In this chapter, I discuss in broad outlines the ingredients of a theory of formal semantics. Specific phenomena are dealt with in detail in other chapters of this volume. So, by necessity the discussion will be at a high level with little in the way of detail. The goal of the chapter is to sketch the desiderata of a theory of semantics and the formal approaches to them. Where possible, I will highlight the choice points and the options selected by different approaches to formal semantics. In section 1, I introduce the aims of a theory of natural language semantics. In section 2, I sketch the necessary components of a formal theory of semantics. Section 3 is devoted to discussion of issues that bear on the choice of rules of semantic composition. In section 4, the structure of denotation domains in a model is discussed. Section 5 examines two proposals for how formal semantic theories might incorporate aspects of the interaction of meanings with pragmatic contexts.

1 Introduction

The study of the semantics of natural language concerns itself with the aspects of meaning that are linguistically determined. Evidence for the meanings of linguistic expressions come from a variety of sources. One primary source of evidence is the way that sentences relate to each other semantically. For example, judgements of entailment, consistency and contradiction between sentences provide crucial evidence concerning their meaning. In some cases, these relations between sentences depend crucially on properties of the language system and others appear to depend instead on information derived from cognitive systems outside of language.

(1) a Kim bought a sweater and Chris read a book.
    b Kim bought a sweater.
(2) a Katrina is a wombat.
    b Katrina is a marsupial.

A theory of semantics of natural language seeks to explain English speakers’ intuitions about the entailment relationship between sentences (1)a and (1)b. A theory of semantics should also be compatible with an account of intuitions concerning the relation between (2)a and (2)b, but we expect that conceptual knowledge about taxonomies will also play a crucial role.
It is commonly assumed that it is not enough for a theory of semantics, however, to simply give an account of a set of relations between sentences. Language has external significance; speakers use sentences of various types to convey and acquire information about the world, as well as to accomplish actions that change the world. From this perspective, a theory of semantics should specify a connection between linguistic expressions and aspects of the external world. The fundamental concept in this connection is truth. Truth and falsity specify particular relations between sentences and the world. A common idea (the correspondence theory) is that a sentence is true if there is some sense in which the meaning of the sentence corresponds to the facts of the world. There are many ideas about how to specify what is meant by correspondence (e.g. theories about truth makers, see the debate in Armstrong 1989 and Lewis 1992 et seq.). A related notion concerning the connection of language and world is reference, which connects terms to objects in the world. We will return to this and related notions below.

Given this connection, another primary source of evidence for a theory of linguistics meaning is speakers’ intuitions about the truth and falsity of sentences in various situations. There is an important connection between these intuitions and the intuitions concerning the semantic relatedness of sentences discussed above. That is, the intuitions concerning semantic relations can be seen as based on intuitions about truth in possible circumstances/situations/worlds. A sentence \( \varphi \) entails another sentence \( \psi \) just in case in any situation/circumstance/world in which \( \varphi \) is true, \( \psi \) is true. Two sentences \( \varphi \) and \( \psi \) are consistent if and only if there is a situation/circumstance/world in which both \( \varphi \) and \( \psi \) are true. Such basic considerations lead to the idea that truth conditions play an important role in a speaker’s linguistic competence concerning meaning. A truth condition provides the necessary and sufficient conditions under which a sentence is true.

(3) Sentence \( \varphi \) is true if and only if ________________________________

If a speaker had such knowledge of the truth conditions of all the sentences of her language, then she would be capable of the kind of judgements discussed above – at least in principle. One might know the conditions under which a sentence is true but not be able to obtain the information relevant to making an actual judgement of truth or falsity – indeed such information may not be obtainable. A common view is that such knowledge is also sufficient to ascribe knowledge of meaning to a speaker.

2 Formal semantics

I reject the contention that an important theoretical distinction exists between formal and natural languages.

Richard Montague, EFL

A theory of formal semantics seeks to give a theory of semantics of the sort described above using a particular set of tools. Montague (1970, 1973) pioneered the interpretation of natural language through the tools of logic and analytic philosophy. First, as set out in the quote above, formal semantics employs notions from the study of formal languages. A formal language is nothing more than a set of strings over a finite vocabulary. It is common, however, to study formal languages that are generated by a set of syntactic rules. In the case of the analysis of natural language, the language is given – it is the output of the faculty of grammar. Natural languages have important syntactic structure. As mentioned above, semantic competence in a language
consists of knowing the truth conditions for the sentences of the language. It is well known, however, that natural languages are unbounded in the number of well-formed, interpretable sentences that they contain. Consequently, an adequate theory of semantics competence – which seeks to discover what is in the head that permits knowledge of meaning – must be **compositional**. A compositional theory specifies the meaning of complex expressions as a function of the meanings of its parts. Hence a theory of formal semantics must begin with a specification of a formal language $L$ and its syntactic description.

There is a fact of the matter about the syntactic structure of natural languages. So, ideally, the formal language specified will adequately model the structure of the language being analyzed. There are many different views on the syntactic structure of natural language. The spectrum ranges from theories that take interpretation to be the most important piece of evidence for structure to theories that recognize non-semantic restrictions on structure and countenance evidence for syntactic structure that is not derived from interpretation. The former theories posit a transparent connection between language structure and interpretation; sentences are assembled to be interpreted and can be interpreted directly (Direct or Surface Compositionality, see Barker and Jacobson 2007). The evidence considered by the latter theories often leads to situations in which there are syntax-semantics mismatches. Within these latter approaches, there are many possible responses to syntax-semantics mismatches. Some approaches posit a mechanism by which sentences are paired with more than one syntactic analysis. One of these may be specified as the analysis that is relevant to interpretation (this is the sentence’s Logical Form, see May 1977, 1985) or the interpretation could depend on features of more than one of the structural analyses. Alternatively, a theory might postulate readjustment rules within the semantics that change the meanings of parts to be deployed in cases of mismatches (Type-shifting, Partee 1987). These alternatives are not mutually exclusive and are often deployed in one and the same theory.

The formal language selected $L$ serves as input to a mechanism that pairs sentence of the language with truth conditions. As said above, the pairing cannot be direct – a language speaker cannot have an infinite list of sound meaning correspondences in her head. So, a compositional strategy mediated by syntactic structure must be adopted. An important approach to this problem is to describe a function that assigns **semantic values** to all the syntactic units of a language, including its sentences. This process involves a number of component parts. The input to this function is our language $L$, but now we must specify its possible outputs. To do so is to specify a model for interpreting the language. A **model** consists of (i) a specification of semantic domains – the possible semantic values for syntactic units, and (ii) a function that maps the basic units of the language $L$ to objects in the semantic domains. Thus, a model specifies both what there is as far as the language is concerned – that is, its ontology – and an interpretation of the minimal units of the language. The minimal units are the vocabulary items of the language. These are the words or morphemes of the language.

The semantic domains of the model will typically include a set of truth values as well as a domain of individuals. There are different approaches to the former domain, but generally the set is assumed to be $\{0,1\}$ where 0 customarily stands for falsity and 1, truth. Sets containing three or four values are also common. The domain of individuals, on the other hand, contains those individuals that serve as the referents of individual constants and terms. These domains can be extended to domains of more complex objects using set theory. For example, one might specify a domain of sets of individuals or of functions from individuals to truth values. As we shall see, these semantic denotation domains differ in the extent to which they are structured. The set of truth values $\{0,1\}$ is naturally ordered by the less than or equal to relation on natural numbers. This structure has linguistic significance. The domain of individuals, on the other hand, might plausibly be viewed as unordered, though
we will see reason to reconsider this below. Finally, a fully intensional semantics must also include domains for indices of interpretation, including possible worlds and times. These may serve as denotation domains for terms of the language or only contribute indirectly to the more complex intensional domains as in Montague (1970, 1973).

The interpretation function, conventionally written in circumfix form as $\langle \cdot \rangle$, maps syntactic units to elements of the semantic domains. The interpretation function carries a model $M$ as a parameter, $\langle \cdot \rangle^M$. Among the basic vocabulary expressions, some, the logical/functional expressions, receive the same interpretation in all models. Others, the non-logical/content expressions, receive varying semantic values in different models. Within a compositional theory, the semantic values of syntactically complex expressions will be calculated recursively from the meanings of these basic expressions using a restricted set of semantic composition principles. Montague proposed a system of rule-by-rule interpretation in which each syntactic rule of the formal language has a specific corresponding rule of interpretation. Others have argued for an abstraction of interpretation principles away from the specific syntactic structures; see for example Klein and Sag’s (1985) view of Type-driven interpretation. Indeed, Frege (1980 [1892]) proposed that the only necessary principle of semantic composition is function application. In a theory that assigns functions as semantic values, the rule of function application says that if an expression is composed of two expressions such that one denotes a function and the other an object in the domain of the first, then the whole expression denotes the output of applying the function to the object.

(4) Function application

If an expression $\alpha$ is composed of immediate constituents $\beta$ and $\gamma$ where $\langle \beta \rangle$ is a function whose domain includes $\langle \gamma \rangle$, then $\langle \alpha \rangle = \langle \beta \rangle(\langle \gamma \rangle)$.

A Fregean approach to predication in simple sentences like Kim smokes would look as follows, assuming a model in which $D_e$ is the domain of individuals (“e” for “entity”) and $D_t$ the domain of truth values. Suppose the model specifies semantic values like those in (5) for the vocabulary items Bill and smokes.

(5)

\begin{align*}
\text{a} & \quad \langle \text{Kim} \rangle^M = \text{Kim} \\
\text{b} & \quad \langle \text{smokes} \rangle^M = f: D_e \rightarrow D_t \\
& \quad \text{for all } x \in D_e, f(x) = 1 \text{ if and only if } x \text{ smokes}
\end{align*}

(6)

\begin{align*}
\langle \text{Kim smokes} \rangle^M &= \langle \text{smokes} \rangle^M(\langle \text{Kim} \rangle^M) \\
&= 1 \text{ if and only if Kim smokes}
\end{align*}

(In fact the entry in (5)b is more of a rule for associating semantic values with the expression *smokes* than the actual semantic value in a given model. In a particular model, the semantic value of *smokes* will be a function of the kind $[\text{Bill} \rightarrow 1, \text{Sue} \rightarrow 0]$. In such a case the output of the calculation is a definite truth value as opposed to a statement of truth conditions.)

The precise set of composition principles is a matter of debate. Generally speaking, there is a tendency wherever possible to minimize the set of composition principles and build peculiarities into the meanings of particular lexical items. The lexicon is the repository of arbitrary connections that must be learned (although the lexicon has its own organization). Still, semanticists differ on the give and take between syntax and semantics. A simple syntax may necessitate a more complex set of composition principles; on the other hand, complicating the syntax can yield a simpler semantics. A strong theory of Logical Form may get by with little more than function application, while a directly compositional theory incorporates a broader array of composition principles.
Natural languages exhibit **ambiguity**, the phenomenon whereby a surface string of the language is associated with more than one meaning. It is worth considering how to deal with ambiguity in an analysis of language that uses formal languages. A familiar kind of ambiguity is lexical ambiguity, whereby a single string may have multiple meanings because the string contains an item that has two meanings. Within a formal language, this kind of ambiguity is not difficult to model. An ambiguous vocabulary item is analyzed as two separate expressions with identical pronunciation. So, strings that contain these homophous expressions are in fact not identical as they contain different vocabulary items. It is also not difficult to deal with structural ambiguity. In structural ambiguity, a single string of vocabulary items has two (or more) different syntactic analyses. That is, the string can be derived through different rule applications in the syntax. The semantics as we have described it is sensitive to syntactic structure/derivation. Consequently, the semantics may assign different interpretations to a string under its different analyses. Other putative cases require a different approach. Consider the sentence below.

(7) He influenced Chris.

This sentence can be used to express different propositions. In one context, it may express that Kim influenced Chris, in another context, that Mark did. A semantic theory needs a way to incorporate this dependence on the context of use.

A common way (cf. Cooper 1979, inspired by Montague) to approach this is to relativize the interpretation of an expression to a function that represents the intentions of a speaker to use a pronoun to refer to a particular individual. This function is a **variable assignment**. Within the language, some expressions, such as pronouns, are singled out as variables. These variables are introduced into the language bearing a natural number as an index. The variable assignment then is a partial function from natural numbers to individual in the models domain of individuals. The variable assignment is a parameter of interpretation in addition to the model, representing a feature of the context of use. This leads to a rule like the following for the interpretation of pronouns.

(8) If α is a pronoun bearing index i and g a variable assignment whose domain includes i, then $\langle \alpha \rangle^g_M = g(i)$

There is an alternative to this view that suggests that pronouns that receive their interpretation from context are not free variables whose value is supplied by a variable assignment, but instead denote identity functions.

(9) If α is a pronoun, then $\langle \alpha \rangle^M = f: D_e \rightarrow D_e$

for all $x \in D_e$, $f(x) = x$

Note that in sentence (7) above, on this view, the pronominal subject and the predicate could not semantically compose by function application. The subject denotes a function from individuals to individuals and the predicate influence Chris denotes a function from individuals to truth values. Such a view then requires an additional principle of composition. The idea is that sentences with pronouns that receive their interpretation from context do not denote truth values. Instead, such sentences denote a function from individuals to truth values. A truth value is obtained by applying this function to a salient individual. Such a function can be obtained from the parts described above via function composition.
The pronoun takes an individual as an argument and gives an argument as output. This individual can in turn serve as input to the predicate, which maps it to a truth value.

The two approaches are quite similar in effect. It is possible to view the variable-based approach as deriving functions from variable assignments to truth values as the denotations of sentences. A truth value is only obtained when such a function is applied to an appropriate variable assignment; that is, one whose domain includes the indices of all free variables. The difference between the two proposals depends on the way they embed with an analysis of further phenomena including bound variable readings of pronouns and the interpretation of displacement in syntax.

At this point it is worth noting another way in which function composition can be implemented in the semantics. Rather than adding a principle of composition that allows for different ways of combining the meanings of two expressions, we can introduce a unary operation that lifts the type of one expression and allows for function application to take place. Geach’s Rule (Geach 1972) is such a rule. The Geach Rule maps a function \( f \) to a new function that takes functions into \( f \)’s domain and maps them to functions into \( f \)’s range. At this point, it will be useful to introduce some pieces of notation that will facilitate formulating the Geach Rule. First, below we extend the notation for denotation domains to functional domains. A domain of type \( \langle \sigma, \tau \rangle \) contains functions from \( D_\sigma \) to \( D_\tau \). Second, we introduce the lambda notation for naming functions.

With this notation we can formulate the Geach Rule more perspicuously.

For any function \( f \in D_{\langle \sigma, \tau \rangle} \), \( G_\rho(f) \) is the function \( g \in D_{\langle \rho, \sigma \rangle, \langle \rho, \tau \rangle} \),
where \( g = [\lambda h : h \in D_{\langle \rho, \sigma \rangle} . [\lambda y : y \in D_\rho . f(h(y))] ] \)

For example,

\( G_\rho((14)a) = [\lambda h : h \in D_{\langle \rho, \epsilon \rangle} . [\lambda y : y \in D_\epsilon . f(h(y))] ] = [\lambda h : h \in D_{\langle \rho, \epsilon \rangle} . [\lambda y : y \in D_\epsilon . h(y) \text{ influenced Chris}] ] \)
\( G_\rho((14)b) = [\lambda h : h \in D_{\langle \rho, \epsilon \rangle} . [\lambda y : y \in D_\epsilon . h(y) \text{ influenced Chris}] ](\lambda x.x) = [\lambda x : x \in D_\epsilon . x \text{ influenced Chris}] \)

3 Rules, binding and quantification

3.1 Binding

As mentioned above, pronouns have uses in which their interpretation does not depend on the context of utterance, but instead depends on syntactic context. Consider for example the “bound variable” readings of pronouns. In sentence (15) below, the caller argument of
the main predicate and the child argument of the relational noun mother co-vary and range
over the relevant students that the subject quantifies over. In this case, we say the pronoun
is bound by the quantificational subject. A formal theory of semantics must determine how
such connections of binding are established.

(15)  a) Every student called his mother.
b) For every student x, x called x’s mother.

Recall that on the variable-based theory, the pronoun carries an index and depends on the
variable assignment for its interpretation. Consequently, what we need is (i) to supplement
the structure with another index that makes the connection between binder and bindee and
(ii) to formulate a new rule that manipulates the variable assignment as directed by the bind-
ing index. In the formulation of this rule, I follow Büring’s (2003) recipe for beta binding.
Büring, in turn, follows the Derived VP Rule of Partee (1975). On this view, β indices are
freely insertable in the syntax.

(16) Logical Form for (15)
Every student β1 [called his1 mother]

The interpretation of the beta binder is as follows. Note that beta binders do not change the
type of their sisters. Instead, they modify the variable assignment that under which their sister
is interpreted and identify the argument of the predicate with interpretation of the bound vari-
able under the modified assignment.

(17) Beta Binding
\[
\llbracket \beta_n \text{ XP} \rrbracket^g = \lambda x. \llbracket \text{ XP} \rrbracket^{g[n \rightarrow x]}(x)
\]

(18) A modified variable assignment
g[n \rightarrow x] is the same as g except that g[n \rightarrow x] maps n to x.

Applying this analysis to (16), we obtain the results below.

(19) \llbracket \beta_1 \text{ [called his1 mother]} \rrbracket^g = \\
\lambda x : x \in D_e. \llbracket \text{ called his1 mother} \rrbracket^{g[1 \rightarrow x]}(x) = \\
\lambda x : x \in D_e. x \text{ called x’s mother}

This is the appropriate type for an argument of a quantifier and has established the appropriate
connection between the co-varying argument positions. Following the insights of Generalized
Quantifier Theory (Barwise and Cooper 1981), a quantificational noun phrase denotes a set of
sets. In type-theoretic terms, a generalized quantifier denotes a function of type \( \langle \langle e, t \rangle, t \rangle \). For
example, with our quantifier every student above has the following semantic value.

(20) \llbracket \text{ every student} \rrbracket = [\lambda f : f \in D_e. \text{ for every student } x, f(x) = 1]

Similar results can be obtained in variable-free framework through the use of wider array
of compositional principles and/or type shifts. The Geach Rule is an effective way of pass-
ing along dependence on an input, but cannot on its own account for the kind of binding
that we observe in the example (16). Rather we need a rule that maps a function that maps
individuals to functions on individuals to a function that can take functions from individuals to individuals to a function on individuals. This rule, which is quite similar in form to the Geach Rule and is given the name Z by Jacobson (1999), is given below.

\[(21) \quad Z \text{ Rule}\]

For any function \( f \in D_{\langle e,\sigma,\tau,\sigma,\tau,\tau \rangle} \), \( z(f) \) is the function \( g \in D_{\langle e,\sigma,\tau,\sigma,\tau,\tau \rangle} \), where:

\[ g = [\lambda h: h \in D_{\langle e,\sigma,\tau,\sigma,\tau,\tau \rangle}, \lambda x: x \in D_e, f(h(x))(x)] \]

\[(22) \quad a \quad [the \ mother] = \lambda x. x’s \ mother\]
\[ b \quad [called] = [\lambda x: x \in D_e. [\lambda y: y \in D_e. y \ called \ x]] \]
\[ c \quad z([called]) = [\lambda f: f \in D_{\langle e,\sigma,\tau,\sigma,\tau,\tau \rangle}, \lambda x: x \in D_e. x \ called \ f(x)] \]
\[ d \quad z([called])([the \ mother]) = [\lambda x: x \in D_e. x \ called \ x’s \ mother] \]

### 3.2 Syntactic displacement

An adequate theory of semantics must also have mechanisms for dealing with the phenomenon of syntactic displacement. Displacement refers to the phenomenon whereby a syntactic constituent (often a phrase) occurs in a position distinct from that typically occupied by constituents serving the same grammatical function. For example, a noun phrase may serve as the object of a verb, providing the verb with an argument that it requires, but appear in a position that is distinct from the canonical position for objects in the language. A typical and simple example from English is the case of topicalization. Consider the sentence in (23a) below. The verb *admire* requires an object, as can be seen from (23b). And the usual position for an object is after the verb, as seen with the accusative marked object pronoun in (23c).

\[(23) \quad a \quad \text{Tom, every student admires.}\]
\[ b \quad *I \ admire. \]
\[ c \quad I \ admire \ him. \]

The challenge for semantics is to allow the noun phrase *Tom* in this sentence initial position to saturate the internal argument (admiree role) of the verb. One response is to see in such constructions a situation formally similar to that of the binding of pronouns. Under such a view, displacement of a noun phrase results in a variable (with index) being inserted in its canonical position, for example in the object position in (23a). This insertion is thought to be a reflex of movement of the noun phrase from object position to the sentence initial position. Such movement presupposes a theory that assigns multiple syntactic representations to a sentence. When the noun phrase lands a binding index is inserted between the noun phrase and its new sister. Again, I follow Büring (2003), who distinguishes movement (\( \mu \)) indices from other binding indices, like the \( \beta \) indices used in (16). (For a theory that identifies binding and movement indices by arguing that all binding requires movement, see e.g. Heim (1993/1998), developing ideas of Reinhart 1983.)

\[(24) \quad \text{Logical Form of (23a)} \]

\[ \text{Tom } \mu_3 \ [ \text{every student admires} \ t_3 ] \]

Such structures are interpreted using the rule below. Note that unlike the beta-binding rule, the mu-binding rule does change the type of the constituent it applies to; a beta-binder maps a predicate of type \( <e,\sigma> \) to \( <e,\sigma> \) for any type \( \sigma \), a mu-binder maps a predicate of type \( \sigma \) to \( <e,\sigma> \).
A variable-free approach takes a different tack on displacement structures. Once again, the addition of function composition plays a role in the analysis. When a predicate lacks an argument to apply to function composition may apply, so long as there is an appropriate type of function around for the lacking predicate to compose with. A full appreciation of an approach of this kind must be supplemented with an appropriate syntax, as may be found in Categorial Grammar, cf. Carpenter 1998 and Steedman 2000, for example. In such a theory, in our example (23a), \( \langle \text{admire} \rangle \) may either apply to an argument to its right or else compose with a function to its left. Suppose \( \langle \text{admire} \rangle \) is a function from individuals (De) to functions from De to truth values \{0, 1\}; that is, \( \langle \text{admire} \rangle \) is type \(<e,<e,t>>\). When, in (23a), \( \langle \text{admire} \rangle \) does not find an argument to its right to apply to it may then compose with the subject quantifier to its left. As shown in (20) above, the quantifier is type \(<<e,t>,t>\).

This function may then apply to the topicalized object Tom yielding the truth value 1 if every student admires Tom and 0 otherwise.

### 3.3 Quantification

Closely related to the issue of how to treat syntactic displacement is the issue of how to treat quantifiers that serve as the non-final arguments of multi-place predicates. The simplest examples involve quantifiers in the object position of transitive verbs. In such cases, neither the verb nor the object quantifier may apply to the other. The types do not match for functional application (\(<e,<e,t>>\) and \(<<e,t>,t>\)).

One possibility is suggested by theories that allow for and interpret syntactic movement. In such theories it is possible to postulate an additional level of syntactic representation, the Logical Form mentioned above. Since movement is a possible way to map from a structure representing the basic grammatical functions of a sentence to its surface representation, as in topicalization, it is hypothesized that movement may also map a surface representation to its Logical Form (May 1977 et seq.). No additional interpretive principles are required. We have already sketched the mechanisms needed to interpret movement.

\[
\text{Kim admires every student.}
\]

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(30) \[ \forall x. (\lambda x. \text{Kim admires } t_2) \] = 1 iff \[ \forall x. (\lambda x. \text{Kim admires } x) \]

Variable-free theories that posit direct compositional analyses of the surface representation cannot make use of the concept of “covert” movement, even though such theories possess mechanisms for interpreting displacement – function composition, as we have seen in (27). In such cases, a strategy of type-shifting must be adopted (see, for example, Hendriks (1993)). Either the type of the verb or the type of the quantifier must be adjusted to allow for semantic composition through function application. A common choice is to lift the type of the quantifiers. In this case, it would be necessary to lift the quantifier from \(<<e,t>,t>\) to \(<<e,<e,t>>,t>\).

(31) Type-shifting operation
For any quantifier \(\alpha\) such that \(\exists\alpha \in D_{<<e,t>,t>}\),
\(TS(\alpha) = \lambda f \in D_{<<e,<e,t>>,t>} . \lambda x \in De. (\alpha(f(x))) = 1 \)

The type-shifted quantifiers may now apply to a transitive verb yielding a one-place predicate that may either apply to an individual denoted by the subject, or serve as the argument of another quantifier. This rule in (31) resolves the type mismatch in this case but will not suffice as a general solution to the problem. This rule must be a member of a family of rules that will allow for interpretation as the first argument of any n-place predicate and additional rules are required to bring about different scope orders in sentences that contain two or more quantifiers.

Choosing between these two perspectives in particular with respect to quantifier interpretation can be a difficult matter. A phenomenon that is often discussed in this regard is antecedent contained deletion (ACD). An example is given in (32)b. This sentence involves the omission of the overt material “read” present in (32)a.

(32) a Bill read every book that Fred read.
    b Bill read every book that Fred did.

Advocates of quantifier raising have argued that ACD involves verb phrase ellipsis – that what is deleted in (32b) is \( [VP \text{ read } t] \). There is no similar VP in the sentence to license elision of this VP – unless the object quantifier moves and leaves a trace.

(33) every book that Fred \textbf{read} \[ t \] [ Bill read \( t \) ]

Advocates of variable-free approaches on the other hand argues that what is missing is just a verb. Jacobson (1992) advocates an approach on which the gap is a free preform anaphoric to a transitive verb. More recently, Charlow 2008 proposes a binding account of ACD within variable-free semantics. See Chapter 18 in this volume for further in-depth discussion of quantification.

### 3.5 Interim summary

In this section, I have endeavoured to give a sketch of how a formal theory of semantics is set up including the specification of a language and its syntax, the specification of a model that includes domains, and in some detail the selection of a set of recursive compositional rules.
hope to have given some sense of how all three of these depend on one another. Certain kinds of approaches to the syntax (CCG vs. LF, for example) favour certain kinds of compositional principles and will require different kinds of entities in the model. In the next section, I will discuss the linguistics significance of the structure of the denotation domains that form part of the models.

4 Structure of models

A linguistically significant property of the models that semanticists propose for natural languages is the structure of the denotation domains they determine. As discussed above, models contain domains of truth values and individuals (and potentially many more including worlds, times, events, numbers/degrees . . .), as well as functional domains defined from the basic domains. The basic domain of individuals may be viewed as an unordered set, or as one that is structured by a part-whole relation. Other domains are naturally viewed as structured. For example, the domain of truth values, when viewed as the set \{0,1\}, can be seen as a set of numbers ordered by the less than or equal to relation \(\leq\). Similarly, the functional domain \(D_{<e,t}>\) of function from individuals to truth values may be seen as a set partially ordered by the subset relation \(\subseteq\) (speaking loosely here, given the one-to-one correspondence between functions of type \(<e,t>\) and subsets of \(D_e\)). In fact, both of these domains may be viewed as having the full structure of Boolean algebra with fully defined operations of meet, join and complementation. In the domain of truth values, these operations are truth-functional conjunction, disjunction and negation. In \(D_{<e,t>}\), the operations are intersection, union and set complementation.

4.1 Monotonicity and orderings

The structure imputed to denotation domains has important consequences for a theory of semantics. There are, for example, expressions of natural language that show a sensitivity to the structure of denotation domains and how they are or are not respected by operators of language. Negative polarity items (NPIs) are such a case. A negative polarity item, like English \textit{ever}, must be in an environment that is somehow negative. Sentential negation is the paradigmatic case.

(34) a  *Bill will ever go to Hartford.
   b  Bill won’t ever go to Hartford.

Ladusaw (1979) argued convincingly that NPIs are sensitive to a formal property of expressions – monotonicity – that is defined in terms of the structure of the model. This case is discussed in Chapter 17 of this volume. I refer readers there for details.

4.2 Plurals: semilattices

Following Link (1983) it is common to attribute to the domain of individuals the structure of join semilattices. A join semilattice is a set of individuals closed under an operation \((\sqcup, \text{join})\) that is associative, commutative and idempotent. This operation corresponds to a non-Boolean interpretation of \textit{and} as in the coordination \textit{Mary and Bill}. The operation maps to individuals, Mary and Bill, to their sum, \(\text{Mary} \sqcup \text{Bill}\). Sum individuals are necessary to deal with the phenomenon of collective predication. This algebraic structure determines a part-whole ordering on the domain of individuals.
For any two individuals $a, b \in D_e$:
$$ a \sqsubseteq b \text{ if and only if } a \sqcup b = b $$

(36) Bill and Mary are a couple. ($\neq$ Bill is a couple and Mary is a couple.)

Given this ordering, there are individuals that are designated as **atoms**. The atoms of a join semilattice are the individuals that are minimal with respect to the order $\sqsubseteq$.

The ordering of individuals in the domain $D_e$ is linguistically significant, because there are predicates of natural language that are sensitive to the part-whole structure of individuals. For example, a predicate like *have blue eyes* may be predicated of a noun phrase that denotes a complex sum individual, but the truth of the statements is always dependent on the properties of the atomic parts of the sum individual. Link (1983) proposed that the extension of *have blue eyes* is a subset of the atoms of the domain.

$$ \langle \text{blue-eyed} \rangle \subseteq \{y : y \text{ is an atom } \& \ y \in D_e \} $$

This would not allow such a predicate to apply to sum individuals. To allow for this in statements like (38), a pluralized version of the predicate must be supplied. To this end, Link proposes the pluralizing * operator.

(38) Mary and Bill have blue eyes.

(39) *$P = \text{the closure of } P \text{ under } \sqcup$*

Given this operation, the predication will be true just in case the atomic parts of the sum individual $Mary \sqcup Bill$ are in the extension of the unstarred extension of *have blue eyes*.

$$ *\langle \text{blue-eyed} \rangle(Mary \sqcup Bill) $$

### 4.3 Islands

It would take us too far afield to discuss these proposals in detail, but many interesting semantic accounts of islands have been given by appealing to the structure of denotation domains. Szabolcsi and Zwarts (1993) proposed that some weak island effects could be explained by the fact that some operations, like negation, are not defined in certain denotation domains, like the domain of manners, cf. (41).

(41) *How did Fred not behave?*

This is reminiscent of Landman’s (1991) claim that individual negation and disjunction do not exist because the domain of individuals lacks full Boolean structure. Fox and Hackl (2006) additionally suggest that negative degree questions are unacceptable due to a crucial property of the structure of the degree domain that interacts with maximalization in questions. The relevant property is density of the ordering on degrees.

(42) *How tall isn’t Bill?*

 Abrusán (2011) revisits both phenomena with a novel view on how exhaustification in questions interacts with the structure of the denotation domains of degree and manner; see section 5.2 below for further discussion of exhaustification.
5 The relation of semantics to context

In this section, we consider how interaction with context enters into the formulation of formal theories of semantics. The reader is referred to Chapters 10 and 11 of this volume for in-depth discussion of the issues.

5.1 Dynamic semantics: context change potentials

There are several problems that theories of formal semantics have faced that have forced a rethinking of the relation between the semantics of a statement and the context in which it is used. One such problem is the phenomenon of intersentential anaphora involving indefinite antecedents. Consider the sequence of sentences in (43).

(43) A woman entered the room. She sat down.

The first sentence introduces existential quantification over women. The second sentence contains a pronoun that one might be tempted to say “refers” back to the woman from the first sentence. The first sentence, however, does not pick out a unique woman who can serve as the referent of the pronoun.

An obvious alternative would be to analyze the relation between the quantifier in the first sentence as one of variable binding, like the cases discussed above in section 3.1. In predicate logic terms, one might suggest the right scope interpretation is the following.

(44) $\exists x$ [ woman(x) $\land$ entered(x) $\land$ sat_down(x) ]

This does appear to be an adequate sketch of the meaning of (43). Unfortunately, the project of assigning (43) such an interpretation faces a rather large obstacle. That obstacle is that quantifiers are generally not allowed to take scope beyond the sentences in which they are contained.

(45) No woman entered the room. She sat down.

[Infelicitous without additional context.]

(46) $\neg\exists x$ [ woman(x) $\land$ entered(x) $\land$ sat_down(x) ]

There is no way to interpret this sequence of sentences in the way that seems to be required in the case of (43). The problem, then, is how can an indefinite/existential quantifier like a woman take scope over a sequence of sentences when a quantifier like no woman may not?

An important response to this, as well as to some related problems in anaphora and presupposition, was to change our fundamental notion about what the meaning of a sentence is. Under traditional views, the meaning of a sentence is identified with its truth conditions. Those truth conditions contain information that, through the utterance of a sentence with the intention to perform a certain speech act, may have an effect on the knowledge and beliefs of the participants of a conversation. Such knowledge and belief form a part of the context in which a sentence is uttered. Under the proposed view, the meaning of a sentence should not be seen as the static truth conditions, but rather as the dynamic effect that the utterance of the sentence has on context. Hence the approach is referred to as dynamic semantics; see Dekker 2012 for a recent approach and Chapters 3 and 10 of this volume for additional discussion.
Under the view of dynamic semanticists there is a big difference between expressions like *no woman* and *a woman*. The former is quantificational in the usual sense, but the latter is not. Rather indefinites like *a woman* are simply variables, which in the context of a dynamic semantics will have the effect of the intersentential scope required by (43). There are many different approaches to dynamic semantics from the Discourse Representation Theory of Kamp (see Kamp and Reyle (1993)) to the Dynamic Predicate Logic of Groenendijk and Stokhof (1991). In this section, I will sketch a version of dynamic semantics due to Heim (1982, 1983). Under this view the meaning of a sentence is its Context Change Potential (CCP), a function from contexts to the contexts modified by the content of the sentence.

To see how to specify the meaning of a sentence as the effect that it has on context, we must have some notion of context in mind. Heim makes use of Stalnaker’s view. Stalnaker (1974) suggests that at any point in a conversation there is a set of propositions – the Common Ground – that represents the mutually held beliefs of the participants of the conversation – or at least what they are disposed to act as if they believe. This set of propositions determines a set of worlds – the Context Set – in which all of the propositions of the common ground are true. Heim adopts the Context Set as the relevant representation of context for dynamic semantics. In Heim’s contexts, each world is paired with a variable assignment, a function from natural numbers into the domain of individuals.

The context potential of a sentence $\phi$ then is written as “+$\phi$” which may then be applied to a context $c$, conventionally written as “$c+$+$\phi$”. Let us now consider what the context change potentials of sentences involving indefinites would look like. According to Heim, indefinites are just variables that bear an index. That index must be new in the context (Heim’s novelty condition). In other words, the context may impose no restrictions on the index of the indefinite. For example, in a context in which the index 2 is new, the CCP of the first sentence of (43) may be as follows.

(47) $c + A\, woman_2$ entered the room
    = $c \cap \{<g,w>: g(2) \text{ is a woman in } w \text{ and entered in } w\}$
    = $c'$

This sentence removes from the context $c$ any assignment of world pairs in which the individual assigned to 2 is not a woman or did not enter. Thus, a new context $c'$ is created that has only kept the pairs $<g,w>$ of $c$ where $g(2)$ is a woman who entered in $w$. This revised context then is the input context to all succeeding sentence in the discourse. The second sentence of (43) may now be analyzed as below. As a pronoun, the index on *she* must have already been used in context, as it was in this case by *a woman* (Heim’s familiarity condition).

(48) $c + she_2$ sat down
    = $c \cap \{<g,w>: g(2) \text{ sat down in } w\}$

When this CCP is applied to $c'$ it further restricts the assignment world pairs, so that the output context, call it $c''$, only contains pairs $<g,w>$ such that $g(2)$ is a woman in $w$ entered in $w$ and sat down in $w$. In this way, a dependency arises between the expressions in the two separate sentences. All that we need now is to understand where existential force comes from. In Heim’s theory, this follows from the definition of the proposition determined by a context.

(49) Let $c$ be a set of assignment world pairs. Then, the proposition determined by $c$ is
    $\{w: \text{for some } g, <g,w> \in c\}$
So, the output context after update by (43) is a proposition that will entail that there exists an individual who is a woman and entered and sat down.

Compare the case of a quantifier like no woman. The quantifier is introduced with an index j. This index must also be novel in the context. Note, however, that updating c with a CCP containing no woman had no effect on the restrictions imposed on the index j in the output context.

\[
(50) \quad c + \text{no}_j x_j \text{ is a woman} = \\
\{ <g,w> \in c : \text{for every a, if } <g[a/j],w> \in c + x_j \text{ is a woman}, \text{ then} \\
<g[a/j],w> \notin c + x_j \text{ is a woman} + A \}
\]

Consequently the CCP of a quantified statement of this kind does not present the possibility of taking scope over and binding pronouns in succeeding sentences. There are alternative analyses of such anaphoric possibilities. The most common is the non-dynamic E-type pronoun approach; cf. Cooper (1979), Heim (1990), Elbourne (2006).

The dynamic approach to semantics and, in particular, the support that it has drawn from the phenomenon of presupposition projection has been a matter of much recent debate. Schlenker (2007), Fox (2008) and George (2008) have presented approaches to the problem of presupposition projection that do not involve the typical mechanisms of dynamic semantics, such as context change potentials. A primary argument in favour of these proposals is that they offer a depth of explanation of the phenomenon that had not been achieved by dynamic theories. Schlenker (2009) has in more recent work reassessed and refined aspects of the dynamic approaches to achieve more explanatory theories.

### 5.2 Scalar implicatures: exhaustification operators

Another important phenomenon concerning the interaction of semantics and context that has received much recent attention is that of scalar implicatures. A scalar implicature derives from the assertion of a sentence containing an expression that belongs to a quantity scale. Grice (1975) was the first to discuss the calculation of implicatures. Horn (1972) formulated an influential approach to scalar implicature proposing the existence of conventionalized scales (often called Horn scales) that give rise to implicatures. The general idea is that when a speaker uses an item from a scale (see 51b), their use of that item is automatically compared with statements they could have made with the item’s scalemates.

\[
(51) \quad \text{a} \quad \text{Bill read some of the articles.} \\
\quad \text{b} \quad <\text{some, many, most, all}> \quad \text{Scale} \\
\quad \text{c} \quad \text{Bill didn’t read all of the articles.} \quad \text{Scalar implicature}
\]

If they could have chosen an item that would have resulted in a stronger statement (all in 51c), they should have used that item (if the additional information is relevant). Given that they did not choose to make the stronger statement (and it was relevant), hearers are entitled to infer that the speaker does not know the stronger statement to be true – or even that the speaker knows it to be false. This line of reasoning relies on Horn’s (1972) scales and Grice’s (1975) rules for conversation including maxims enjoining speakers to make their contributions as informative as appropriate and to say only that for which they have adequate evidence.

Another perspective has arisen suggesting that, in fact, scalar implicatures are calculated in parallel with the recursive compositional rules of semantics. Here I will discuss only one argument that has been given in favour of this view (cf. Chierchia et al. 2010). Hurford
(1974) observed that disjunctive statements are infelicitous when one of the disjuncts entails the other. Since being a dog entails being an animal, the following sentence is infelicitous.

(52) "Mary saw an dog or an animal.

It has been noticed, however, that in some cases that involve scalar items, disjunctions that have one disjunct entailed by the other can indeed be felicitous. Consider the case of “some or all”. Solving all of the problems means solving some of them, and yet (53) is felicitous.

(53) Bill solved some or all of the problems.

Chierchia et al. 2010 argue that this sentence is felicitous because it does in fact satisfy Hurford’s constraint. They argue that this is so because there is a reading for the first conjunct that is not entailed by the second. Typical uses of some result in a “not all” implicature. Chierchia et al. suggest that a grammatical mechanism is able to apply that makes this implicature part of the meaning of the first disjunct. They propose, specifically, that there is an exhaustification operator O, similar in meaning to only that introduces implicatures. Under such a view, it is possible to embed O under the scope of other operators.

(54) Bill read (only) some of the articles.

(55) [O[Bill solved some of the problems] or [Bill solved all the problems]]

“Bill solved only some of the problems or he solved all of them.”

The proposal is controversial and has been the topic of much recent debate. Alternative proposals that do not make use of exhaustification operators have been advanced. For criticisms of this approach and more Gricean alternative see Geurts (2009).

6 Conclusion

In this brief point, I have attempted to give a sketch of the tools and concerns of formal approaches to semantics. Many issues remain to be resolved: the appropriate character of the syntactic input, the dividing line between semantics and pragmatics, the role of models in semantic theory. On the last point, see Zimmermann (2012). One important development within the field of formal semantics has been the attempt to extend the use of these tools to the analysis of less well-studied languages. Important contributions in this domain include Matthewson (2008) et seq. on the cross-linguistic study of the semantics-pragmatics interface and Schlenker et al. (2013) on the formal semantics of sign languages.

Further reading


References


George, Benjamin 2008. Predicting presupposition projection: some alternatives in the Strong Kleene tradition. Ms. UCLA.


Szabolcsi, Anna and Frans Zwarts 1993. Weak islands and an algebraic semantics of scope taking. 


**Related topics**

Chapter 1, (Descriptive) Externalism in semantics; Chapter 16, The semantics of nominals; Chapter 17, Negation and polarity; Chapter 18, Varieties of quantification.