students who were having difficulty understanding the previous explanations.

Thus far, the core of the collective emerging argumentation looks like Figure 10.3. Notice that the warrants and the data are very similar in their content. Toulmin (1969) explains that it is not the content of what is said that necessarily characterizes statements as warrants, but rather the function that the statements serve. Initially, explaining one’s

<table>
<thead>
<tr>
<th>The core</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data 1 and 2:</strong></td>
</tr>
<tr>
<td><strong>Claims 1 and 2:</strong></td>
</tr>
<tr>
<td><strong>Warrants 1 and 2:</strong></td>
</tr>
</tbody>
</table>

*Figure 10.3 The Core of the Collective Emerging Argumentation.*
method of measuring served merely as data for the claims (i.e., the resulting measure) that each student made. However, when challenged by the teacher to articulate the difference between the two methods more explicitly, the students explained their procedures again, with the intention of making the difference in the methods more explicit. Their explanations served the function of detailing how the resulting measures (their claims) related to the method they used (the data).

It is important to note that, although the teacher and the students had tried to make their interpretations more clear, the focus of the discussion had concerned only how the students counted their paces when they measured and the different results that their methods obtained. Yet, the teacher’s intention was to lead a discussion in which measuring became an activity of covering space. In the next part of the dialogue, the teacher pushed the mathematical agenda (that of measuring becoming about covering space) by asking a pivotal question:

Teacher: What do you think about those two different ways—Sandra, Alice, or anybody else? Does it matter? Or can we do it either way? Hilary?

Hilary: You can do it Alice’s way or you can do it Sandra’s way.

Teacher: And it won’t make any difference?

Hilary: Yeah, well, they’re different. But it won’t make any difference because they’re still measuring but just a different way and they’re still using their feet. Sandra’s leaving the first one out and starting with the second one, but Alice does the second one and Sandra’s just calling it the first.

Phil: She’s 15 [he is referring to the total number of feet Sandra counted when she paced]. Alice went to the end of the carpet [he means the beginning of the carpet]. Sandra started after the carpet. Hers is lesser ‘cause there’s lesser more carpet. Alice started here and there’s more carpet. It’s the same way, but she’s ending up with a lesser number than everybody else.

Alex: She’s [Sandra’s] missing one right there. She’s missing this one right here [he points to the first taped space]. She’s going one but this should be one cause you’re missing a foot so it would be shorter. Since you leave a spot, it’s gonna be a little bit less carpet.

The question that the teacher posed: “Does it matter?” raised the level of the discussion from talking about which paces the students counted to whether it made a difference which method was used. Such questions served the function of eliciting, according to Toulmin (1969), the backing for the core of the emerging argumentation. The teacher’s question prompted the students to verify which method they believed was valid for their purposes. As seen by the explanations above, several students argued that, if one measured as Sandra did, part of the rug would not have been counted (i.e., they would have missed a spot). Critical to this emerging discussion is the fact that the students used the record of pacing to support their interpretations by pointing to the tape on the rug as they referred to the space they counted as “one.” In this way, reasoning with tools was an integral part of the collective meaning that emerged during this discussion.

Eliciting backings is crucial for supporting the evolution of increasingly sophisticated mathematical ideas. If the preceding discussion had remained at the level of warrants, any method of measuring would have been acceptable. However, the teacher and the students negotiated what counted as an acceptable measuring interpretation by providing backings for the emerging argumentation. As it happened, backings involving not missing any space were judged as acceptable in this class, and the group’s collective goal of measuring was to cover amounts of space as determined by the length of the item.
After these discussions, the students explained their measuring in whole-class discussions by counting with the method shown in Figure 10.2(b). Further, they no longer made their warrants explicit or provided backings to justify why they measured that way and none of them objected or challenged anyone’s interpretation any further. As noted earlier, when warrants become implicit and there is no longer a need to provide backings in public discourse, we claim that a mathematical idea (i.e., measuring is an activity of covering space) functions as if shared. This constitutes the first criterion for documenting when a particular mathematical idea can be characterized as a collective activity:

- **Criterion One:** When the backings and/or warrants for an argumentation no longer appear in students’ explanations (i.e., they become implied rather than being stated explicitly or called for), no member of the community challenges the argumentation, and/or if the argumentation is contested and the student’s challenge is rejected, we consider that the mathematical idea expressed in the core of the argument stands as self-evident.

This criterion is consistent with the one developed by Yackel (1997), in which she contends that mathematical practices are established when students no longer need to provide the supports (i.e., warrants and backings) for their conclusions. When we attempted to use this criterion to analyze the collective activity of the students in the differential equations teaching experiment, we found that it was useful, but insufficient, for our analysis. The students in the differential equations class used backings infrequently to support their arguments; therefore, we often were unable to use the first criterion to analyze their learning. This constraint enabled us to develop a new criterion for documenting when mathematical ideas become a part of a group’s collective activity. In the next section, we use examples of students’ argumentations in the differential equations design experiment to describe this new criterion.

**Criterion Two: The Shifting Function of Argumentation Elements**

The example we use to illustrate the second criterion for documenting collective activity occurred during a 15-week classroom teaching experiment in an introductory course in differential equations that was taught from a reform-oriented approach (see also Rasmussen et al., 2004, 2005; Stephan & Rasmussen, 2002). According to our mathematical ideas charts from Phase Two (see the sample given in Table 10.1), the idea that the slopes at a certain population (P) value remain constant across time (t) for autonomous differential equations first emerged as a topic of argumentation in the second class period. However, from the discourse below, we see that no backings were provided by the students to support why two slopes at the same P value across time should be the same, and the teacher did not push for one in this case. The problem that the students had solved involved trying to estimate the number of rabbits predicted by the rate of change equation \( \frac{dP}{dt} = 3P \) at half-year increments and quarter-year increments. The students had argued that the solution function starting with an initial population of ten rabbits would be an exponential curve, and the teacher drew the graph shown in Figure 10.4 on the board.

The teacher then asked the students what the graph of the solution would look like if they started with an initial population of 20 rabbits instead of ten. In the following discussion, the students began to think about the structure of the space of solution functions by relating the slope on one solution curve (with the initial condition of 10) to the slope on the curve with the initial condition of 20.
Teacher: What if you start at 20?
Andy: It would increase quicker. There’s more rabbits so you would start off, they’d have more babies, it’d just go faster like further \textit{sic} up on the 10 curve.
Rick: If you just look at the graph you have already, when you’re at 20, it’s already a steep increase.
Teacher: So, at 20, you already have a steep increase over here [she points to where \( P \) is 20 on the 10 curve].
Rick: It’s just the same thing.
Teacher: So this kind of increase here might be more like a similar increase here. This rate of change [she marks a slope line on the 10 curve] is similar to the rate of change here [she marks the same slope line at \( P = 20 \) and \( t = 0 \)] (see Figure 10.5).

Using Toulmin’s (1969) model of argumentation, Andy’s claim was that “it” increases quicker. The data for his conclusion were that there were more rabbits (20, not 10), so the population here would have more babies. He then went on to make a more complex conclusion; namely, that “it’d just go faster, like further \textit{sic} up on the 10 curve.” Rick expanded on Andy’s argument by claiming that the slope or rate of change is just the same at \( t = 0, P = 20 \), as it is on the 10 curve at the same \( P = 20 \) height. The data and warrant for this emerging argument were coconstructed by Rick and the teacher when Rick explained that one can “just look at the graph . . .” and the teacher pointed to the graph on the board as Rick explained the method.
At this point, neither the teacher nor the students challenged the legitimacy of this argument; that is, why it should be self-evident that the slopes at the same P value will be the same across time. Therefore, because no backing was provided during this argumentation and it was the first instance that such an argument was given, we cannot claim that the invariance of slopes across time functioned as if shared at this point, the second class session of the semester. Rather, we continued to look at the mathematical ideas charts to see if this idea, which we had placed in the “keep-an-eye-on” column, emerged again from the students and if backings or challenges to the argument were present. The most we can claim in this episode is that the students were beginning to create and organize a slope field by finding patterns in the slopes of the solutions.

The idea that slopes remain invariant across time re-emerged on the fourth day of class as the students filled out a slope field for the differential equation \( \frac{dP}{dt} = 3P(1 - P/100) \). During the whole-class discussion of this problem, the teacher asked John to share his group’s insights. John came to the front of the classroom and said that his group discussed that the slopes would be the same all the way across the t axis for any particular P value. To support his claim that they would, John drew on an argument from previous class periods in which they had discussed the solutions to the equation \( \frac{dP}{dt} = 3P \). In terms of Toulmin’s (1969) model, we see that a prior argument (for \( \frac{dP}{dt} = 3P \)) served as the warrant for the second part of John’s argument for the equation \( \frac{dP}{dt} = 3P(1 - P/100) \):

**John:** We were kind of thinking about it for awhile and . . . I looked back at our original question, what we were doing like even before the rabbits, when we had \( \frac{dP}{dt} \) was just equal to 3P and we were just trying to . . . we had the 10, we had the 30 and 50, and we were just going with 3P and we had no cap [he draws the axes and indicates 10, 30, and 50]. . . . When we discussed it in class, we said that the 10, the rate of change, is going to be very slow if we just started off with ten rabbits [he draws the solution function corresponding to the initial condition of 10, similar to that shown in Figure 10.4]. And we said that the rate of change immediately is going to be increasing a lot more when we start off at 30 [he draws in the solution function corresponding to the initial condition of 30]. When we look back on it, when we started at time, when we started at ten rabbits [he points to a population of 10 at time zero on his drawing], and we got to, say, the three years or whatever that it went by and we finally got to 30 rabbits [he draws in a slope mark at population 30 on the solution corresponding to the initial condition of 10], even though we started off with 30 rabbits over here [he gestures to the slope at the initial condition of 30], it had the same slope as that 10 did at time like two years. So I applied this or our whole group applied this to our problem now.

In this part of the argument, John made a claim that “the slopes are the same.” For this argument, John’s evidence to support his claim involved describing that the rate of change at 10 is slow and the rate of change at 30 is faster, and he drew a graph on the board to inscribe these ideas. The warrant for how the evidence relates to his claim that the slopes will be the same all the way across was that if you go along the 10 curve until you get to the point where \( P = 30 \), you can draw in the slope and notice that it is the same as the slope where \( P = 30 \) on the 30 curve (empirically-based reasoning). Rather than give a backing for this argument, John proceeded with the remainder of his argument for \( \frac{dP}{dt} = 3P(1 - P/100) \) as if he took it for granted that the other students agreed with his findings:
John: That they're initially going to start off and have a little different slopes but they're all going to be, kinda, when it reaches a certain point, they're all going to have the same slope at certain numbers. When 15, when they finally get to 30, that's going to have the same slope as 30 starting off at time zero. So we kinda all decided that all the slopes are going to be the same.

According to Toulmin’s (1969) model of argumentation, the structure of John’s argument can be summarized as shown in Figure 10.6. John provided no backing for why anyone should believe that this argument holds true. As a matter of fact, no backing was provided for the prior argument about why the slopes should be the same for a given P for the equation \( \frac{dP}{dt} = 3P \), especially because this invariance does not hold for all equations. However, another student, Jen, summarized his contribution by stating:

Jen: “So, basically, he’s saying that the rate of change is only dependent on the number of rabbits, not the time.”

In order to assess whether the students accepted this contribution, the teacher asked then if they agreed with Jen’s comment. They argued that the slopes would be invariant horizontally because the rate of change equation does not have a variable t on the right-hand side of the equation; if you “replace the P with a t on the right-hand side,” the slopes would be the same vertically. Therefore, another way in which the students justified John and Jen’s conclusions was by using the rate of change equation to make predictions about the invariance of slopes. Another student took the argument to a higher level by saying:

Andy: Another thing we found out that like since all the graphs’ slopes are the same, it’s just like you’re sliding this whole graph over one. Like going over here, toward 15 [he gestures to grab the curve for the initial population just above 0], it’s like the exact same thing. If you slide it over one more time, you get the 30 graph; another time, you get the 45. So if you know the graph,

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**Figure 10.6** John’s Argument for the Invariance of Slopes.
you can kinda predict what happened in the past, a little bit before your time zero because the graph is the same for all of them. You just pop it back for whatever your time interval was between the different 15 and 30 populations.

Teacher: So let’s put the shifting of the graph left and right out as a conjecture.

In the examples above, we see evidence that the notion that slopes would remain invariant across time for autonomous differential equations functioned as if shared. This evidence comes from analyzing the argumentation structures, both John’s and Andy’s contributions, as well as how their arguments were dealt with by the community. The claim that Andy made above built on John’s conclusion that the slopes were invariant. Andy used the previous conclusion of invariant slopes as data to conclude something new; one could merely slide the whole graph left or right. Here, we see an example of a case where the conclusion that was debated previously now functioned as the data for a more sophisticated conclusion (see Figure 10.7).

Therefore, according to our second criterion for documenting collective activity (restated below), the notion that slopes are invariant across time for autonomous differential equations functioned here as if it were shared.

- **Criterion Two:** When any of the four parts of an argument (the data, warrant, claim, or backing) shift position (i.e., function) in subsequent arguments and are unchallenged (or, if contested, the challenges are rejected), the mathematical idea functions as if it were shared by the classroom community. For example, when students use a previously justified claim as an unchallenged justification (the data, warrant, or backing) for future arguments, we conclude that the mathematical idea expressed in the claim becomes a part of the group’s normative ways of reasoning.

As the measurement and differential equations examples illustrate, analyzing the structure and function of students’ argumentations is interpretive in that the researcher must infer the intentions of the speaker(s) as they make contributions in the flow of the conversation. Therefore, for the methodology to be strong and credible, safeguards must be in place in order to ensure the trustworthiness of the analytic process. We discuss issues of trustworthiness as well as generalizability in the next section.

**Generalizability and Trustworthiness of the Methodology**

Of central concern about any methodology are issues related to the generalizability and trustworthiness of the approach. Generalizability in teaching experiments (and in

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**Figure 10.7** A Shift from Conclusion to Data.
design research more generally), as emphasized by Steffe and Thompson (2000), is an issue about the usefulness of the products from these interventions in settings other than the one in which they were built. The initial setting in which we developed the methodology for documenting collective activity was a first-grade, elementary school class. Our attempts to analyze the measurement practices in this class led us to invent a methodology that draws on argumentation theory. The first incarnation of the methodology involved the creation of the first criterion in which warrants and backings drop out of students’ arguments. Our efforts to use this criterion in a new classroom teaching experiment—differential equations—proved fruitful in that (a) we learned that our methodology was useful for understanding the collective activity of a different group of students and a different mathematical domain, and (b) we were able to refine the methodology by creating criterion two. Thus, we found that the methodology was useful in more than one context (i.e., it was generalizable), and, in turn, we used the new context to strengthen the methodology. Although the methodology to date is relatively rigorous and stable, it is, of course, open to further revision as researchers use it in their own contexts.

We view these instances of testing and revising in other contexts not as negating the generalizability of the methodology, rather as strengthening it. Because of these diverse settings, we argue that our methodology would be effective for documenting collective activity in almost any mathematics course in which there is genuine argumentation. We conjecture that the methodology also would be useful in science classes when there is genuine argumentation. We do not think, however, that the methodology we describe would be useful in classroom environments where there is no genuine argumentation (e.g., a classroom in which the teacher dominates the discourse).

Another type of usefulness of the methodology extends to the instructional innovators involved in the original design research study. In particular, it has been our experience that analyses of classroom mathematical practices provide new inspirations for revising and refining the instructional theories underpinning the intervention (e.g., see Yackel et al., 2003). Classroom mathematical practices provide an account of the quality of the mathematical experience of the community, as well as of the processes (i.e., argumentation, tools, gestures, etc.) that support their emergence. In addition, Cobb (2003) argues that a classroom mathematical practice analysis describes the evolution of the mathematical content as experienced and created by the participants. Thus, the documentation of the collective activity can ultimately impact teachers and students not involved in the original intervention but who benefit from the research-based, instructional interventions. The benefit is seen as teachers attempt to adapt the instructional theory, comprised primarily of documented mathematical practices, to create their own instructional environments that are commensurable with the initial classes.

We turn next to the issue of the reliability or trustworthiness of the methodology for documenting collective activity. Our method is made reliable by the fact that we employed an interactive approach to reliability (Blumer, 1969) in which two, and sometimes three, researchers conducted all three phases of the method. For example, in Phase One, we each created argumentation schemes and then compared and defended our schemes with each other, ending ultimately with an agreed-upon set of schemes. Multiple viewpoints are crucial for deciding which interpretation holds the most viability, especially in cases where conflicting interpretations are involved (Lincoln & Guba, 1985). A complementary approach to reliability is to seek quantitative measures of interrater reliability.

Lincoln and Guba (1985) also argue that finding consistencies with other research strengthens reliability. Our methodology grew out of the work of other researchers who
were seeking to establish methods for documenting the social contexts of classrooms (Cobb & Yackel, 1996). We also drew on Yackel (1997), Krummheuer (1995), and Toulmin (1969) to create our criteria for documenting collective activity and found overlap in our methodology with the work of Forman et al. (1998). We argue that trustworthiness is strengthened in our methodology by prolonged engagement with the subjects (from several weeks to the entire semester), multiple observations (analyzing every whole-class argumentation in every class period), triangulation of sources (videos, artifacts, field notes, interviews, etc.), and member checking (the teacher was one of the researchers involved in the analysis in the differential equations study).

Finally, we take a step back and examine the methodology as a legitimate way of assessing students’ learning. In this chapter, we used Toulmin’s (1969) model to analyze the arguments that students create during mathematics class. Mislevy (2003) argues that Toulmin’s scheme can be used in a broader way—namely, to judge the effectiveness of an educational researcher’s methodology for assessing learning. Therefore, we employ Toulmin’s scheme at a metalevel to examine the three-phase method for assessing collective learning in order to determine the legitimacy of our means of assessment. As shown in Figure 10.8, Toulmin’s scheme of data, claim, warrant, and backing is used to structure the argument that the method itself makes.

The data for the method are the argumentation log from Phase One and the patterns that emerge from Phase Two of the analysis. The claim or conclusion is the collective activity; that is, all the as-if-shared ways of reasoning. The warrant, the logic that allows us to make these claims from the data, is the two criteria for determining when particular ideas begin to function as if shared. In other words, these two criteria function as the license we use to go from our raw argumentation data to our conclusions about what ideas functioned as if shared in the classroom. Finally, we see three types of backing that validate the core of the method. At a theoretical level, knowing is inseparable from

![Figure 10.8 The Structure of the Methodology.](#)
discourse, and, therefore, argumentations are a valid way of discerning learning. This theoretical position is well developed by, among others, Toulmin (1969), Wittgenstein (1958), and, more recently, Sfard and Kieran (2001). A second backing for the core of the method is the fact that the method has been useful in a variety of content and grade level domains. A third backing, one that is emerging still for us, is the comparability of collective activity across several instantiations of the same instructional intervention with different teachers.

The analysis of the structure of the method for documenting collective activity provides what Kelly (2004) refers to as the “argumentative grammar” for the method. Kelly explains that

an argumentative grammar is the logic that guides the use of a method and that supports reasoning about its data. It supplies the logos (reason, rationale) in the methodology (method + logos) and is the basis for the warrant for the claims that arise.

(2004: 118)

Thus, the structure detailed in Figure 10.8 begins to move the three-phase approach from method to methodology. Design research studies have the potential to offer the field new research methods. These new research methods, such as the one we developed in this chapter, will increase their currency when accompanied by an argumentative grammar.

Conclusions

In the previous sections, we outlined a three-phase, methodological approach for documenting the collective activity of a classroom community of learners. Central to the methodology are a systematic use of Toulmin’s (1969) argumentation scheme over extended classroom lessons and the application of well-defined criteria for determining the normative ways of reasoning at the classroom level. We stress that the collection of classroom mathematical practices that result from such an analysis depicts the evolving mathematical activity of a community, which is potentially different from the intellectual achievement of each individual in the classroom. Documenting the collective activity of a classroom is significant in its own right because it offers insight into the quality of the students’ learning environment, an environment in which students are active agents in the construction of understandings. Indeed, Freudenthal (1973) argued that mathematics should be a human activity first and foremost. The methodological approach that we developed in this chapter offers a way to document mathematics as a human activity—an activity whose experiential richness can be traced systematically through genuine argumentation.

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Notes

1 In conversations, individuals negotiate meaning through an exchange of words until, at some point, they come to a mutual agreement about the meaning of an idea. When an agreement is struck (most often implicitly) among individuals so that the conversation can progress, we say that it is as if the individuals share the idea. We do not make the stronger claim that the individuals share the idea, but that they act as if they share the idea in order to proceed with a different or more elaborate argument. Other researchers have used the term “taken-as-shared” to refer to this phenomenon.

2 In practice, the most difficult contributions to categorize are the distinctions between warrants and backings. In our experience to date, distinguishing between these two, although difficult, is not critical in order to use this methodology. The two criteria that we developed for deciding when an idea functions as if shared tend not to necessitate distinguishing between the warrants and the backings. The first criterion involves the supporting statements (the warrants and backings) dropping out of students’ argumentations. Because they both drop out, it is not crucial to have made the distinction between the two. For the second criterion, we have seen conclusions typically shift function in subsequent argumentations and serve as data for more elaborate support (warrants and backings). We have not seen warrants or backings shift function in subsequent argumentations, and therefore, the distinction between the two is not paramount. We are exploring ways in which making the distinction between warrants and backings can have implications for the methodology.

3 The fact that backings appeared infrequently in students’ argumentations is consistent with Toulmin’s (1969) theory in which he states that backings often are not required to continue an argumentation. A person does not have the time to give the backing for every argument he or she makes. If backings were required, a conclusion might never be reached.

4 An autonomous differential equation is one that does not depend explicitly on the independent variable, usually time. For example, dP/dt = 2P – t is an autonomous differential equation, whereas dP/dt = 2P + t is not. In general, an autonomous differential equation has the form dP/dt = f(P). Moreover, because autonomous differential equations do not depend explicitly on time, the slopes of solutions to such equations in the t–P plane are invariant along the t axis. For example, the slope of the function P(t) = 20e^t at any particular P value is the same as the slope of the graph of the function P(t) = 10e^t at that same P value. As a consequence, the graph of the functions P(t) = 10e^t and P(t) = 20e^t is horizontal shifts of each other along the t axis. The reader may recall that, unlike algebraic equations that have numbers as solutions, solutions to a differential equation are functions. For example, the function P(t) = 10e^t is a solution to the differential equation dP/dt = 3P because the derivative of P(t) is 3*10e^t, which is the same as 3*P(t); that is, the function P(t) = 10e^t satisfies the equation dP/dt = 3P. Similarly, the function P(t) = 20e^t is a solution to dP/dt = 3P. In general, P(t) = ke^t is a solution to dP/dt = 3P for any real number k.

5 A slope field for a differential equation dy/dx = f(t,y) is plotted by selecting evenly-spaced points t1, t2, ..., tn along the t axis and a collection of evenly-spaced points y1, y2, ..., yn along the y axis; at each point (ti, yi), a small line with a slope f(ti, yi) is drawn. Slope fields offer a way of sketching graphs of solutions to differential equations, even when the analytic form of the solutions is unknown or unobtainable.

References


