Varieties of quantification

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1 What is a quantifier in natural language?

In his study of the word *both*, the philosopher M. Glanzberg addresses this question in the following way.

We often think about quantifiers via intuitions about kinds of thoughts. Certain terms are naturally used to express singular thoughts, and appear to do so by contributing objects to the thoughts expressed. Other terms are naturally used to express general thoughts, and appear to do so by contributing higher-order properties to the thoughts expressed. Viewed this way, the main condition on whether a term is a quantifier or not is whether its semantic value is an object or a higher-order property. At least, these provide necessary conditions. [. . .] We also often think about quantifiers in terms of a range of linguistic features, including semantic value, presupposition, scope, binding, syntactic distribution, and many others. [. . .] *Both* appears quantificational by some linguistic standards, and yet appears object-denoting by standards based on intuitions about the kinds of thoughts it expresses. It can appear this way, I shall argue, because the notion of quantification in natural language is in fact the intersection of a number of features, which do not always group together in the same ways, and do not always group together precisely in accord with our intuitions about expressing singular and general thoughts.

(Glanzberg 2008: 208)

On this view, which we also share, the subject matter of the study of quantification in natural language is defined and redefined as research progresses; it is not defined by some antecedently conceived distinction in philosophical logic.

Montague (1974), a classic that launched the enterprise of compositional formal semantics, offered an approach that does not force one to start with assigning expressions like *Arach, the dragon, every dragon, both dragons*, and their brothers to different and irreconcilable sides of the quantificational barricade. Montague’s theory therefore explained how the above expressions can be grammatically interchangeable in the majority of sentences in which they occur and yet contribute to the truth conditions of those sentences in their own ways. The approach, which crucially used generalized quantifiers as noun phrase denotations, dominated the 1970s and 1980s, and was extended to capture the commonalities in quantification over individuals, events and times (*always, usually, twice*), and worlds (*must and may, necessarily, probably, and possibly*).
The second major wave of research, starting in the 1990s and continuing as we speak, has uncovered complex and thorough-going differences in the behaviour of the above expressions; among them, the differences hinted at in the Glanzberg quote above. Three major classes of expressions emerged: distributive universals, extra-wide scoping plain indefinites, and modified numeral or counting quantifiers. Their ranks have been joined by event-related quantification, exemplified by verbal pluractionality and numeral phrase pluralization. Although many of the new results, once obtained, could be formalized using generalized quantifier theory, other methods have proved more conducive to discovery and to compositional analysis.

The third major wave, starting around 2000, is focusing on quantifier-phrase-internal and even quantifier-word-internal compositionality.

The discussion will track these strands of research. More detailed discussion can be found in Szabolcsi (2010); the relevant chapters will be pointed out.

2 Uniformity: generalized quantifiers

Consider the pairs *Arach* vs. *every dragon*, *on the 1st of May* vs. *always*, and *actually/in the actual world* vs. *obligatorily/must*. The members of each pair have similar grammatical behaviour. In addition, the first member of each pair refers to a particular individual, time, or world, and the second member is classically quantificational. It would be desirable to have an interpretive strategy that is compatible with all these similarities without obliterating the differences. Generalized quantifier theory offers such a strategy. A generalized quantifier is a semantic object (i.e. not a quantifier phrase). It is a set of properties of individuals, events, times, or worlds. Generalized quantifiers serve as a common denominator for the denotations of diverse expressions of the same category, and moreover offer a mechanics of interpretation that is uniformly applicable across several different categories.

(1) *Arach* belches fire.
the property of belching fire is an element of the set of properties that *Arach* has

(2) *Every dragon* belches fire.
the property of belching fire is an element of the set of properties that *every dragon* has

(3) *On the 1st of May* Ladro snacked.
an event of Ladro’s snacking is an element of the set of events that occurred on the 1st of May

(4) *Always, when he is at home*, Ladro snacks.
an event of Ladro’s snacking is an element of the set of events that occur whenever Ladro is at home

(5) Ladro *actually/in fact* thieves.
the fact of Ladro’s thieving is an element of the set of facts that obtain in the actual world

(6) Ladro *must* thieve.
the fact of Ladro’s thieving is an element of the set of facts that obtain in every world in which Ladro does what he is obliged to

Using the lambda operator to perform abstraction, the prose in (1) and (2) can be formalized as follows, in a simplified notation in the style of Heim and Kratzer (1998); analogous
representations can be devised for the adverbs and modals in (3) through (6), and for their interaction with other scope-bearing expressions. Double-bracketing, as in $[[\text{Arach}]]$, signifies the denotation of the enclosed expression. $x, y, z$ are variables over individuals (type $e$), $P$ is a variable over properties, extensionally, sets (type $\langle e, t \rangle$). Notice that the verb phrase’s denotation is asserted to be an element ($\in$) of the subject’s denotation, irrespective of whether the subject is a name or a universal.

(7) $[[\text{Arach}[\text{belch_fire}]]] = \lambda x . \text{belch_fire}(x)=True \in \lambda P_{(e,t)} . P(\text{arach})=True$

(8) $[[\text{every dragon}[\text{belch_fire}]]] = \lambda x . \text{belch_fire}(x)=True \in \lambda P_{(e,t)} . \forall \text{dragons } y . P(y)=True$

The same denotations $[[\text{Arach}]]$ and $[[\text{every dragon}]]$ are at work when these expressions occur in the direct object position. The object and the subject denotations combine with properties one at a time. Suppose we have,

(9) Ladro tricked every dragon.

(10) Some troll tricked every dragon.

In the interpretation of (9), the property $\lambda x . \text{belch_fire}(x)=True$ (being one that belches fire) that occurs in (7) and (8) is replaced by $\lambda x . \text{trick(ladro, x)}=True$ (being one that is tricked by Ladro), so that this property is attributed to every dragon. What about (10)? If we use the analogous property $\lambda x . \text{for some troll } y . \text{trick(y, x)}=True$, then being tricked by some troll or other is attributed to every dragon. Here the subject finds itself within the scope of the object, because the denotation of the subject is incorporated into the definition of the property that is asserted to be an element of the generalized quantifier that the object denotes. The scope relation between quantifier phrases is mediated by the relation between generalized quantifiers and their elements.

(11) Generalized quantifiers and their elements: operators and their scopes

A quantifier phrase denotes a set of properties. Its scope is that stretch of the sentence that denotes a property that is asserted to be an element of that set.

The “subject within the scope of the object” reading just discussed has the following semantic constituent structure:

(12)

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[subject] [verb] [object]
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We haven’t said how the properties $\lambda x . \text{trick(ladro, x)}=True$ and $\lambda x . \text{for some troll } y . \text{trick(y, x)}=True$ are assembled. This is a somewhat technical point which different theories will implement differently: with quantifying-in (Montague 1974), Quantifier Raising (May 1985, Heim and Kratzer 1998), type-lifting applied to verbs (Hendriks 1993), type-lifting applied to quantificational expressions in a continuation semantics (Barker 2002), and so on. Let’s say that in the first step $\lambda x . \text{trick(z, x)}=True$ is asserted to be an element of $[[\text{Ladro}]]$ or $[[\text{some troll}]]$, i.e. the property of tricking the individual $x$ is attributed to Ladro or some troll; subsequently (and this is somewhat sloppy) the variable $x$ is abstracted over to form the desired properties.
If, instead, the denotation of the direct object, \([\text{every dragon}]\) combines with \(\lambda x. \text{trick}(z, x) = True\) (the property of being tricked by the individual \(z\)), then \(z\) can be subsequently abstracted over, forming \(\lambda z. \text{for all dragons } y. \text{trick}(z, y) = True\) (the property of tricking every dragon). If this derivation is chosen, the object \textit{every dragon} will find itself within the scope of the subject. Now the semantic constituent structure will be this:

\[
(\text{subject}) \quad (\text{verb}) \quad (\text{object})
\]

Generalized quantifier theory does not only provide a uniform interpretation mechanism for a large and varied set of expressions. Generalized quantifiers are semantic objects well suited for the interpretation of the Boolean compounding (conjunction, disjunction, negation) of quantifiers. For example,

(14) \([\text{every dragon and some thief}] = [\text{every dragon}] \cap [\text{some thief}] = \lambda P_{(v,t)} (\text{for all dragons } x. P(x) = True) \text{ and for some thief } x. P(x) = True\)

(15) \([\text{on May 1st but not always}] = [\text{on May 1st}] \cap (\sim [\text{always}])\)

Generalized quantifiers also facilitate detecting and studying linguistically important semantic properties, again across categories. Consider the licensing of negative polarity items (\textit{any more . . .}):

(16) No dragons belched any more fire.
(17) Arach never belched any more fire.
(18) *Every dragon belched any more fire.

The rough consensus is that downward entailment (entailment from supersets to subsets) is the semantic property that unites NPI-licensors. This property can be conveniently defined using the denotations we are assuming. Let \(e\) be a variable over events (type \(v\)), and \(Q\) a variable over event-properties (type \(\langle v, t \rangle\)).

(19) An expression \(E\) is downward entailing iff \(B \in [E]\) entails \(A \in [E]\), whenever \(A \subseteq B\).

(20) \(\lambda x. \text{fly}(x) = True \subseteq \lambda x. \text{move}(x) = True\)

(21) \([\text{no dragons}]\) is downward entailing, because
\(\lambda x. \text{move}(x) = True \in \lambda P_{(v,t)}\), for no dragons \(y. P(y) = True\) entails
\(\lambda x. \text{fly}(x) = True \in \lambda P_{(v,t)}\), for no dragons \(y. P(y) = True\).

(22) \([\text{never}]\) is downward entailing, because
\(\lambda e. \text{Arach}_\text{move}(e) \in \lambda Q_{(v,t)}\), for no \(e. Q(e) = True\) entails
\(\lambda e. \text{Arach}_\text{fly}(e) \in \lambda Q_{(v,t)}\), for no \(e. Q(e) = True\).

(23) \([\text{every dragon}]\) is not downward entailing, because
\(\lambda x. \text{move}(x) = True \in \lambda P_{(v,t)}\), for all dragons \(y. P(y) = True\) does not entail
\(\lambda x. \text{fly}(x) = True \in \lambda P_{(v,t)}\), for all dragons \(y. P(y) = True\).
We have just shown that \( \langle \text{no dragons} \rangle \) and \( \langle \text{never} \rangle \) have the semantic property that licenses NPIs, but \( \langle \text{every dragon} \rangle \) does not. Notice that if we were using predicate logic, we might be able to write formulae whose truth conditions match those of the full English sentences, but we would not be able to assign explicit denotations to subsentential expressions and to check their semantic properties.

It is important to see that all the above can be done if we take the denotations \( \langle \text{no dragon} \rangle \), \( \langle \text{every dragon} \rangle \), and so on to be given; we do not need to know whether they are composed of parts, what the parts are, and what they mean. But in many cases we can go further. The part of the quantifier phrase that determines its denotational semantic properties is what generalized quantifier theory calls a (semantic) determiner. Semantic determiners denote relations between two sets, i.e. properties. For example:

\[
\begin{align*}
\langle \text{every} \rangle (A)(B) &= \text{True iff } A \text{ is a subset of } B. \\
\langle \text{no} \rangle (A)(B) &= \text{True iff the intersection of } A \text{ and } B \text{ is empty.} \\
\langle \text{more than one} \rangle (A)(B) &= \text{True iff the intersection of } A \text{ and } B \text{ has more than one element.}
\end{align*}
\]

This description suggests that the sets \( A \) and \( B \) are on a par. But in fact the relations denoted by natural language determiners are special in that they are restricted by one of the sets, \( A \): in checking the truth of \( \langle \text{det} \rangle (A)(B) \) one never needs to look beyond the set \( A \). Elements of \( B \) that are not in \( A \) do not play a role, nor do things that are in neither \( A \) nor \( B \). (Technically, restrictedness is a combination of conservativity and extension; see Barwise and Cooper (1981).) It turns out that this set \( A \) is the denotation of the nominal segment of the noun phrase, and thus the semantic asymmetry in the roles of the two sets nicely matches the asymmetry in syntactic constituency.

\[
\begin{array}{c}
\text{GQ} \\
\langle \text{scope} \rangle \\
\langle \text{determiner} \rangle & \langle \text{restrictor} \rangle
\end{array}
\]

It is to be noted that not all noun phrases denote generalized quantifiers; the dependent character of those underlined in (26) prevents them from doing so; they require extensions of the theory. The same holds for meanings that cannot be built step by step but require, instead, for two or more quantifiers to form a unit and combine with a relation in one fell swoop, illustrated in (27).

\[
\begin{align*}
\langle \text{the dragons are proud of } \{\text{themselves/each other}\} \rangle \\
\langle \text{every dragon lives in } \{\text{a different/the same} \} \text{ cave} \rangle [\text{than/as the other dragons}]. \\
\langle \text{Zwerg and Ladro saw two dragons each} \rangle
\end{align*}
\]

\[
\begin{align*}
\langle \text{Six bridegrooms led six brides to the altar.} \rangle \\
\langle \text{“six couples got married”} \rangle \\
\langle \text{Different people have different tastes.} \rangle \\
\langle \text{“pairs of people do not/need not have the same taste”} \rangle
\end{align*}
\]

3 Diversity in the behaviour of quantifiers

The uniform grammatical treatment afforded by generalized quantifier theory is a great advantage, and we have just seen that it correctly preserves differences between expressions that are directly denotational in nature, e.g. whether the expression is downward entailing or not. But at least two natural questions arise now. First, suppose we find differences in the behaviour of noun phrases. Do they fall out from the differences between the generalized quantifiers they denote, in combination with whatever syntax or pragmatics independently contribute to the meaning of the sentence? Second, does the internal composition of semantic determiners matter more than the picture in (24) and (25) suggests?

At least three interesting classes have emerged from the study of differential behaviour: plain indefinites, distributive universals, and modified numerals or counters; distributive numerals then form a bridge to the domain of pluractionality. In what follows we survey some of the most important findings. What is their theoretical consequence for generalized quantifier theory? It turns out that the findings do not follow from the insights that the theory offers, although they could often be encoded as “annotations” on the pertinent lexical items. That means that generalized quantifier theory remains a useful tool for grammars that are satisfied with the granularity that it offers. But it is limited in its ability to guide the research leading to new discoveries, and so current work is often not framed in its terms. See general discussion in Szabolcsi (2010: chapter 6).

The following convention is used in the presentation of examples. When a reading is prefixed with OK or #, that indicates that the given reading is available or unavailable, and does not imply anything about whether the example has other readings. This allows us to list and annotate only those readings of the examples that are relevant to the discussion.

3.1 Plain indefinites

The examples below involve plural indefinites, because only those can illustrate the full range of relevant facts. Definites share many properties with plain indefinites.

Indefinites can take unbounded (“extra-wide”) scope, i.e. existential import may extend outside the clause, even the island, that the indefinite is contained in.

(28) If Arach suspects that two dwarves snuck in, he gives out a roar.
    OK if Arach suspects that there are two dwarves that snuck in . . .
    OK if there are two dwarves that Arach suspects snuck in . . .
    OK there are two dwarves such that if Arach suspects . . .

Indefinites support discourse anaphora that parallels their existential import. For example, the following discourse is acceptable, but it requires two dwarves to have matrix-clausal existential import in the first sentence:

(29) If Arach suspects that two dwarves snuck in, he gives out a roar. {The two dwarves/they} had given him trouble in the past.
    OK there are two dwarves such that if Arach suspects . . .
Notice that *they* above can refer to just the two particular dwarves, like *the two dwarves* does. It does not have to refer to all dwarves that may have snuck in.

On the other hand, whichever interpretation of (28) we choose, the sentence only entails Arach’s giving out of a single roar (per suspicious situation), not one roar per dwarf, altogether two (per suspicious situation). In other words, no matter how broad its existential import, *two dwarves* in (29) cannot make a roar referentially dependent. The unbounded “existential scope” (existential import and discourse-referent introducing ability) of plain indeterminates contrasts with their clause-bounded “distributive scope”.

Within its own clause the plural indefinite is capable of inducing referential variation, observing the hierarchy Subject > Indirect object/Adjunct > Direct object, when neither it nor its target is partitive (see Beghelli (1997) for further details). In (30), quadruples of elves easily vary with dwarves but not with dragons; triples of dragons easily vary with dwarves and/or elves; pairs of dwarves do not vary at all.

(30) Two dwarves reported three dragons to four elves.

Plural indeterminates also support collective and cumulative readings:

(31) Two dwarves make a good team.
(32) Two dwarves slayed five dragons between them.

Among singular indeterminates, especially the type of *a certain dwarf* has unbounded existential import and supports (singular) discourse anaphora as *two dwarves* does, but given singularity, the question of inducing referential variation and supporting collective or cumulative readings does not arise.

(33) If Arach suspects that *a certain dwarf* snuck in, he gives out a roar. *The dwarf/he* had given him trouble in the past.

In the past two decades the standard treatment of these properties of plain indeterminates has had two crucial components. Their “existential scope” is due to existential closure, effected in a structure-building manner; given that it is syntactically unbounded, it cannot be a product of movement. *Two dwarves* itself denotes a set of two-dwarf pluralities; one of these pluralities is selected by a choice function variable; existential closure applies to this variable. (Alternatively, the value of the choice function variable is supplied by the context and “existential scope” is just an inference.) Their “distributive scope” is due to a distributive operator on the predicate. In other words, plain indeterminates are not operators; whatever operator-like behaviour they exhibit is due to the ministrations of some silent helper.

The distributive operator is notated as * and its working is defined in (35).

(34) Two dwarves had a beer.

\[ \exists f. f \text{ is a choice function and } *\left[ \text{had_a_beer}\right](f(\left[\text{two}\right](\left[\text{dwarves}\right])))=\text{True} \]

(35) \[ *P(x)=\text{True if } P \text{ holds of every atomic part of } x; *P(x)=\text{False if } P \text{ does not hold of any atomic part of } x; \text{undefined otherwise.} \]

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However, various problems have been identified in connection with the use of choice functions in the treatment of unbounded existential scope. Heim (2011) concludes,

If Schwarz (2001, 2004) is right, we may have to concede what Fodor and Sag (1982) and most subsequent authors wanted to avoid: indefinites are existential quantifiers that enjoy a greater degree of scopal mobility than other kinds of quantificational DPs.

(Heim 2011: 1022)

3.2 Distributive universals

The label serves to set apart every dragon and each dragon from all dragons (a generic of sorts; compare *All dragons are grouchy today with Every dragon is grouchy today) and from all the dragons (a definite plural; compare All the dragons collided with *Every dragon collided). Singular universal would also be a possible preliminary label, although it is not obvious how it would subsume every five dragons. We start with data and then turn to analyses.

Distributive universals of the each dragon type do, and those of the every dragon type do not, take extra-clausal distributive scope:

(36) Ask someone whether every dragon is dangerous.
    # for every dragon, ask someone whether it is dangerous

(37) Ask someone whether each dragon is dangerous.
    OK for each dragon, ask someone whether it is dangerous

Neither type supports singular discourse anaphora. However, both have unbounded existential import with respect to their nominal restrictions and support matching plural discourse anaphora, like indefinites do.

(38) {Every / Each} dragon was sitting on a heap. *It was guarding treasure.

(39) Ladro imagined that {every / each} dragon was at home. But they were out.
    OK there was a set of dragons, viz. the set of all dragons, and Ladro imagined that they were at home; but those dragons were out.

Neither type supports collective readings, nor cumulative ones, at least not when in subject position:

(40) *{Every / each} dragon is a good team.

(41) {Every / each} dragon ate ten ponies.
    # the dragons ate ten ponies between them
    OK the dragons ate ten ponies each

The mere fact that every / each dragon supports the distributive “ate ten ponies each” reading is not very remarkable; compare:

(42) {Two dragons / most of the dragons / at most two dragons} ate ten ponies.
    OK . . . ate ten ponies each

Much more interesting are the following facts, which seem to be unique to distributive universals.
Distributive universals enable the sentence-internal reading of singular a different cave. The plurals in (44) only support a contextual reading of it.

(43) {Every / each} dragon lives in a different cave.
   OK . . . in a cave that is different from where the other dragons live
(44) {All the / most / ten / more than five} dragons live in a different cave.
   # . . . in a cave that is different from where the other dragons live
   OK . . . in a cave that is different from one mentioned earlier

Distributive universals support a reading of matrix questions that can be answered with a list of pairs, not only with an individual or functional answer. A word of warning is in order here. Some speakers’ initial response may be that (46) can also be answered with a list of pairs and thus the # is incorrect. However, in (46) the list is not a grammatically conditioned answer, merely a cooperative way of spelling out a cumulative answer, and indeed many speakers only accept it if the wh-expression is semantically non-singular (plural what dishes, or unmarked what).

(45) What dish did {every / each} dwarf want?
   OK Zwerg wanted stew, Kerdil a pie, . . .
(46) What dish did {the / most / ten / more than five} dwarves want?
   # Zwerg wanted stew, Kerdil a pie, . . .
   OK A roast.
   OK Their own favourite dishes.

The above generalizations are discussed extensively in multiple articles in Szabolcsi (1997a). Distributive universals support unproblematic “donkey-sentences”, i.e. sentences in which an indefinite within the restriction of the quantifier is referred back to with a singular pronoun in the scope of the quantifier (the main clause), as in (47). In contrast, speakers find the plural examples in (48) either unacceptable or confusing in situations where dwarves own multiple donkeys (Kamp and Reyle 1993; Kanazawa 1994, 2001).

(47) {every / each} dwarf who owns a donkey feeds it.
   OK every dwarf feeds whatever donkey(s) he owns
(48) ?{The / most / ten / more than five} dwarves who own a donkey feed it.

Brasoveanu (2008, 2010) pulled together the above observations and added new ones, initiating an interesting new line of research. One suggestive example is as follows:

(49) Every dwarf who loaded a donkey with provisions fastened its burden to its back.

We paraphrased (47) using “whatever donkey(s) he owns” – but (49) does not easily lend itself to that treatment. In particular, the interpretation must synchronize individual donkeys, their burdens, and their backs. Brasoveanu (2012) observes that a similar synchronization occurs in conditionals, when(ever)-clauses, and classical multiple correlatives in South-Asian and Slavic languages:
(50) “whenever a dwarf loaded a donkey with provisions, that dwarf fastened that donkey’s burden to that donkey’s back”

We now turn to analyses. Beghelli and Stowell (1997) proposed that every/each dragon is similar to the dragons in that it contributes only a set of individuals to the interpretation of the sentence. This set is the set of all dragons. It can be computed as the denotation of the nominal restriction of the determiner, dragon(s). Every/each dragon and the dragons differ in how sentences that contain these expressions obtain their distributive readings. Distributive readings of the dragons are due to the * operator that modifies the predicate, exactly as in (34) and (35), but the presence of * is optional. In contrast, every/each dragon must associate with a syntactic functional head Dist, whose semantic content, as its name suggests, is a distributive operator. On Beghelli and Stowell’s analysis every and each themselves are not distributive operators; instead, they force the association of their noun phrases with a clause-level distributive operator, much like “negative morphemes” in negative concord items are themselves not negative (pace their glosses): instead, they force association with an overt or null clause-level negative operator.

(51) Nikto nichego ne videl. (Russian)
    nobody nothing not saw
    “Nobody saw anything”

(52) Personne ∅ a vu rien. (French)
    nobody neg saw nothing
    “Nobody saw anything”

For general discussion, see Szabolcsi (2010: chapter 8).

The significant innovation that Brasoveanu (2008, 2010, 2012) introduces pertains to the encoding of distributivity. On Brasoveanu’s theory, the domain of quantification in sentences with every and each is not a set of individuals, but a set of dependencies between individuals (technically, variable assignments, or sequences). The anaphoric connections, pair-list answers, and internal readings of a different discussed above all live off of those dependencies.

The core of the analysis revives the early 1980s Kamp/Heim analysis of quantificational determiners in terms of case-quantification, following Lewis’s (1975) proposal for adverbs of quantification, where a “case” is an n-tuple of individuals. That analysis was intended to be general, and it was quickly abandoned, because it was marred by the so-called proportion problem. The determiner most (of the) counts donkey-owning dwarves; only adverbs count dwarf–donkey pairs. Therefore the truth conditions differ.

(53) Most (of the) dwarves who own a donkey feed it.
(54) {For the most part / usually}, if a dwarf owns a donkey, he feeds it.

The present reincarnation does not involve proportional determiners, and so the same problem does not arise. The unification of a wide range of phenomena that are specific for distributive universals is a remarkable payoff. The compositional aspects of the analysis are not yet obvious, and may not be simple.
3.3 Modified numeral or counting quantifiers

Kamp and Reyle (1993) accorded modified numerals the same quantificational treatment as distributive universals (box-splitting, in DRT terms); Beghelli and Stowell (1997) and Szabolcsi (1997b) keep them apart both from plain indefinites and from distributive universals. The three-way separation is necessary, because noun phrases such as *at least/at most five dragons, more/fewer than five dragons, five or more dragons, [focus five] dragons*, etc. do not have the kind of unbounded (“extra-wide”) existential scope that plain indefinites have, and they do not license the singular *a different* or matrix pair-list readings that distributive universals do:

(55) If Arach suspects that more than five dwarves snuck in, he gives out a roar.

   OK  if Arach suspects that there are more than five dwarves that snuck in . . .
   #    if there are more than five dwarves that Arach suspects snuck in . . .
   #    there are more than five dwarves such that if Arach suspects they snuck in . . .

(56) More than five dwarves came from a different village.

   #    . . . from villages different from each other’s

(57) What dish do more than five dwarves want?

   #    Zwerg wants stew, Kerdil a pie, . . .

They are also poor clause-internal inverse scope takers, at least in the most frequently studied configuration in (58). In this respect they contrast with distributive universals (Liu 1997, Beghelli and Stowell 1997). But the same does not carry over to (59) and (60), as was observed in Takahashi (2006).

(58) Every dragon saw more than five dwarves.

   #    there are more than five dwarves that every dragon saw

(59) Tovenaar showed every dwarf to more than five elves.

   OK    . . . there are more than five elves to whom Tovenaar showed every dwarf

(60) Tovenaar showed more than five dwarves to every elf.

   OK    . . . there are more than five dwarves that Tovenaar showed to every elf

The most intriguing property of this class is that it lays bare the limitations of a purely truth-conditional account. The class of modified numerals or counting quantifiers cannot be delimited in truth-conditional terms, and even within the class, truth-conditionally equivalent members behave differently. Here is a sampler.

Although *most of the* and *more than 50% of the* are equivalent according to Barwise and Cooper (1981), only noun phrases determined by the latter host binominal *each* (Sutton 1993). Szabolcsi (1997b, 2010: chapter 10) finds that admitting binominal *each* is a good diagnostic of counting quantifiers in English:

(61) *The dragons saw most of the dwarves each.

(62) The dragons saw more than 50% of the dwarves each.

Among counting quantifiers, although *at least two* and *more than one* are logically equivalent, the reciprocal *each other* cares about which is used (Hackl 2002):
At least two dwarves shook hands [with each other].

# More than one dwarf shook hands [with each other].

At most $n$ and at least $n$ are epistemic modal varieties of the plain indefinite containing $n$, in contrast to fewer than $n$ and more than $n$. This may explain their differences in supporting namely-anaphora and in interaction with possibility modals (Geurts and Nouwen 2007):

Ladro invited \{at most two / at least two\} friends, namely Zwerg and Medve.

?Ladro invited \{fewer than four / more than two\} friends, namely Tovenaar, Zwerg, and Medve.

The pony can carry fewer than six sacks.

OK  not able/allowed to carry more than five
OK  able/allowed to carry not more than five

The pony can carry at most five sacks.

OK  not able/allowed to carry more than five
#  able/allowed to carry not more than five

In addition to such contrasts in acceptability, Koster-Moeller et al. (2008), Hackl (2009), and Lidz et al. (2011) showed that people process some of these expressions differently even if they are logically equivalent. What processing strategies experimental subjects use can be detected by presenting the verification task in different ways that make it easier or more difficult to perform, depending on what strategy the subject uses. When asked to verify statements like Most/More than half/More than ten of the dots are blue, experimental subjects were found to persist in using counting strategies that tracked the composition of the expressions, even when that made performing the verification task more difficult. Generalizing, this leads to some version of the following hypothesis:

Interface Transparency Thesis (Lidz et al. 2011: 229):

Speakers exhibit a bias towards the verification procedures provided by canonical specifications of truth conditions.

To be sure, the significance of syntactico-semantic composition that underlies the canonical specifications of truth conditions is probably not particular to counting quantifiers. These expressions make the fact more conspicuous, because they have more varied and more complex structures than some other quantified expressions. Complexity is especially clear in the case of expressions like more than five dragons and the most dragons, which involve comparisons and degree quantification (Heim 2001; Hackl 2009). Takahashi’s (2006) account of the inverse scoping data, cf. (51)–(53), also exploits the syntactic complexity associated with degree quantification. The basic idea is that these expressions comprise two quantifiers that part ways in the course of the syntactico-semantic derivation but may only do so subject to severe constraints. See Szabolcsi (2010: chapters 10 and 11) for further discussion.

Generalized quantifier theory is a good tool for studying those properties of noun phrases that can be captured in truth-conditional terms, but would not be a particularly good tool for studying these newly discovered properties.

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3.4 Distributive numerals and pluractionality

The so-called binominal *each* construction in English is clearly related to both distributive quantification and to counting quantifiers. On one hand, it excludes collective (“all together”) and cumulative (“between them”) readings. On the other hand, as Sutton (1993) observed, it does not suffice for the host to be indefinite; it must be a counting quantifier:

(70) *Three dragons captured {those/most of the} dwarves each.
(71) Three dragons captured {five/few/more than 70%} of the dwarves each.
(72) *Three dragons captured {dwarves/no dwarves} each.
(73) Three dragons captured {one/?a} dwarf each.

The acceptable sentences mean that each dragon captured \( n \) (% of the) dwarves, with the addition that they did not capture the same dwarves; i.e. \( n \) (% of the) dwarves takes narrow scope and exhibits some variation. Although it is interesting that distributivity is marked on the distributed share, in English this seems like the end of the story. In many other languages, however, similar sentences can be true in a wider range of situations. According to Balusu (2005), the Telugu sentence in (74) can be true in any of the situations described in (75).

(74) ii pilla-lu renDu renDu kootu-lu-ni cuus-ee-ru
    these kid.pl two two monkey.pl.acc see.past.3pl
    lit. “These kids saw two monkeys”

(75) OK these kids saw two monkeys each
    OK these kids (jointly or severally) saw two monkeys at each location
    OK these kids (jointly or severally) saw two monkeys at each interval

What we see is that pairs of monkeys can be distributed over event-participants (kids), but also over contextually individuated spatial or temporal parts of the event. To wit, the following is an equally good sentence in Telugu:

(76) renDu renDu kootu-lu egir-i-niyyi
    two two monkey.pl jump.past.3pl
    lit. “Two two monkeys jumped”

(77) OK two monkeys jumped at each location
    OK two monkeys jumped at each interval

Balusu analyzes distributive numeral reduplication as the pluralization of the denotation of a numeral phrase, derived by distribution over parts of an event. Balusu also argues that the reading that (76) shares with English sentences like *The kids saw two monkeys each* does not involve direct distribution over kids; instead, it is a special case in which kids and events correspond one-to-one.

Szabolcsi (2010: chapter 8.4) observes that distributive numeral reduplication belongs to the same family of formal devices as the -ssik suffix (Korean), -nka (Quechua), -na (Basque), po (Russian), zutsu (Japanese), and jeweils (German); see the references there. We add here that, when properly spelled out, the interpretation requires the existence of multiple contextually individuated events (event-parts), and therefore locate the phenomenon in an even larger family, the domain of pluractionality.
The term pluractionality refers to event-pluralization, which may manifest itself in multiple event-participants or multiple occurrences of the event over time or at different locations (Lasersohn 1995). Its marking, by inflection, reduplication, or particles, may appear on verbs, on noun phrases, or on numerals; measure phrase split in Japanese seems like another event-pluralizing device (Nakanishi 2007). In +Hoian, a Khoisan language, the same plural marker applies to verbs and nouns (Collins 2001). Therefore pluractionality offers an especially striking example of the unity of nominal quantification and event quantification.

For both careful cross-linguistic documentation and theoretical analysis, see Matthewson (2000), Zimmermann (2003), Champollion (2012), Cable (2012), and Henderson (2012), among many other pieces in the rapidly growing literature.

4 Compositionality in quantifier words and future directions

Many semanticists intend their research to be guided by the principle of compositionality (see Chapter 24).

(78) The meaning of a complex expression is a function of the meanings of its parts and how they are put together.

In section 2 we saw that generalized quantifier theory gives excellent insights into how the larger constituents (subject, object, etc.) contribute to semantic interpretation. On the other hand, it does not help much with investigating the impact of the internal composition of quantifier phrases; see the discussion in section 3, especially towards the end of 3.3. So different theories offer compositional insights of different granularity. Let us now supplement the above considerations with the following question.

(79) Are (phonological) words the smallest parts that a compositional grammar should take into account? If not, what smaller parts are to be recognized?

Although there is no doctrine that says that word meanings are the minimal building blocks of sentence meanings, in practice semanticists often make that assumption. For example, we readily assign very complex interpretations to quantificational words without specifying how the semantic ingredients are anchored in the components of those words. That practice is probably motivated by the time-honoured lexicalist tradition in syntax. It is therefore of some interest to observe that in the past two decades different lines of research have been converging on the view that words do not have a distinguished status in morpho-syntax; see Distributed Morphology as well as a strand of research in Minimalist Syntax (Julien (2002), Koopman (2005), among others). If that is on the right track, then it does not go without saying that words are minimal building blocks for compositional semantics.

Deconstructing quantifier words has both logical and cross-linguistic motivation, according to Gil’s (2008) article in The World Atlas of Language Structures (WALS):

[S]ome semanticists have proposed deriving the interpretations of universal quantifiers from those of conjunctions. For example, in the Boolean Semantics of Keenan and Faltz (1986), conjunctions and universal quantifiers are both represented in terms of set-theoretic intersections.
How well do such semantic representations correspond to the observable lexical and grammatical patterns of languages? . . . One might suspect that they do not correspond at all well. Thus, in English, the conjunction and and the universal quantifier every are distinct words with quite different grammatical properties.

However, a broader cross-linguistic perspective suggests that there are indeed widespread lexical and grammatical resemblances between conjunctions and universal quantifiers, thereby lending support to the logicians' analyses . . . .

For the purposes of the [WALS] map, conjunctions are taken to include not only forms with meanings similar to that of and, but in addition expressions that are sometimes characterized as conjunctive operators or focus particles, with meanings resembling those of also, even, another, again, and in addition the restrictive only. As for universal quantifiers, these are assumed to encompass not only forms with meanings such as those of every, each and all, but also expressions that are sometimes referred to as free-choice.

(Gil 2008, emphases in the original)

To illustrate, consider the Japanese data from Shimoyama (2006) and Kobuchi-Philip (2009):

(80) a nani-mo “everything/anything” (dep. on stress)
b jyuu-nin-mo-no gakusei “as many as ten students”
c Tetsuya-mo Akira-mo “both Tetsuya and Akira”
d Tetsuya-mo “also/even Tetsuya” (dep. on stress)

In this particular case there is no doubt that nani-mo is not a compositional primitive, since both parts lead independent lives in exactly the same sense as they have within nani-mo. For example (Shimoyama 2007: 146):

(81) [Taro-ga nani-o katta-kara] okotta] hito]-mo heya-o deteitta.
    Taro-Nom what-Acc bought-because got.angry person-MO room-Acc left
    ‘For every thing x, the people who got angry because Taro had bought x left the room.’

On the other hand, the relationship between (80a) and (80b, c, d) is rarely investigated, Kobuchi-Philip (2009) being one of the remarkable exceptions. It appears that compartmentalization is more of a matter of research habits than an empirical or theoretical necessity. In view of Gil’s cross-linguistic observations, unified investigation could be most fruitful. Likeminded work on Japanese -ka, Malayalam -oo, Sinhala da, and Tlingit sá, which appear in indefinites, disjunctions, and questions, has been flourishing in recent years; see Cable (2010), Slade (2011), and references therein.

The quantificational elements more and most have traditionally been analyzed setting aside word boundaries, in generative syntax as well as in recent work in semantics (Heim (2001), Hackl (2009), among many others), and are boosted by new insights from the cross-linguistic patterns of suppletive morphology in comparatives and superlatives (Bobaljik 2012). Szabolcsi (2010: chapter 12; 2015) highlights a range of further results pertaining to compositionality in quantifier words.

Extending compositionality below the word level, especially in quantification as opposed to inflectional morphology, is a new domain of inquiry and no doubt raises many methodological and theoretical questions. But taking this path seems inevitable, even though it must be treaded with appropriate caution.
Further reading


Keenan, Edward L. and Denis Paperno (eds) 2012. Handbook of Quantifiers in Natural Language. Dordrecht: Springer. Questionnaire-based surveys of quantifiers in 17 languages representing altogether 12 language families, written by native speaker or field-worker semanticists.


References


Cable, Seth 2012. Distance distributivity and pluractionality in Tlingit (and beyond). http://www.semanticsarchive.net/Archive/2Y5NzgwZ/.


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Varieties of quantification


Related topics

Chapter 4, Foundations of formal semantics; Chapter 16, The semantics of nominals; Chapter 17, Negation and polarity; Chapter 23, Event semantics; Chapter 24, Compositionality.