INTRODUCTION

In 1973, Stephen Erlwanger published a groundbreaking case study of one sixth-grade student’s mathematical thinking. “Benny’s conceptions of rules and answers in IPI [Individually Prescribed Instruction] Mathematics,” or “Benny” as it came to be called, analyzed clinical interviews to reveal the “rules” Benny had developed to operate on fractions and decimals. Benny was a successful student in IPI, a curriculum in which students individually progress through sequentially ordered behavioral objectives via continuous cycles of assessment and feedback. Erlwanger showed that while Benny demonstrated mastery on the IPI assessments, he had little conceptual understanding of fractions and decimals. Perhaps more importantly, he also argued that Benny’s “rules” could be seen as a sensible effort of the learner to construct meaning out of instructional experiences that made little mathematical sense.

This article was important in several ways. It catalyzed the articulation and development of the constructivist perspective in mathematics education research. “Benny” was a rebuke towards prevailing behaviorist conceptualizations of and methodological approaches to mathematics learning. It was also an existence proof that alternative paradigms for mathematics education research, grounded in cognitive science and developmental psychology, could reveal how students actually think mathematically. In a broader sense, it raised questions that are still fundamental to inquiry into mathematics learning: What constitutes mathematical knowledge or knowing? What is mathematical learning? How is mathematics learned? How should mathematical knowing and learning be studied?

Since “Benny” the field has revisited these questions many times. For example, radical constructivists called into question basic ontological and epistemological assumptions of mathematical thinking and learning—that mathematics exists outside of the learner’s active construction of it and that learning mathematics is the acquisition of an objective representation of that external reality. Cognitive science has focused the field’s attention on the nature, construction, and use of mentally represented mathematical knowledge, and is increasingly studying these processes of learning in situ, in
addition to its historical focus in the laboratory. Work in anthropology, sociology, and cultural psychology broadened the field’s notions of who does mathematics, where and how, thereby opening up avenues of inquiry into mathematical activity as situated in cultural practices. Increasing recognition of the diversity of students in schools today has begun to draw our attention to issues of identity in mathematics learning, particularly as it relates to gender, race, culture, and language. Social psychological research on factors shaping students’ engagement and motivation has recently focused greater attention on the social and emotional aspects of mathematical engagement and learning. The resultant theoretical, methodological and moral considerations are reshaping our understandings of what constitutes mathematics, mathematical thinking and mathematics learning.

Mathematics education research today is diverse in its foci of inquiry, methodological approaches, and epistemological commitments, reflecting the willingness of the field to draw upon theoretical and methodological resources from other disciplines within and outside of educational research. Each of the aforementioned examples, grounded in different theoretical perspectives, can be seen as a thread in the fabric of research on mathematics learning. Tensions and even contradictions exist within this fabric, yet each line of research seeks to shed light on the same questions raised by Erlwanger’s (1973) work, questions that reflect the most fundamental theoretical issues in research on mathematics learning.

In this chapter, we highlight several of the threads that take up timely yet enduringly important questions for mathematics education. In the next section, we briefly discuss the intellectual grounds of research in mathematics learning during the last several decades, describing the predominant theoretical perspectives that have shaped and continue to shape research on mathematics learning. That is followed by syntheses of five current threads of research on mathematics learning: research on the structures and processes of mathematical cognition; research on the role of discourse and language on mathematics learning; research addressing identity and mathematics learning; research on the psycho-social aspects of mathematics learning; and research in neuroscience relevant to mathematics learning. We conclude with thoughts on future directions for research.

THEORETICAL FRAMEWORK

Reflective of the broader intellectual history of inquiry into thinking and learning, the field of mathematics education research over its history has been predominantly influenced by three theoretical perspectives that take contrasting, perhaps divergent, views: the behaviorist perspective, the cognitive/constructivist perspective, and the situative/sociocultural perspective. Of course, any categorization of a landscape as complex as theories of cognition and learning is necessarily simplistic and limited. We offer this formulation to reflect discourse about the dominant theoretical traditions in mathematics education (Gutierrez & Boero, 2006; Lerman, 2000) as well as for reasons of brevity.

In mathematics education, behaviorism dominated research on learning through the first half of the 20th century. Research in the behaviorist tradition, particularly that of Skinner (1953), conceptualized mathematical behavior as observable skills that arise as responses to stimuli, and conceptualized mathematical learning as the formation and strengthening of those stimulus-response associations. Although strictly behaviorist accounts of mathematical learning all but disappeared in the later 20th century
(though neo-behaviorist theory continues to investigate conditions for learning and, increasingly, biological bases of learning; Mowrer & Klein, 2000), a continued legacy of this line of work has been the analysis of mathematical tasks into component subskills. The hierarchical organization of the components constitutes the sequence of skills to be learned in order to achieve mastery of the task (Gagné, 1965). Researchers adopting methods from cognitive science later reconceptualized this analysis of mathematical tasks through cognitive task analysis, and component cognitive processes underlying the completion of mathematical tasks became the focus of investigation.

Beginning in the 1960s, due to the influences of developmental psychology and insights from the emerging fields of information processing and cognitive psychology that highlighted the limitations of the behaviorist paradigm, research on mathematics learning shifted toward inquiry into the mental bases of mathematical thinking and learning. Cognitive research on mathematics learning has proved highly productive and valuable to the field’s understandings of the organization of the knower’s knowledge of specific mathematical content, problem-solving and metacognitive processes, the role of internal and external representations in mathematical sense-making and learning, and the reorganization of knowledge structures in conceptual growth, among other crucially important aspects of mathematical cognition. A key cognitive paradigm in mathematics education research is constructivism, particularly as it emerged from learning and developmental theories of Piaget. This approach assumes that knowledge emerges as an individual’s cognitive structures are constructed and refined through encounters and interaction with his or her physical and social environment. The impact of constructivist perspectives has been to place active exploration, inquiry, and modeling activities at the heart of mathematical learning, in contrast to the previous and sometimes exclusive emphasis on the acquisition of computational and algebraic rules and skills.

As cognitive/constructivist research on mathematical thinking and learning has established itself as the dominant paradigm in mathematics education in recent decades, it has been critiqued from alternative epistemological perspectives, particularly sociocultural perspectives including situated learning and situated cognition. Under this view, mathematical knowledge is situated—located in particular forms of experience—and involves “competence in life settings” rather than or in addition to mathematically specific psychological or mental structures (Lerman, 2000, p. 26). Research on mathematics learning in the situated/sociocultural tradition spans research on out-of-school mathematical practices (e.g., Saxe, 1991) to accounts of mathematical activity as coordinated productions of talk, gesture and inscription (e.g., Hall & Stevens, 1995) to analyses of the practices of mathematics classrooms (e.g., Cobb, Stephan, McLain, & Gravemeijer, 2001). Although some researchers in situated cognition largely rejected purely cognitive accounts of learning and knowing, social constructivism (influenced by Vygotsky, 1978) extended cognitive constructivist accounts by analytically locating individuals in relation to others, objects, activities, settings and histories in conceptualizations of learning.

Much of the current research on mathematics learning reflects influences from these major perspectives, particularly the cognitive/constructivist and the situated/sociocultural. The first three research threads presented in this chapter are situated within these broader traditions. The first section focuses on current directions in research on mathematics thinking and learning in the cognitivist tradition. The second addresses several important conceptualizations of the role of discourse and language in mathematics learning. The third section takes up the construct of identity and synthesizes
literature concerned with the nature and role of identity in mathematics learning. The fourth section addresses research on the psycho-social aspects of mathematics learning, particularly on the concepts of grit and growth mindset. The final thread presents a relatively new yet important field of inquiry, research in neuroscience relevant for mathematics learning.

These threads do not, of course, provide a comprehensive survey of current research on mathematics learning—that is beyond the scope of this chapter (see handbooks on mathematics education research for more comprehensive treatments: e.g., Bishop, Clements, Keitel, Kilpatrick, & Leung, 2003; Lester, 2007). There are many viable ways to structure such a chapter: historically, by mathematical domains (e.g., arithmetic, geometry), by processes (e.g., representation, problem-solving), or by age or grade level. Our choice to foreground theoretical paradigms is grounded in our conviction that advances in the field’s understandings depend on explicit and robust theoretical formulations of the phenomena of mathematical knowing and learning. Additionally, we believe that the diversity of the theoretical perspectives represented in research on mathematics learning is a strength, despite or perhaps due to the tensions that this diversity naturally raises. We hope that our choices capture that diversity.

CURRENT TRENDS AND ISSUES IN RESEARCH ON MATHEMATICS LEARNING

Research on the Organization and Processes of Mathematical Cognition

Historically, research in the nature of mathematical knowledge and learning has been dominated by a variety of cognitive epistemologies, reflecting mathematics education’s strong historical roots in educational and cognitive psychology, and later developments of cognitive modeling with the rise of computational metaphors of cognitive processes. Throughout its history, mathematics education research has attended both to the acquisition of mathematical rules and procedures as well as the development of mathematical meaning and sense-making. A thorough historical consideration would include the early and mid-20th-century work of Judd (1908), Wertheimer (1945), and Brownell (1948), for example, each of whom highlighted the importance of meaningful mathematical learning, largely in contrast to the then-dominant connectionist and behaviorist approaches of Thorndike (1913) and Skinner (1953) that focused on the acquisition of computational skills.

Cognitive researchers have traditionally focused on identifying and typifying knowledge and mental functions such as memory, declarative and procedural knowledge, frames, schemata, routines, heuristics, and the nature of mental (internal) representations. The unit of analysis is the thinking individual, or more specifically, the individual’s mental constructs and processes. Such approaches have been used to study and model arithmetic procedures, mental arithmetic, algebraic skills, and the solutions of word problems throughout the mathematics curriculum.

One reason for the close relation between cognitive research and mathematics has been the perception that the logical and structural constraints of mathematical activity make it perhaps more readily modeled than other cognitive processes. As research has advanced, however, particularly to better accommodate higher levels of mathematical thinking, complex problem solving, conceptual understanding, and sense-making, the cognitive study of mathematical learning has expanded. Schoenfeld (2006) identified a consensus among cognitive researchers that a thorough analysis of an individual’s
mathematical performance requires attention to that individual’s knowledge base, available problem-solving strategies or heuristics, metacognitive processes (particularly self-regulation and monitoring), beliefs and affective variables, and acquired practices typical of the classroom or mathematical community.

Research attention to conceptual understanding and sense-making has examined students’ abilities to invent or discover their own strategies and solutions to mathematical problems when given sufficient opportunities and support. The Cognitively Guided Instruction program (CGI, Fennema et al., 1996; Fennema, Franke, Carpenter, & Carey, 1993) remains a preeminent example of how mathematics teaching and learning in the early grades benefits when teachers are made aware of and attend to students’ ways of thinking about mathematics problems. Fennema et al. (1996) demonstrated that teachers’ learning about students’ mathematical thinking is related to changes in the teachers’ instructional practices and subsequent student achievement. Students showed higher achievement in concepts and problem solving without diminishing computational performance. Cognitive research on student thinking has further inspired, at least in part, the development and dissemination of mathematics reform curricula for middle- and high-school mathematics (Martin et al., 2001), calculus (Darken, Wynegar, & Kuhn, 2000), and differential equations (Rasmussen & Kwon, 2007).

Among cognitive researchers behind mathematics education reform efforts, constructivism has played a dominant role since the late 1980s. Rooted in the research and developmental theories of Piaget (1954, 1970), constructivism posits that individuals generate knowledge and understanding through their experiences and reflection. Thus, knowledge arises from the active mental processes of the learner as new experiences are interpreted through existing knowledge structures. Prior knowledge may be called upon to assimilate experience as familiar, or restructured to accommodate new experience.

Sociocultural critiques challenged cognitive researchers to attend more directly to the role of the sociocultural context of the learner in mathematical thinking. Lave (1988), in particular, argued the inadequacy of then-dominant cognitive approaches to address mathematical learning and transfer by demonstrating apparent discrepancies between adult shoppers successfully navigating price comparisons while grocery shopping, yet performing less successfully on what Lave argued to be equivalent arithmetic tasks presented on paper in standard arithmetic notation. Lave asserted that such evidence revealed the dynamic relation between learner, activity and context, and that mathematics learned in the classroom was not transferred to other contexts as cognitive researchers expected. Advances in cognitive research methods subsequently prompted some cognitive researchers to expand the types of mathematical knowledge and activity studied, motivating them toward moving cognitive studies into classrooms. Jaworski (1994) examined the tensions inherent in studying students as individual, cognizing subjects in the social context of the classroom. Cobb and Yackel (1996) and Izsák, Tillema, and Tunc-Pekkan (2008) have pursued an approach to classroom research that coordinates individual psychological analysis with interactionist analyses of classroom activity and discourse.

Cognitive research has supported the study of mathematics learning through a variety of approaches, from information processing models (e.g., Singley & Anderson, 1989), to Piagetian constructivism (Confrey & Kazak, 2006), to radical constructivism (Steffe & Thompson, 2000a; von Glasersfeld, 1995). It has been highly successful in modeling differences between novices and experts, mapping the complexities of
learning transfer, and revealing learning and developmental trajectories of children in mathematics and in other domains (see, for example, Bransford, Brown, & Cocking, 1999, or Kilpatrick, Swafford & Findell, 2001). A number of mathematics curricula, informed largely (though not exclusively) by cognitive research, have been implemented at a large scale and have shown signs of success in improving students’ conceptual learning without diminishing computational skills (Schoenfeld, 2002).

Despite much success, cognitive research in mathematics learning has considerable room for growth. We identify two specific tasks for cognitive researchers in mathematical thinking and learning that have begun to receive more attention yet remain essential challenges for future research. First, cognitive researchers must develop research methods to take on the challenge of studying mathematical thinking and learning in natural settings. Careful study of students in classroom settings, particularly as students are actively engaged in mathematical activities involving collaboration in problem solving, debates of mathematical ideas, verbal communication of mathematical thinking, and interactions with a teacher can open windows into aspects of mathematical knowledge and learning not observable through laboratory settings or clinical interviews.

An example of primarily cognitive research that has moved into actual classrooms has been that of Rasmussen and colleagues. Rasmussen (2001) began with clinical interviews of undergraduate students studying differential equations, and he outlined a framework for understanding significant conceptual difficulties that students face when learning the subject. He subsequently developed the Inquiry Oriented Differential Equations Curriculum (Rasmussen & Kwon, 2007). Experimental studies suggest that the curriculum, enacted through small group work and whole class discussion, supports students’ increased development and retention of conceptual understanding of differential equations, while matching the procedural competence of traditionally taught comparison groups (Kwon, Allen, & Rasmussen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burch, 2006). These efforts toward rethinking the teaching and learning of undergraduate differential equations have since inspired similar research and curriculum design in other undergraduate mathematics courses including linear algebra (Wawro, Rasmussen, Zandieh, & Larson, 2013) and abstract algebra (Larsen & Johnson, 2013). Taken as a whole, this still-growing body of research has entailed both cognitive and sociocultural research methods, using clinical interviews and real-world classroom settings, to reveal aspects of individual and cooperative learning that have direct implications for classroom instruction.

A second essential task of contemporary cognitive researchers must be to continue probing more deeply the nature of mathematical knowledge and learning in themselves. Some who adopt a radical constructivist perspective preserve particular aspects of Piaget’s work concerning the structure of knowledge and processes of learning (e.g., Steffe & Thompson, 2000a; 2000b). Primary operative cognitive structures are understood to be mental schemes, rather carefully delineated theoretical structures that support goal directed activity. Learning mechanisms are based on Piaget’s notions of accommodation, interiorization, and reflective abstraction. In contrast, an emerging epistemological approach, arising from cognitive research in physics education, is diSessa’s (1993) knowledge in pieces perspective. Although a knowledge-in-pieces perspective shares much with radical constructivism in its constructivist roots and its attention to fine-grained analyses of students’ talk, gesture, and inscriptions, it makes no assumptions about the role or existence of mental schemes or other components of Piaget’s framework. Developed to examine how intuitive or naïve knowledge progresses toward expertise, a knowledge-in-pieces epistemology posits that some forms
of conceptual knowledge are best modeled as complex systems of knowledge consisting of many more fundamental knowledge elements, often sensitive to context, in that different knowledge elements may be called upon in different contextual circumstances (diSessa, 1993; Wagner, 2006). Knowledge-in-pieces researchers assume that part of the research task is to investigate and describe the nature of different knowledge elements and structures, as well as mechanisms of learning. This approach has shown promise for hypothesizing specific types of knowledge constructs such as phenomenological primitives (diSessa, 1993), coordination classes (diSessa & Sherin, 1998), concept projections (diSessa & Wagner, 2005), and meta-representational knowledge (diSessa, Hammer, Sherin, & Kolpakowski, 1991); and it uses the context-sensitivity of such knowledge elements to explain and interpret knowledge transfer (Wagner, 2006). Although these approaches had been familiar to cognitive researchers in physics education, they have only recently emerged within the mathematics education research community (e.g., Izsák, 2005; Jacobson & Izsák, 2014; Pratt & Noss, 2010; Wagner, 2006; 2010).

One potentially promising line of research that has emerged from a knowledge-in-pieces approach concerns the role of meta-representational knowledge (diSessa et al., 1991). Meta-representational knowledge refers to knowledge by which individuals create or evaluate the suitability of representations within problem-solving contexts. diSessa et al. (1991) introduced this work in the context of students’ learning about Cartesian graphs, and Izsák (2003) extended it in a clinical interview study of pairs of eighth-grade students learning to model the behavior of a physical device with algebra. Using fine-grained analysis of students’ extended problem-solving sessions, Izsák revealed how students coordinated meta-representational knowledge with elements of algebraic knowledge as their abilities to algebraically model the given situation grew and changed. Izsák, Caglayan, and Olive (2009) focused the same analytic lens on both the students and the teacher of an eighth-grade algebra classroom, demonstrating that attention to meta-representational knowledge could reveal different perspectives on the part of the teacher and the students that impeded learning and instruction. In this way, the work of Izsák and his colleagues suggests that a knowledge-in-pieces epistemology can reveal aspects of knowledge not evident through other analytical lenses, yet demonstrate very practical consequences in a typical mathematics classroom. The extent to which such work can inform instructional practice remains a significant question for further research.

Research on the Role of Discourse and Language in Mathematics Learning

Reflecting the “turn to discourse” in the social sciences in recent decades, research in mathematics education is undergoing its own “linguistic turn” (Lerman, 2004), with increasing attention to the role that language plays in mathematical activity and learning. Much has been made in discussions about mathematics education reform regarding mathematical discourse not only as an object of study (i.e., What is the nature of the discourse of the mathematical community? What are the structural and semantic characteristics of language use in mathematical activity?), but also as a desired outcome of mathematical learning and a key component of a classroom environment that promotes mathematical learning. The ubiquity of references to mathematical discourse in current educational discourse to growing recognition that language and mathematics learning are inextricably intertwined, and that understanding mathematical learning, assessing
students’ mathematical learning, and designing mathematical learning environments requires examining the role of language in mathematical activity. Research in mathematics education that focuses on language varies widely in terms of foci and methods of analysis, as does the work in linguistics, discourse and conversation analysis, ethnomethodology, and other related fields from which mathematics education researchers draw; this diversity is both a source of fresh insights and a potential obstacle to coherence and depth. Since representing this body of work in its entirety is beyond the scope of this section, we have chosen to highlight four areas—two relatively mature and two emerging lines of work—that represent distinctly different points in the landscape.

The first area of research relevant to the role of discourse and mathematics learning focuses on mathematics as a discursive and textual disciplinary practice. A key starting point for this work is the notion that mathematics is a language—a semiotic system with particular syntactic and semantic entailments that differ from those of “everyday” language. Halliday (1978) described this “mathematics register” as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings” (p. 195). Following Halliday, researchers in systemic functional linguistics have characterized the linguistic features of the mathematics register. The mathematics register is highly technical, including particular uses of everyday words (e.g., set, order), specialist vocabulary that is specifically mathematical (e.g., sine, equation), composite words and expressions with mathematical meanings distinct from everyday usage (e.g., square root, differential operator), precise and specific meanings of linguistic features (e.g., conjunctions such as if, therefore; directives such as assume; modifiers such as clearly, obvious), highly dense nominal structures (e.g., The sum of the squares of two sides of a right triangle), and grammatical constructions that imply logical relationships (e.g., if . . . then: Lemke, 1990; O’Halloran, 2000; Pimm, 1987).

From this perspective, mathematical learning involves the appropriation of the vocabulary and the “styles of meanings and models of argument” (Halliday, 1978, p. 195) that characterize this system. Developing mathematical fluency therefore requires recognizing and shifting between everyday and mathematical registers. Word meaning and grammatical structures are key concerns here, in particular when the same words or structures exist in both mathematical and everyday registers and students must learn to negotiate their multiple, situationally dependent meanings and uses. Research on students’ appropriation of the mathematics register has included attention to what Pimm (1987) has called “semantic contamination” (p. 88), in which word meanings in the everyday register are used in attempts to make sense of new mathematical language. For example, Cornu’s (1981) interview study comparing high school and university mathematics students’ notions of limit revealed that students’ difficulties with limit were due, in part, to their “spontaneous models” for describing limiting behavior, particularly their use of the expression tends to.

While this area of research has provided insights into students’ difficulties appropriating the mathematics register, some have noted that its focus on an idealized form of “mathematics as language” is limiting and have suggested an alternate conceptualization of mathematical discourse that foregrounds the notion of mathematics as social practice (e.g., Moschkovich, 2002). Research in this second area of work, informed by sociocultural theories of cognition and learning, assumes that mathematics is constituted by complex social practices (which are crucially discursive but not limited to specific categories of vocabulary and grammatical structure) and defines mathematical learning as socialization into the practices of the mathematics community, including conjecturing, explaining, justifying, representing, and evaluating mathematical
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ideas, claims and solutions (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Moschkovich, 2003; O’Connor, 1998).

Drawing upon work in sociolinguistics on language socialization (e.g., Schieffelin & Ochs, 1986) and sociocultural participationist theories of cognition (Sfard, 1998), one productive line of inquiry examines how teacher-mediated classroom participation structures create opportunities for student participation in and learning of school-based mathematical practices. For example, several researchers have examined how the discursive move of *revoicing*—the repetition, restatement or reformulation of a student’s utterance by another classroom participant (usually the teacher)—facilitates student engagement and appropriation of mathematical discourse practices (Enyedy et al., 2008; Forman et al., 1998; O’Connor & Michaels, 1993). In one influential study, Forman et al. (1998) drew upon analytical tools in sociolinguistics and rhetoric to investigate the discursive structure and propositional content of mathematical argumentation in an urban middle school classroom. The study showed how teachers’ revoicing moves orchestrate classroom discussion in ways that recruit and ratify students’ contributions, position students as authors and legitimizers of knowledge, align students and their contributions with one another and with the mathematical content, expand and refine students’ contributions into more conventional forms of mathematical discourse, and socialize them into mathematical discourse practices such as conjecturing, justifying, argumentation, and evaluating ideas. While this work focuses on teaching, it has contributed to understandings of student learning by demonstrating how particular participation structures and discursive moves in classroom interactions support students’ appropriation of mathematical practices.

A third area of research relevant to the role of discourse in mathematics learning focuses on how ordinary language resources—features or structures of ordinary talk as identified in linguistics, sociolinguistics, pragmatics, ethnomethodology and conversation analysis among other disciplines—are employed by interactants in mathematical activity. Researchers in this area view mathematical activity as discursive and interactional accomplishments and take the perspective that doers and learners of mathematics routinely and systematically deploy features of talk-in-interaction to accomplish mathematical work (Stevens & Hall, 1998). The range of this work is broad; to illustrate, we highlight one line of inquiry focused on pronoun use in mathematical talk. Rowland (1992) first demonstrated how the pronoun “it” functions as a *conceptual deictic*—a linguistic pointer to (mathematical) concepts or ideas which are as yet unnamed in the course of the interaction and whose meanings are to be inferred by the context of the utterance. For example, he shows how one child, while working on a multiplication task, uses the pronoun “it” to name the notion of commutativity—a concept that is useful for making sense of the task but that she has not formally encountered. Her use of this deictic pointer allows her to “share and discuss a concept which [she] possesses as a meaningful abstraction, yet is unable to name” (p. 47), in this way serving as a resource for mathematical sense-making. While his study was perhaps limited by the focus on a single, somewhat unique case, Rowland’s use of linguistic methods of analysis to illuminate mathematical functions of elements of language in micro-interaction suggests that close examination of structures of talk can reveal how more complex mathematical processes are accomplished in interaction and how those structures can be leveraged in the learning of increasingly sophisticated mathematical activity. For example, Rowland’s subsequent work (1999, 2000) as well as later analyses of students’ mathematical talk in problem-solving or sense-making activities (Edwards, Farlow, Liang, & Hall, 2009; Jurow, 2004) show how structures of talk (such
Finally, studies of discourse and language have illuminated dynamics of power and ideology manifest through language in mathematics learning and teaching contexts. These studies have shown how the discourse of mathematics classrooms (including "reform" mathematics classrooms) are often reflective of the practices of dominant groups. These classroom discourse practices can alienate or exclude some students, in particular those from cultural groups or with class backgrounds whose discourse practices differ from the dominant discourses in their school contexts (e.g., Zevenbergen, 2000). For example, Lubienski (2000a, 2000b) examined the influence of class on students’ experiences in a middle grades reform-based mathematics classroom. Drawing upon multiple sources of qualitative data, including surveys, interviews, observations, and student written work, she found that lower SES and higher SES students responded differently to and had different understandings and opinions of several reform-oriented instructional practices, including the use of open-ended problems and discussion-intensive classroom pedagogy. In particular, the discursive practices of the reform classroom seemed more closely aligned with middle class students’ ways of learning and knowing than those of lower and working class students, raising important questions about whether and how reforms intended to serve all students support students with different class backgrounds. In another example, Wagner and Herbel-Eisenmann (2008) used critical discourse analysis (Fairclough, 1989) to analyze a large corpus of classroom transcripts to examine how a specific discursive move—the use of the word “just,” common in mathematics classroom talk—invited or suppressed student dialogue. Their analysis shows how teachers’ use of the term positioned students in relation to mathematics, often shutting down dialogue, and how it seemed to shape students’ perceptions of their authority and legitimacy to take mathematical initiative. Like Walkerdine and Lucey’s seminal work on early childhood mathematical development (Walkerdine, 1988; Walkerdine & Lucey, 1989), these studies point to the power of language to exclude and oppress—a power that is often invisible to those exercising it—and the particular power that mathematics as a discourse holds as a form of cultural or linguistic capital (Bourdieu & Wacquant, 1992).

The diverse body of work pertaining to the role of discourse in mathematics learning has provided insight into the processes that constitute the doing and learning of mathematics as socioculturally situated and interactional activities. However, the relationships between mathematics as language, mathematical talk and interaction, and mathematical learning are complex; the field has only begun to identify and unpack key constructs and relations through which they can be understood. Consequently, these studies are often very specific and focused on single cases of a particular phenomenon, and, as such, can be limited in scope. In addition, across this field of work, there is often a lack of clarity in how key ideas are conceptualized, resulting in some theoretical incoherence. This is problematic not only for developing theory and building new knowledge, but also for those seeking to translate insights about the role of discourse in mathematics learning into meaningful contributions to practice.

**Research Addressing Identity and Mathematics Learning**

As sociocultural theories of learning have gained prominence in mathematics education research, the concept of identity has received increased attention from the field, both theoretically and empirically. In this body of work, different definitions and
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methods have been used to study identity, with the only broad area of agreement being that identity should be conceived as neither purely individual, nor purely social. The definitions currently in use have important empirical and theoretical implications, and open up different possibilities for research. Although there are many approaches we could discuss here (e.g., Castanheira, Green, Dixon, & Yeager, 2007; Cobb, Gresalfi & Hodge, 2009; Sfard & Prusak, 2005; Solomon, 2007; Walshaw, 2005), we limit ourselves to three definitions and analytic strategies for studying identity as it relates to mathematics learning. For each approach, we present the definition, and describe a representative study. We close this section by discussing the strengths and weaknesses of these various approaches to the study of mathematics learning.

Nasir and Hand (2008) use the term practice-linked identities to describe “identities that people come to take on, construct, and embrace that are linked to participation in particular social and cultural practices” (p. 147). Practice-linked identities are simultaneously social (reflecting the practice) and individual (integrated into an individual sense of self). In their analysis, they consider both the moment-to-moment shifting of identity (sometimes called positioning; Davies & Harré, 1990; Holland, Lachiotte, Skinner, & Cain, 2001) and longer-term trajectories of identity as a relatively stable sense of self (Erikson, 1968).

Nasir and Hand conducted ethnographic studies of two different practices—a high school basketball team, and a high school mathematics classroom—drawing on participant observation, ethnographic fieldnotes, and structured interviews with young African American men who participated in both of the practices. Their analysis considered how each of these practices afforded opportunities for participants to develop practice-linked identities. They found that the school and the team differed in the ways participants were offered: a) access to the domain itself (basketball or mathematics); b) integral roles in the practice; and c) opportunities for self-expression. The structure of the basketball team’s practice afforded deeper engagement (in all aspects of the sport, including mathematical aspects) than the mathematics classroom.

A second strand of research in this area considers the interplay between mathematics identity and racial identity, with a special focus on African American learners (Martin, 2000, 2006, 2007). A mathematics identity is the set of “dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives” (Martin, 2007, p. 150). These identities are shaped by one’s experiences, how one perceives oneself, and also how one is seen by others. One’s racial identity is one’s sense of self as a racialized being—one again, partially reflecting one’s own self-perceptions, as well as the perceptions of others. This concept of identity differs from the notion of practice-linked identities in that while practice-linked identities are conceived as being primarily local, the mathematics and racial identities are conceived as going beyond local contexts or practices to develop in a relatively enduring fashion over the life-span.

For his analysis of these forms of identity, Martin (2007) drew on lengthy interviews with adults and adolescents in which he asked participants to describe their life experiences with mathematics in and outside of school and to articulate their attitudes and beliefs about mathematics. These interviews were then analyzed thematically. Based on this analysis, Martin argued that although mathematical and racial identities are often considered to be separate, in fact they reflect two “intersecting realms of experience” (p. 146). Interview narratives showed that for the interview participants, all of whom were African American adults who grew up in the US, racial identities have influenced
mathematics identities, and vice versa. When faced with mathematical or educational challenges, interview subjects used aspects of their racial identities—specifically, ongoing struggles for racial justice—to help overcome the educational barriers they faced. Interview subjects argued that mathematics as a domain was often unwelcoming of or unimportant to African American people, and they spoke of the ways achievement in mathematics affected their racial identities as well. Through these interview studies, Martin found that successful African American students of mathematics (middle school students and adult learners alike) drew on their African American identities as sources of strength to support their learning.

In contrast with Martin’s long-term view of identity, several recent studies of identity in mathematics classrooms have investigated shifts in identity over very short time periods (DeJarnette & González, 2015; Esmonde & Langer-Osuna, 2013; Wood, 2013). Wood (2013) argues that studies that take a ‘macro’ view of identity gloss over the complexities and contradictions inherent in identity as it shifts from moment-to-moment. Instead, Wood defines micro-identity as “the position of a person in a moment of time” (p. 780), and presents a study of a single class period in which a student, Jakeel, took on three different micro-identities: mathematical explainer, mathematical student, and menial worker. These micro-identities differed in the degree to which they afforded meaningful opportunities to learn.

Studies of micro-identities have shown that these identities may be transient or they may “thicken” (Wortham, 2004) into a long-term identity. Thickened and relatively stable identities can still shift when the classroom context shifts—from whole class discussion to group work, for example, or from one group to another (Esmonde & Langer-Osuna, 2013). Even a change in task can prompt a shift in micro-identity (Esmonde, 2009).

These three distinct, yet related, ways of conceptualizing identity offer different possibilities for research and for educational practice. Wood (2013) argues that the micro time scale is important because all learning happens at the micro scale, and fluctuations in micro-identity can have significant impacts on opportunities to learn. Martin (2007) focuses instead on events that have a long-term impact on one’s identity and even life trajectory. Nasir and Hand (2008) combine the micro and the macro, and situate identity within a single collective practice (e.g., a single mathematics classroom vs. a life-long mathematical education). A persistent challenge for studies of identity (and educational studies, more broadly) has been to integrate our understanding of identity at the micro and macro time scales (Lemke, 2001).

These conceptions of identity are productive for our understanding of learning because they clarify the relationship between participation in collective practices and individual development. All three of these frameworks address how broad social structures shape opportunities for participation, and make space for particular kinds of individual and practice-linked identities. They try to address the question of how several students may sit in the same classroom and yet come away with completely different learning experiences.

A general weakness of research in this area is that, perhaps as a result of the proliferation of definitions and stances towards the concept of identity, it is difficult to compare results across studies to develop a coherent body of knowledge on this topic. Further, studies have rarely addressed the learning of specific mathematical topics or processes. For example, algebra and calculus seem to have a particular status in the public eye, so that success in algebra or calculus becomes symbolic of intelligence. It is unknown whether different mathematical topics have different affordances for identity development, though this is an intriguing possibility.
Research on Psycho-Social Aspects of Mathematics Learning

An upcoming area of research is the focus on the psycho-social aspects of mathematics learning, particularly on the concepts of grit (Duckworth, Peterson, Matthews, & Kelly, 2007) and growth mindset (Dweck, 2006). Grit is defined as the passion and perseverance towards long-term goals and this quality has been shown to be a strong predictor of various outcomes, including adolescents’ GPA, retention in West Point cadets, and success in the National Spelling Bee (Duckworth & Quinn, 2009). In addition to predicting individual differences in student achievement, grit has also found to be a positive predictor of teaching effectiveness, defined as increased academic performance of students, for novice teachers in low-income school districts (Robertson-Kraft & Duckworth, 2014) and Teach For America (Duckworth, Quinn, & Seligman, 2009). The authors even suggest using ‘grittiness’ as selection criteria for hiring new teachers. According to Duckworth et al. (2007), stamina is the key to the success. The authors claim that, “Whereas disappointment or boredom signals to others that it is time to change trajectory and cut losses, the gritty individual stays the course” (p. 1088).

One major critique of grit is that there is little information about how to develop or change one’s level of grit. Because the grit narrative focuses so much on the personal qualities of an individual, it has been criticized for blaming students for not succeeding (Socol, 2014). At present, there has been no published research about how to create learning environments that would promote students to be gritty. Moreover, another critique of grit is that the theory advocates that a student should persevere even when he or she views the task at hand as not interesting, relevant, nor meaningful. However, if a student decides to not be “gritty,” educators are much better served by creating contexts that address that.

Unlike grit, the research around growth mindset primarily focuses on the factors that contribute to the development of a growth mindset, including aspects that relate to the student lens as well as the learning context. Building on the theory of learned helplessness (Diener & Dweck, 1978; Dweck, 1975), Carol Dweck’s (2006) conception of a fixed mindset connects a learners’ beliefs about their own intelligence as being unchangeable with the behaviors observed in learned helplessness, such as a tendency to give up after failures or to avoid challenges. Students can have the knowledge and strategies to be successful and may have a desire to learn math, but they may not productively persist if they think they just can’t “cut it” in math class because they are not “math people.” On the opposite side of the spectrum, learners are said to be in a growth mindset when they hold the belief that it is possible to learn.

Blackwell, Trzesniewski, and Dweck (2007) studied the trajectories of 373 students during the transition from seventh grade to eighth grade. They observed that even though students with fixed and growth mindsets started with similar prior background mathematics achievement, students with a growth mindset started to have greater success in mathematics early in seventh grade and the gap between the groups continued to grow through the end of eighth grade. They observed that this divergence in mathematical achievement was mediated by three beliefs: the orientation towards learning goals, a belief that effort was essential for learning mathematics, and the use of mastery-oriented strategies when faced with a challenge.

The research frames these mindsets as malleable and influenced by the environment rather than enduring personality traits. Much of the research has focused on how to directly intervene on students’ mindsets by teaching students about the plasticity of the brain. These studies show that relatively brief, one-time interventions can shift
students to a growth mindset and have enduring effects on students’ mathematical achievement (e.g., Blackwell et al., 2007; Good, Aronson & Inzlicht, 2003).

Similar to the criticism lobbed at grit, the narrative around growth mindset may suggest that students’ views are invalid and that students should always radiate a growth mindset no matter what situational cues are present. For learners, messages that promote a fixed mindset are prevalent. For example, American cultural beliefs around education emphasize the importance of innate ability or being effortlessly “smart” (Rattan, Good, & Dweck, 2012), whereas other cultures focus on the role of effort in mathematics learning (Stevenson & Stigler, 1992). Additionally, teachers’ low perceptions of students’ ability have been shown to negatively impact students’ self-conceptions as mathematics learners (Upadyaya & Eccles, 2014a; 2014b).

There is emerging research focusing beyond shifting students’ conceptions of themselves from a fixed mindset to a growth mindset. Instead, insights from this line of research can inform classroom practice to create mathematics learning environments and learning opportunities that promote students’ growth mindsets.

**Detracking Mathematics Students**

Placing students into different tracks of mathematics messages the importance of ability rather than effort. Detracking provides opportunities for all mathematics students to grow and learn together and Boaler (2002, 2011) has found that mathematics students in untracked schools performed significantly higher than those students in schools with distinct tracks.

**Establish Norms for Help-Seeking**

In classrooms that emphasized that all students could succeed and promoted opportunities for collaboration between students, rather than competition, Walton, Cohen, Cwir, and Spencer (2012) observed higher retention rates and achievement.

**Reframe Failure**

Attempts to comfort learners with phrases like “Well, not everyone can be good at math” may instead lower a learner’s expectations for himself (Georgiou et al., 2002; Rattan et al., 2012). Studies suggest that messages that indicate that the class has a high standard and that the student can reach those standards with the support of the teacher and the class (Cohen, Steele, & Ross, 1999) are more effective at supporting growth mindset and achievement.

Although there is potential for research on the psycho-social aspects of learning to advance mathematics education, there is also a danger that the narrative could be used to heap the responsibility for learning and maintaining the desired mindset onto students. As this field develops, we hope to gain insights into how to better foster environments that support students’ beliefs that it is possible for them to learn mathematics.

**Research in Cognitive Neuroscience Relevant to Mathematics Learning**

The current bridge between cognitive neuroscience and education has been described as “one way” (Turner, 2011) in that neuroscience research has been translated to inform education; however, there have been fewer neuroscience studies directly evaluating the
effects of different approaches to learning. Much of neuroscientific research is not concerned with learning; instead, neuroscience research excels at studying cognition, or more properly, brain structure and function. This research is based on an implicit definition of learning as a change in either the quantity or speed of brain function (i.e., reduced or increased activation in a particular part of the brain), or a shift in the network of brain areas that work to perform a particular task (i.e., a qualitative change in terms of which areas of the brain are active during the task).

Learning has often been studied by proxy—for example, by comparing the brain function of younger vs. older people (Rivera, Reiss, Eckert, & Menon, 2005) or comparing people researchers call “prodigies” vs. people they categorize as “normal” (Pesenti et al., 2001). In both of the studies cited here, researchers found that increasing expertise in particular mathematical tasks was associated with a shift in brain function; when comparing the brain function of the two groups, different areas of the brain were activated, or similar areas were activated but to greater or lesser degrees.

Another approach to investigating learning has been to train a seemingly homogenous group of participants (most often adults) in particular tasks, and then compare brain function on the trained vs. untrained tasks. For example, in one study, participants memorized a set of multiplication facts, and subsequent fMRI scans demonstrated that different areas of the brain were activated for facts that were recalled versus facts that were untrained and were, therefore, presumably computed on the spot (Ischebeck et al., 2009).

Whichever approach one takes, the question about what neuroscientific research can tell us about the more complex processes of teaching or schooling remains, as yet, unanswered (Bruer, 1997). The tasks that have been studied are usually broken down into basic components, and simplified such that participants can expect to solve each in a few seconds. The selection of tasks is further limited because head movements (e.g., moving the lips for speech) can interfere with imaging techniques. A typical experiment involves asking a participant to perform the same (usually quite basic) task dozens, sometimes hundreds, of times, while their brain function is being scanned. These experimental trials are usually preceded by dozens of trials in which the scanner is not used; this is to habituate the participant to the task so the images of brain function capture only (or mostly) the functions relevant to the task itself (e.g., adding one-digit numbers) rather than functions related to the technology or the context (e.g., which button to push).

Despite these methodological limitations, the popularity of neuroscience has led to the creation of so-called “neuromyths”—that is, common beliefs about the brain that have been erroneously turned into facts (e.g., that we only use 10% of our brains or that people are either left-brained or right-brained; Susac & Braeutigam, 2014). These neuromyths are so pervasive that they have even been used as the basis for teaching and learning methods, despite the lack of evidence for these claims (Goswami, 2006). This is not to say that all neuroscience research should be disregarded; however, we caution against making quick conclusions solely based on neuroscience. Rather, neuroscience research on learning should be considered in conjunction other evidence when making decisions about education, as the productive interaction between different fields will yield the most valuable insights (De Smedt et al., 2011).

Although the majority of neuroscience research was not explicitly designed to inform learning and teaching, there are insights that can help us better understand the underlying processes of mathematical learning when considered alongside other data about learning and teaching. In this section, we report key findings from neuroscientific
research in three areas: seemingly universal aspects of number processing, individual differences in brain function that might help to explain achievement differences, and cultural differences in brain function.

Universals in Brain Function

Early studies of patients with lesions and subsequent number processing impairments indicated that the parietal cortex was essential for mathematical learning and performance (e.g., Gerstmann, 1940). As the methods in neuroscience research have become more sophisticated, a more nuanced understanding of mathematical processing and learning has emerged. Dehaene, Piazza, Pinel, and Cohen’s (2003) synthesis of the research posits that there are three circuits for number processing in the parietal lobe. These networks were identified based on a meta-analysis of studies of functional brain activity.

The first is a quantity network, defined by activation in the horizontal segment of the intraparietal sulcus, which is involved in many number processing tasks that require accessing semantic representations of numbers, such as mental arithmetic and number comparison. Dehaene et al. (2003) theorized that the second parietal network accessed during number processing is linked to tasks that involve verbal processing and is characterized by activation in the left angular gyrus, a crucial region for general reading and language processing (Simon et al., 2002). Activation in the third parietal network, specifically in the posterior superior parietal lobule, is observed during tasks that require number manipulations. This region is also activated during other processing tasks that involve attention-orienting, not only number processing tasks (Wojciulik & Kanwisher, 1999). Looking across a variety of functional neuroscience studies, number processing involves differentiated networks, not a single area of the brain, and taps into capacities that extend beyond mathematics.

Individual Differences and Mathematics Achievement

When designed appropriately, neuroscience studies can illuminate subtle underlying processing differences between groups of learners and these findings can be used to inform teaching and learning. This is especially true for the study of dyscalculia or mathematical learning disabilities. In these studies, dyscalculia is defined simply as a deficit in mathematics learning that does not affect other areas. Neuroscientists study brain structure and/or function to determine whether and how individuals diagnosed with dyscalculia have systematically different brain function from their typically developing peers.

For example, De Smedt, Holloway, and Ansari (2011) studied 10–12 year old children with low and average levels of arithmetical competence, as measured by a standardized arithmetic fluency test. Their brain activity was compared during simple addition and subtraction tasks within the fMRI scanner. There were no significant differences in the behavioral data; however, the fMRI data revealed that children with low arithmetical competence had significantly higher activation in the right intraparietal sulcus when solving problems with a small problem size. Given that this region has been associated with processing of numerical magnitudes, over-activation might suggest that these children were using compensatory strategies that were less efficient, such as counting rather than retrieving the answer from memory.

Research has also shown that individuals differ in their ability to estimate number from a visual or auditory array and that these differences correlate with school
achievement from the very beginning of schooling (Halberda, Mazzocco, & Feigen-
son, 2008). It is unknown whether this estimation ability influences math achievement,
math achievement influences estimation ability, or some other factors influence both,
but in children diagnosed with dyscalculia, the right intraparietal sulcus was not acti-
vated in the same way as it was for typically developing children (Price et al., 2007).
This suggests that differences in the functioning of this area of the brain may be a root
cause of this type of learning disability.

Cultural Differences

Although neuroscience does at times focus on universals, we know that the human brain goes through enormous changes from the moment of birth and remains plastic
throughout the life span. The brain changes because of human experience in cultural communities; the development of the brain is therefore “biologically cultural” (Rogoff,
2003, p. 63). As Rogoff points out, some universals are also profoundly cultural; for example, the use of language may be universal across human cultural groups, but the
ways in which language is used are undoubtedly cultural.

Differences in language can also influence the nature of brain function for particular mathematical tasks. In one study comparing the ways in which Chinese speakers and
English speakers represented numbers, it was found that different parts of the brain
were used by the different groups (Tang et al., 2006). The results indicated that “the
different biological encoding of numbers may be shaped by visual reading experience
during language acquisition and other cultural factors such as mathematics learning
strategies and education systems, which cannot be explained completely by the differ-
ences in languages per se” (p. 10775). That is, verbal language, reading practices, and
schooling experiences all influenced the way information was encoded in and retrieved
from the brain. These and similar findings highlight the importance of considering the
ways in which everyday human activity influences learning. In other words, it is critical
for future neuroscientific research to focus further on processes of learning, and not
just cognition, in order to understand how the brain functions.

FUTURE DIRECTIONS FOR RESEARCH
ON MATHEMATICS LEARNING

The intent of this chapter has been to highlight threads of research that have broad-
ened and deepened our understandings of mathematics learning. These threads reflect
a theoretical and methodological diversity that enriches the development of knowledge
but that also creates challenges for research in mathematics education. We now turn to
three of these challenges and propose avenues of inquiry that may address them.

The Complexity of Context in Research on Mathematics Learning

The role of context in mathematical cognition and learning has been an enduring
issue within research in mathematics education. In part, the challenge arises from the
accountability of the field to understanding not only how people learn mathematics in
a universal sense, but also how students, with histories and identities, learn mathemat-
ics in their school and classroom contexts. Consequently, research in both the cog-
nitive/constructivist and situated/sociocultural traditions has sought to understand
aspects of context in mathematics learning—for example, the situational sensitivity
of knowledge in studies of transfer or the role of culture and race in students’ mathematical engagement and learning. Through our review of the current research, we identified a few directions for future research that seem productive for furthering the field’s theoretical understandings of mathematics learning and context, particularly those that may be important for addressing the current challenges facing mathematics learning and teaching.

First, in our discussions of research on discourse, identity, and neuroscience, we outlined some research findings that focus on single social categories, like race or ethnicity. Greater attention should be paid to issues of intersectionality—in particular, how the interplay of race, gender, socioeconomic status, language proficiencies, and so on affects mathematics learning and how students experience mathematics in particular contexts of learning. Mathematics education research might be well served to consider theoretical frameworks outside of mathematics education—for example, current formulations of Critical Race Theory—that draw upon theoretical paradigms of intersectionality in the recognition that the dynamics of race manifest in and through gender, sexuality, class, ethnicity, and culture (e.g., Crenshaw, 1991).

Second, research on mathematics learning needs to better address how the socio-political contexts of schooling interact with how students experience and learn mathematics. For example, while studies have long included school and community categories such as urban, rural, and so on in their research design, little progress has been made in understanding how the urban or rural context shapes students’ mathematical experiences and, thus, their learning. Indeed, there is little consensus about what constitutes an urban school or district (Chazan et al., 2013) and even less attention paid to rural communities. Further, the significant body of research that has been conducted on urban schooling and urban students’ learning has not, by and large, examined how mathematics is experienced and learned in urban settings. For example, how does the “high stakes” status of mathematical performance—for students as well as teachers and schools—shape the perceptions of mathematics and mathematics identities of students in large urban schools serving high minority and high poverty populations? How do urban youth experience school mathematics as relevant in their futures and how does this shape their mathematical participation and learning? Broadly speaking, this work necessitates understanding how mathematical performance and expertise are constructed in the social, economic and political discourses that frame current debates in educational policy and then how policies emerging from those debates affect how school mathematics is structured, taught, and learned in specific settings.

**Disciplinary Diversity**

As illustrated by the studies reviewed in this chapter, the field of research on mathematics learning is truly multi-disciplinary. The range and complexity of the phenomena relevant to mathematics learning necessitate a diversity of theoretical and methodological tools. Sometimes driven by challenges of understanding contextual forces (e.g., how race and culture shape mathematical learning) or by advances in other disciplines (e.g., findings from social psychology, neurological imaging technologies), researchers often draw upon resources outside of the traditional boundaries of mathematics education research grounded in psychology and cognitive science. However, as has been said of educational research more generally, researchers in mathematics education have not always had an adequate understanding of the intellectual histories...
and disciplinary groundings of theories and methods borrowed from other fields. As a result, methods such as “discourse analysis” are sometimes cited or employed inappropriately, and theoretical notions such as “community of practice” get taken up in so many different ways that their original purpose, context, and characterization are muddied. In such cases, what is lost is the methodological rigor and theoretical robustness that allow for the reliability and comparability of findings. That is not to suggest that tools should not be repurposed. Rather, tools are created for particular purposes and within particular intellectual contexts. Thus, using them to build knowledge within mathematics education and across disciplinary boundaries requires that we educate ourselves about those intellectual traditions.

**Theoretical Bridging**

Much has been written about the need and potential for bridging the seemingly divergent theoretical perspectives that inform current research on learning (e.g., Green, Collins & Resnick, 1996; Sfard, 1998), particularly those theoretical perspectives that center on individuals’ cognition and learning versus those that focus on the social or the situated nature of cognitive activity. Two decades ago, Greeno, Collins and Resnick (1996) suggested that cognitive, behaviorist, and situative views could be seen as analyzing learning at different levels of aggregation (i.e., individual cognitive structures and processes, individuals’ behaviors, and activity systems within which individuals participate and interact with others and with material resources). They suggest that theoretical coherence might be accomplished by developments that show how these levels are nested.

In current research, researchers are pushing on the boundaries of their theoretical frameworks in order to examine phenomena in different contexts or at different units of analysis. Cognitive researchers are seeking to understand students’ learning by taking into account the social processes that structure individuals’ cognitive activity; research in the situative/sociocultural tradition has sought to characterize individual students’ trajectories of learning within communities of practice and arising from interaction. However, it remains to be determined whether and how a kind of “grand unified theory” of learning can be developed, or is even desirable, given the apparent incommensurability of some fundamental assumptions and commitments. We do not necessarily believe that overarching theoretical unity is necessary or achievable within a field so diverse and multi-disciplinary; however, in order for the field to progress and for new knowledge to relate to and build upon existing work, researchers need to be explicit about their theoretical and analytical assumptions and provide robust formulations of theoretical constructs. In this way, greater coherence may be achieved as points of resonance and contrast are more clearly visible.

Finally, progress toward greater clarity and coherence is critical if the field is to be relevant to practitioners and policymakers. Too often, research on mathematics learning is seen as irrelevant to or difficult to realize within today’s classrooms and schools—even more so when district, state and federal organizations are seeking interventions that are effective at large scales and for diverse populations of students. In order to better communicate what has been learned about mathematics learning and how to support it, and to translate these findings into tools, practices and policies that are meaningful for students, practitioners and policymakers, we must understand and build upon connections across theoretical perspectives, levels of aggregation, and methodological approaches.
NOTE

1 Comparing different groups of people is central to some areas of research in cognitive neuroscience. In such comparisons, researchers frequently compare one group that they describe as “normal” to another group that is in some way deemed “abnormal” (e.g., individuals with learning disabilities, individuals deemed to be prodigies, etc.). Throughout this review, we adopt the language used by the researchers.

REFERENCES


