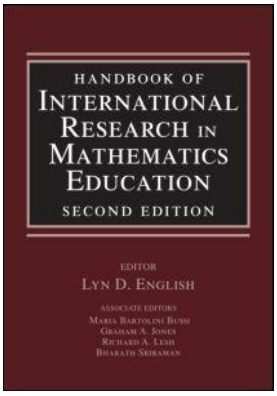

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From mathematics learning out-of-school to multicultural classrooms

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15 From mathematics learning out-of-school to multicultural classrooms

A cultural psychology perspective

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INTRODUCTION

The relationship between the learning and uses of mathematics in and out of school has been discussed in recent decades from several distinct perspectives. Two distinct phases are discernible in this research. First, until the early 1990s, research was primarily driven by a desire to understand learning and uses of mathematics in non-traditional (often non-Western) contexts. The focus of the research in this first stage was on relationships between culture and mathematics cognition. From an educational point of view, the agenda was driven by a need to understand difficulties non-Western groups faced with Western-like mathematics introduced with schooling (e.g., Cole, 1977; Cole, 1995). More recently, globalization and the unprecedented levels of migration shifted the major focus of this research to multicultural settings within Western societies. In many Western countries, immigration is substantially changing the ethnic and cultural composition of the school population. This diversity within schools poses major challenges to systems of education developed to serve the needs of homogeneous monocultural groups (Abreu & Elbers, 2005; Cole, 1998). The European Society for Research in Mathematics Education (ERME) responded to this challenge by creating a special working group focused on research on mathematics learning in multicultural settings (Gorgorió, Abreu, Cesar, & Valero, 2005; Gorgorió, Barton, Elbers, & Favilli, 2003).

Insights into the nature of mathematics learning in out-of-school contexts were initially data driven, grounded in ethnographic-type descriptions and linked to different disciplines. For example, two disciplines that have approached the problem from a sociocultural stance are ethnomathematics education (D'Ambrosio, 1985) and developmental psychology (Nunes, Schliemann, & Carraher, 1993). As illustrated in Table 15.1, studies along these lines provided complementary perspectives in the analysis of mathematics learning outside school.

Developments in cultural psychology in the last decades provide a useful framework to integrate theoretical ideas, methodologies and empirical findings emerging from different but complementary disciplines, such as the ethnomathematics and developmental psychology referred above. The next section outlines the main focus of cultural psychology and how it will be used in this chapter as a framework to review past research in out-of-school mathematics and inform discussion around current issues of mathematics learning in multicultural classrooms.

CULTURAL PSYCHOLOGY: A FRAMEWORK TO REFLECT ON PAST RESEARCH AND FRAME THE AGENDA FOR THE FUTURE

Within the discipline of psychology, the renewed interest in cultural psychology is linked to a shift in the focus of the studies from products to processes. Studies on mathematics learning have played a central role in this development (Lave, 1990). Efforts to understand the relationship between mathematical thinking and culture can be traced to the interest of Western science in testing the universality of cognitive development and to the expansion of Western style of schooling in other societies (Cole, Gay, & Glick, 1968). The latter included mathematics as one of key subjects in the elementary curriculum. Cole (1977, 1995) recounted his first task in (cross)-cultural research as being "to figure out why Liberian children seemed to experience so much difficulty learning mathematics" (Cole, 1995, p. 23). He also noted that his "graduate training was in the tradition of American mathematical learning theory, which at that time entailed the use of algebra and probability theory to provide a foundation for discovery of presumably universal laws of learning." This happened about 40 years ago, and as he confessed, he "knew almost nothing about the teaching of mathematics, and even less about Liberia" (Cole, 1995, p. 24). As now seems obvious, Cole and his group soon grew skeptical about their own knowledge and methods. The contradiction between the logic of thinking that Cole and his colleagues were trying to investigate and the logic used by the Liberians was illustrated in a well-cited anecdotal episode (Lucariello, 1995). In the course of a classification study, which required participants to sort objects, it was observed that they tended to use functional groupings. The participants justified their sorting on the basis of their understanding of how "smart" people would undertake the task. The researchers then asked them how they thought a "less smart" person would undertake the task. Following this request the objects were grouped in taxonomic categories. Obviously, there were distinct types of logic in operation: the logic of taxonomy in the mind of the researcher and the logic of functionality in the mind of the participants.

Cole's example at first glance exposes the inadequacy of the research procedure used. However, as cautioned by Rogoff and Lave (1984), this does not allow us to jump to the conclusion that variation between cultural groups could be explained solely in terms of inadequacies of research procedures, either in tests and tasks or in the difficulties of communication between researchers and participants. These explanations would reduce the differences in performance to differences in "display" (Shweder, 1990) and presuppose that in ideal circumstances of testing the differences should disappear. Still implicit in such a view is the idea of cognition as a process located in an autonomous individual mind. Cole and his colleagues also observed that differences could be linked to the tools used as mediators. To their surprise, the differences were not only linked to a tool being available, but to a more complex organization of the tool itself. Hand spans and foot lengths were tools used both in the United States and by the Kpelle of Liberia. When Cole and his colleagues tested both Americans and Kpelle in tasks that required estimating length with hand spans and foot lengths, the Americans performed better despite the familiarity of both groups with the tools. Further analysis revealed that although both groups were familiar with these length measures, the Kpelle did not relate

Table 15.1 Two perspectives on outside school mathematics

	Ethnomathematics education (D'Ambrosio, 1985)	*Developmental psychology* (Nunes et al., 1993)
Level of analysis	Sociogenetic level: Historical and anthropological analysis of the mathematics of different sociocultural groups.	Ontogenetic level: Analysis of the individual's psychological processes involved in learning and using mathematics in specific sociocultural contexts.
Focus of analysis	Relationship between social-political order and individual learning: How the value social groups attribute to certain forms of mathematics mediates its transmission and appropriation.	Relationships between culture and cognition: How specific cultural tools mediate mathematics cognition.

different measures in a system, thus making the task more difficult to accomplish. In contrast, the Americans were familiar with well-articulated systems, such as inches, feet and yards that enabled them to translate between the measures. Observations of this kind revealed the need for a shift from studying products to studying processes. This shift is at the heart of the movement from cross-cultural to cultural psychology (Cole, 1995). It marked the starting of a new era in psychological approaches to mathematics learning outside school. The revised theoretical and methodological foundations led to the emergence of new research programmers, such as the series of "street mathematics" studies carried out by Nunes and her colleagues in Brazil in the 1980s (Carraher, Schliemann & Carraher, 1988; Nunes et al., 1993). Research in "everyday cognition," "situated cognition," and "mathematics learning in and out-of-school" flourished in the 1980s, with seminal research works being published (see, for instance, Lave, 1988; Rogoff & Lave, 1984; Saxe, 1991).

A basic claim shared by the principal proponents of cultural psychology (see Stiegler, Shweder, & Herdt, 1990; Wertsch, 1991) is that the adoption of such a perspective offers the possibility of following an alternative agenda that focuses on accounting for diversity in human psychological functioning. Of course, understanding variations between groups and individuals has been at the core of psychology, but the mainstream approach searches for its sources in the universality of the mind and the presence or absence of capacities, properties, traits, and so forth. Cultural psychology searches for the sources of diversity in socioculturally specific experiences. As Engeström noted, "the challenge is to go beyond traditional psychological conceptions which place motives inside the individual" (1999, p. 255). This standpoint does not deny the existence of universals linked to the biological make-up of humans (Bruner, 1996; Cole, 1996; Nunes & Bryant, 1996). Instead, it attempts to pay more attention to issues that appear to have been neglected since Psychology was established as a science by Wundt (Cole, 1995).

Proponents of cultural psychology approaches also share the view that understanding diversity requires attention to the interplay between the individual, society and culture. For analytical purposes, Much (1995) suggested that the person, the society, and the culture can be seen as "the three systems of cultural psychology." She argued that each can be seen as a system in terms of its organization and dynamics.

- The first system "is a person, with a distinctive biological make-up and unique history of experience."
- The second system—"is a 'society', more precisely, the local social structures (for example, the family and other institutions) of a society or culture." And,
- The third system—"is culture in its symbolic sense, culture as a representational system, the collective symbol systems and institutionalised meanings for interpretation and organisation of experience and action in local social contexts" (Much, 1995, p. 100).

Although each can be seen as a system, they are not independent of each other. Instead, as Much stressed, they are mutually constitutive or co-create each other. For instance, there is content overlap among the three systems. A personal identity, for example, may entail the mastering of certain representational systems as part of one's culture and also positions in social structures, such as roles and status.

Rogoff's (1995) three planes model to account for human development in sociocultural contexts is also based on the idea that interplay between different parts of a system can be studied separately, but are mutually constitutive. Her three planes of analysis focus on the personal, the interpersonal, and the community. These planes bear resemblance to Much's three systems. To Rogoff, the community is the institutional- cultural plane of the activity; the interpersonal includes interaction with others directly or through the social organisation of cultural activities; the personal is the plane of the individual's change (in "which individuals transform their understanding of and responsibility for activities through their own participation" Rogoff, 1995, p. 150). In her view, however, these planes are part of a whole (activity or event), and each plane can be taken as the main focus of analysis (the foreground) while the others remain in the background. Recent reviews of sociocultural approaches to mathematics learning led me to believe that an analysis addressing relationships between different systems or planes of activity can shed new light on ways of interpreting and studying critical issues. For instance, it might help to clarify the emergence of individual diversity among mathematical learners and in particular whether this diversity originates from the interplay between sociocultural and person systems (Abreu & Cline, 2003; Abreu & Elbers, 2005; Gauvain, 1998; O'Toole & Abreu, 2005).

If mathematics learning is examined from a cultural psychology perspective, the mutually constitutive systems or planes need to be addressed. In particular, it will be useful to clarify the ways in which the interplay between the systems is studied. This is the approach I follow in my review of the literature. I use Much's three systems, combined with the notion that cultural psychology emphasizes mediated action in a context (Cole, 1996, p. 104), as a framework to present the review and my own thinking on mathematics learning in out-of-school contexts and multicultural classrooms. The next part of the chapter reviews research that has focused on the mediating aspects of:

- The cultural system (the role of cultural tools, both mental and physical);
- The social system (the role of social interactions and other types of social processes);
- The person system (the role of the individual in the re-construction of the cultural and the social at the level of the person system).

The decision to follow a descending order from Much's third system to the first is adopted to reflect the movement in the cultural psychology of mathematics education from an understanding of mathematics of particular cultural and social groups to the understanding of the person as a participant in sociocultural practices.

FOCUS ON THE CULTURAL SYSTEM

Approaches in the study of the impact of a cultural system on the learning and uses of knowledge vary, depending on the way culture is conceptualized. Valsiner (1989) raised this issue by asking the question "What is culture in the minds of psychologists?" (p. 502). He attempted to answer this question by pointing out how the underlying view of culture in mainstream cross-cultural studies has led to its treatment as an independent variable. That is, culture has been understood as "something that is, in its essence, shared in a qualitatively similar manner by all (or almost all) members of the given culture (as a population, society, or an ethnic group)" (p. 503). For Valsiner, this treatment of culture overlooks its historical-developmental dimension. Cultural psychologists have certainly moved beyond the notion of culture as an independent variable, but this does not mean they have a shared definition of culture. Cole (1995) suggested caution in the use of the term *culture* and referred to the search for a generally accepted definition as a "hopeless enterprise" (p. 31).

Most studies that have explored issues related to mathematics learning in out-of-school contexts, at the level of a cultural system, have emphasized the historical, practical and socially organized nature of the activity. These include studies by anthropologists such as Lave (1988) and Brenner (1983, 1998a, 1991), mathematics educators (Bishop, 1988a, 1988b; D'Ambrosio, 1985; Knijnick, 1993, 1996) and developmental psychologists (Cole et al., 1968; Nunes et al., 1993; Saxe, 1991). A major influence on psychologists working in this area has been Vygotsky's notion of cultural mediation (van der Veer, 1996). A central focus of the psychological studies has been on understanding what are the tools or artifacts used in specific social practices and their role in mediating mathematical action and thinking. As noted by Resnick, Pontecorvo, and Säljö (1997) in situated theories of cognition,

> The concept of tool is expanded (…) beyond the conventional view of a tool as a physical artifact. Not only physical artifacts but also concepts, structures of reasoning, and the forms of discourse that constrain and enable interactions within communities qualify as tools. Vygotsky (…) originally distinguished tools from signs, or language. However, subsequent influential developers of theories of socially situated cognition (…) have suggested that many kinds of thinking, as well as physical actions, are carried out by means of tools. (p.3)

According to Cole, the relevance of "thinking about culture as a medium constituted of historically cumulated artifacts which are organized to accomplish human growth must be demonstrated by its ability to help us understand the processes of learning and development." (1995, p. 35). He seemed to have no doubts that he achieved these demonstrations in his empirical investigations. There were two distinct phases in the investigation of cultural influences on mathematics learning in out-of-school contexts. In the first phase, scholars traveled to other countries to investigate the mathematical thinking and learning of foreign people. Although not always made explicit, this traveling was often politically motivated. The sponsoring of this type of research was linked to the intention of Western countries to support the introduction of their style of schooling in non-schooled cultures (see, for instance, Cole et al., 1968). The second phase involves the researchers' return to their home countries. This also has a politically motivated dimension, coming from the researchers themselves, who realized that the lessons learned abroad could throw light on the learning of diverse groups in their own country. It was also linked to the need in industrialized countries to increase the level of mathematical education of the whole population (Zaslavsky, 1990). Key studies from these two phases are reviewed below.

Cross-cultural studies

Some of the pioneer studies of mathematics in out-of-school contexts were carried out on non-Western cultures (Brenner, 1983; Gay & Cole, 1967; Lave, 1977; Pettito & Ginsburg, 1982; Saxe, 1982; Saxe & Posner, 1983). These studies can now be considered a landmark in view of the realization of psychologists that cultural differences are not necessarily associated with deep cognitive differences. They also persuaded psychologists to rethink their research methodology. Instead of taking school knowledge as the reference and formulating tasks from this perspective to see whether individuals transfer to other settings, the researchers engaged in ethnographic observations in outside school contexts. In these studies one can see, along with some traditional cross-cultural concepts, the emergence of new constructs such as apprenticeship, distinct arithmetic systems, and strategies that mark a new era of research in out-of-school mathematics. One also can see how unexpected findings have challenged views about key constructs that have been used to explain mathematical thinking and learning in and out-of-school. Among the key constructs (linked to a universalistic, culture-free, view of mathematical cognition and learning) challenged were concrete versus abstract dichotomy and the notion of transfer.

Challenging the Concrete versus Abstract Dichotomy

About four decades ago, Piaget (1966) wrote: "… it is quite possible (and it is the impression given by the known ethnographic literature) that in numerous cultures adult thinking does not proceed beyond the level of concrete operations, and does not reach that of propositional operations, elaborated between 12 and 15 years of age in our culture" (p. 309). Abstract thinking for Piaget was characterized by the independence of the form (abstract) from the content. Ability to reason abstractly in this perspective presupposes the existence of specific cognitive structures. Basic mechanisms that will allow individuals to construct these cognitive structures were considered endogenous to human nature. Thus, in theory individuals will be able to achieve a level of abstract thinking independent of the culture they live in.

Vygotsky also distinguished knowledge between different cultural groups in terms of the concrete versus abstract. However, his view of the origins of these forms of thinking was distinct from that of Piaget. He held that abstract thinking demanded a higher cognitive ability, and as such was acquired through sociocultural mediation. Thus, the ability of individuals to reason in concrete and in abstract terms would be a reflection of the historical development of their sociocultural groups. Collaboration between Vygotsky and Luria led to a series of studies in Uzbekistan where this view was articulated. In the foreword to Luria's (1976) book, Cole stressed that

> His [Luria] general purpose was to show the sociohistorical roots of all basic cognitive processes; the structure of thought depends upon the structure of the dominant types of activity in different cultures. From this set of assumptions, it follows that practical thinking will predominate in societies that are characterised by practical manipulations of objects, and more 'abstract' forms of 'theoretical' activity in technological societies will induce more abstract, theoretical thinking. (p. xiv–xv)

Thus, it should not come as a surprise that until recently the concrete versus abstract dichotomy was used as an important criterion to distinguish between mathematics learning in and out-of-school. This notion was challenged when the studies consistently failed to identify the presence of any formal cognitive structures in individuals living in non-Western societies, who nevertheless demonstrated high competence when dealing with their local practices. Piaget (1972) attempted to revise his theory to accommodate these findings. New insights emerged when researchers stopped trying to establish the existence of particular cognitive structures, through the use of Piagetian-type tests, to investigate the strategies actually used in the solution of practical arithmetical problems.

The focus on strategies rather than structures led to new emphasis on differences in the use of mathematics in and out-of-school. According to Nunes (1992b) the differences were not along a concrete—abstract dimension, but were to do with the existence of multiple arithmetic systems in a single culture. Initial support for this assumption was based on Reed and Lave's (1981) observations of differences in strategies and types of errors depending on whether tailors used the system linked to tailoring or to school. Reed and Lave argued that the use of tailoring based tools (use of counters, such as fingers, pebbles, marks on paper) was associated with a manipulation-of-quantity strategy. Errors in this case tend to be of small magnitude. On the other hand use of school tools was associated with a manipulation-of-symbols strategy. Similar findings were observed in other studies (such as, Grando, 1988; Nunes et al., 1993). The link between strategy and the systems available in particular cultures is in line with Vygotsky's notion of cultural mediation. Vygotsky, however, tended to conceptualize out-of-school forms of knowledge as more concrete when compared with the scientific and abstract forms linked with school, a distinction that became questionable when strategies were analyzed. Viewing cognitive and psychological functioning as deeply embedded in historical contexts had enabled researchers to demonstrate that uses and learning of mathematics in and out-of-school cannot be defined in oppositions (such as abstract versus concrete). As argued and demonstrated by Khan (2002) "School sites can become sites for concrete thinking just as theoretical understandings are affected in the concrete workings of everyday situations, and differences are not always as marked as these dichotomies seem to suggest" (p. 312).

Challenging the construct of transfer

The findings in research on strategies used to deal with mathematics problems outside school also led to a questioning of the relationship with school mathematics. For it has long been assumed that the secret of school mathematics lay in its power of transfer. This assumption, however, needed to be rethought. In 1987, Lauren Resnick wrote that

> Schooling is coming to look increasingly isolated from the rest of what we do (…) part of the reason for this isolation may be that schools aim to teach general, widely usable skills and theoretical principles. That is their *raison d'etre*. Indeed, the major justification offered for formal instruction is — usually — its generality and power of transfer. (p. 15)

Out-of-school practices in non-Western cultures provided an environment suitable for empirical testing of the power of transfer of school related skills. In these settings, researchers could easily find people with different degrees of exposure to schooling from none to advanced levels. This situation also meant that the learning of mathematical skills required for specific crafts and professions were often embedded in the apprenticeship. Within the same group it was possible to find people who were going to vary both in terms of levels of skill in the profession and in terms of their levels of schooling. Various researchers took advantage of this naturally occurring situation to disentangle the effects of schooling on cognitive development (Greenfield & Lave, 1982).

A classic example is the work of Jean Lave among the tailors of Monrovia, Liberia. As an anthropologist, she spent several months observing the work of masters and apprentice tailors. This enabled her to gain access to the arithmetic tailors used, such as estimating size, in inches, of the waistbands of pairs of trousers. Lave then used this knowledge to develop a strategy to study the impact of schooling and tailoring on mathematical skills. Her strategy consisted of devising arithmetical tasks that varied according to their degree of familiarity with tailoring or schooling practices. She then applied the tasks to tailors who also varied in two dimensions: (1) none to 10 years of schooling; and (2) a few months to 25 years of tailoring experience. Lave observed specific effects. Schooling contributed more to the performance in school-oriented tasks and tailoring to the tailoring-oriented tasks. On the basis of these findings, Lave concluded that "It appears that neither schooling nor tailoring skills generalise very far beyond the circumstances in which they are ordinarily applied" (Greenfield & Lave, 1982, p. 199). This study was only a starting point for a challenge of the view that schooling has general cognitive effects, which would transfer and generalise across practices (Lave, 1988). Evidence from other studies supported Lave's ideas of the context-specific nature of cognition (for a review see, LCHC, 1983). Two decades after research in out-of-school contexts began to cast doubts on the existence of transfer and generalization as mechanisms supposedly located in the mind, a theory of how learning is carried out and applied across practices is not yet available (Engeström, 1999), although some interesting alternative conceptions have started to emerge (Noss, Pozzi, & Hoyles, 1999; Saxe & Esmonde, 2005).

Research that followed an ethnographic approach not only challenged the notion of transfer but also demonstrated that inclusion of cultural practices in analyzing mathematical cognition was complex. The influence of cultural practices in cognition could not be explained in terms of general cultural effects. This became apparent in Gay and Cole's observation of the contrasts in performances of the same individuals in the same content domain (e.g., measurement). For instance, adult Kpelle did better in tasks estimating the measurement of volume than in those measuring length. The Kpelle had not acquired a general ability to carry out estimations of measurement. Instead, they had acquired specific skills closely related to the nature of the thinking and mediating tools involved in the practices. The same was found to be true of the performance of poorly educated American adults. That is, their ability to estimate was not general, independent of what they were required to estimate (e.g., amount of rice), nor was it independent of the measuring tool used or of the situation.

Saxe (1982, 1991, and Saxe & Esmonde, 2005) also illustrated the specificity of the impact of cultural practices in cognition by studying the introduction of Western-style currency in the Oksapmin community (Papua New Guinea). He found that people with little participation in commercial activities continued to use traditional counting systems based on body parts, whereas the others who actively participated in economic exchanges had adopted a hybrid counting system combining body parts and numerical representation. In short, developmental cross-cultural research that follows an ethnographic approach has led to a dismantling of established ideas on the nature of human mathematical cognition. This movement in the field of mathematics learning has been paralleled in other areas of human development (Eckensberger, 1995; Woodhead, 1999).

Everyday mathematics in the context of western societies

The insights gained into non-Western cultures have encouraged outside school research in the researchers' own cultures (Brenner, 1998b; Carraher, Carraher, & Schliemann, 1982; de la Rocha, 1986; Lave, 1988; Lave, Murtaugh, & Rocha, 1984; Masinglia, 1994; Masinglia, Davidenko, & Prus-Winniowsaka, 1996; Murtaugh, 1985; Scribner, 1984). This shift in emphasis is acknowledged as a change in strategy. Cole (1995) makes this explicit when he stated that "instead of engaging in cross-cultural research, we began to focus on children in our own society." In the 1980s, this research was conceptualized in terms of "everyday cognition" (Rogoff & Lave, 1984). "Everyday" was initially adopted as the contrasting term to "laboratory" or "test situations." Some quotations from Rogoff (1984, p. 2) that reflect this contrast are:

> Subjects who perform poorly on logic or communication problems in a *test situation* often reason precisely and communicate persuasively in more *familiar contexts.*
>
> Observations that children's capacities appear quite different in their *familiar* environments than in the *laboratory.*
>
> Young children routinely have difficulty in referential communication *tasks*, yet in *everyday situations* they adjust their communication.

A careful reading of Rogoff's chapter suggests that her goal was not to create a theory of differences in psychological functioning between laboratory and everyday settings but to demonstrate that cognition is not context-free. "Everyday" was (and still is) used to differentiate not only research laboratories from other settings but also school versus outside school settings (Brenner, 1998a; Civil, 1995; Nunes, 1992b; Schliemann, 1995; Zack, 1998). According to Lave (1998), the key issue is the interpretation of "everyday." She distinguished between a functionalist view in which "the label 'everyday' is heavy with negative connotations emanating from its definition in contrast to scientific thought" (p. 14) and a practice theory view, in which "the everyday world is just that: what people do in daily, weekly, monthly cycles of activity" (p.15). Thus, Lave argued that "a schoolteacher and pupils in the classroom are engaged in 'everyday activity' in the same sense as a person shopping for groceries in the supermarket after work and a scientist in the laboratory" (p. 15).

Lave's conceptualization of everyday contexts and how they have contributed to the structuring of learning and uses of knowledge has been supported by findings from the Adult Maths Project, conducted in the United States. She started this project in 1978, apparently immediately after the end of her investigations among the tailors in Monrovia, which took place between 1973 and 1978 (Lave, 1990). The Adult Maths Project included a supermarket shopping study designed to investigate uses of "well-learned and routine" arithmetic (de la Rocha, 1986; Murtaugh, 1985). Examining the mathematical performance of a group of adults shoppers in three settings—routine supermarket shopping, best-buy simulation experiment, and school-arithmetical tests—the findings showed a gap between the performance in the school-like tests (average 59% correct answers) and in the supermarket (98%) and best-buy experiment (93%). Analysis of relations between performance in the three situations and schooling showed that years of schooling was a good predictor of performance in the school-like tests but bore no statistical relationship to performance in the other two situations. Lave concluded that these results were against the logic of learning transfer, which is based on the generality and power of transfer of the school procedures to other situations. She used these findings to expand the notion, first put forward in her studies in Liberia, of the inadequacy of the mechanism of transfer to account for the relationship between knowledge acquired in different contexts. As an alternative, she proposed that variations in the performance of the same person across contexts could be explored in terms of the following:

- Interrelations between situations, occasions and activities, which might shape arithmetic practice: For instance, use of particular measurement systems in supermarkets might require the person to compare and convert measures.
- Relations between the problem solver and the problems: An important dimension of this relationship seems to be the degree the practice allows the person to be in control. Experiencing control can be subjective, but is also shaped by the way activities are structured. Lave observed that at the supermarket, one has the option of abandoning the arithmetic for other types of solutions. The same option might not be available in a mathematics classroom.
- The prominence of the mathematics in the activity that unfolds in different settings: In mathematics classrooms and numerical tests the prominence of using mathematical tools is built into the situation. This might not be the case in supermarket shopping. For instance, some shoppers can establish priorities in terms of a quality, such as organic food. Giving prominence to quality, comparing prices might not be important, thus no calculations are carried out.

The above observations led Lave to conclude that success and failure in mathematics might best be understood in terms of relations between persons, their activities, and contexts rather than solely in terms of cognitive strategies. Lave's analysis offered some understanding of variations in the uses of the mathematics by adults that apparently have already "acquired" competence.

Replacing the Concrete versus Abstract Dichotomy by the Notion of "Tool Kit"

Without entering into a discussion of the concrete versus abstract nature of mathematical knowledge (for a re-conceptualization see, Hoyles, Noss, & Pozzi, 2001), I want to emphasize that this dichotomy seems inappropriate for distinguishing between out-of-school and school mathematics (Nunes, 1992b; Schliemann, 1995). Historically, mental calculations used by unschooled people were viewed as an example of concrete (practical) thinking. Analysis of strategies linked to oral and written arithmetic challenged this view (Nunes et al., 1993). Using a repeated measures design, Nunes and her colleagues asked working-class Brazilian children to solve mathematical problems in three situations: simulated store problems, word problems, and computation exercises. As predicted from previous studies (Carraher et al., 1982), the children were more likely to choose oral strategies in the simulated store situation and written strategies in the computation exercise. Accuracy also varied as a function of the type of sum and strategy. For addition, the difference in correct answers between oral and written strategies was small, but for the other three operations—subtraction, multiplication and division—the difference was quite marked. For instance, accuracy in oral subtraction was 62% compared with 17% in written strategies. How is it that the same children offered two different performances in problems apparently similar?

Nunes (1992a) suggests that part of the answer can be linked to properties of the two systems that then mediate problem solving in different ways. She analyzed the solution of the computation 252-57. Adelson, who solved the problem orally, provided the following account: "57 minus 52 equals 5. 200 take away 5 equals 195." Angela wrote it down in the traditional vertical column format and explained: "12 minus 7, 12 minus 7, let me see how much, 7, 8, 9, 10, 11, 12. It's 5 (writes down 5). Five minus five equals nothing. Two minus nothing equals two." According to Nunes (1992a), the strategies of Adelson and Angela illustrate the following differences between oral and written arithmetic:

Oral arithmetic

- Preserves the relative value of number;
- Proceeds in the order we speak, from large to small numbers;
- Allows different types of manipulation, such as dealing with different values and then adjusting;
- Preserves the meaning of the situation.

Written arithmetic

- Sets the relative value of number aside;
- Usually follows the opposite order in which we speak;
- Given numbers are strictly adhered to in problem solving;
- Sets the meaning of the situation aside.

Nunes's analysis highlighted the fact that common errors in both written and oral calculations were not linked only to the functioning of the mind, but inherently linked to specific organizations of systems of representations—to the tool that mediated the person's mathematical action. Cultural practices provide tools that both expand and impose limits on the operations the person can carry out (Nunes, 1992b; Nunes & Bryant, 1996; Schliemann, 1995). In this sense, systems of representations, such as mental tools, can be seen as sharing some properties with tools used to operate in the physical environment. For instance, using a bike as a means of transport compared with using a car shapes the action in different ways. Indeed, the analogy between mental and physical tools was used by Vygotsky to illustrate his claim that human cognition is mediated by tools and signs (Scribner & Cole, 1981; Vygotsky, 1978).

Findings such as the above suggested that to understand human performance, it is necessary to look at the interaction between the agent (person carrying out mental or physical action) and the cultural tool. As Wertsch (1998) noted, the way we attribute competence often obscures the role played by tools. He gave the example of asking someone to do a multiplication with large numbers and afterward asking the same person how the solution was obtained. A common reply would be, "I just multiplied….", and the person demonstrates writing the algorithm. But who solved the problem, the person alone? Certainly not; he or she used a cultural tool—an algorithm for multiplication—to mediate the solution of the problem. Thus, for Wertsch it is more appropriate to say, "I and the cultural tool I employed" solved the problem. The same applies to Nunes' examples. Adelson used an oral arithmetic procedure as a tool and Angela a written algorithm. One cannot guarantee that Adelson would have solved the same problem if asked to use the written tool. He would have needed to have it available in his "tool kit."

In Wertsch's (1991) view, a "tool kit" approach offers a more appropriate way of explaining variations in performances between groups. He argued that it enables one to replace the "metaphor of possession" (p. 14), which looks for differences in terms of "having" or "not having" mental capacities, such as for higher order abstract thinking, by a "tool kit metaphor." In his perspective, "A tool kit approach allows groups and contextual differences in mediated action to be understood in terms of the array of mediational means to which people have access and the patterns of choice they manifest in selecting a particular means for a particular occasion" (p. 94). He goes further, claiming that

> As long as the metaphor of possession shapes the debate, a basic issue—the different uses or functions of a tool—escapes the attention of those involved, and they often find themselves in the somewhat ridiculous position of claiming that there are no differences between groups that are obviously different. If the argument is formulated in terms of tool kit analogy, however, with the understanding that different groups may employ similar tools in different ways, much of this confusion can be avoided. (p. 95)

There are various aspects in Wertsch's concept of "tool kit" that need to be considered. The first is the properties inherent to specific tools. The second is the reasons that inform a particular selection of tools. Finally, the processes that explain different uses of similar tools need consideration. The first aspect has been widely researched as illustrated above, but the other two need attention. In my opinion, they are linked to gaps or unresolved issues in a culturally sensitive approach to learning that I attempt to address below.

Issues on the way cultural tools mediate cognition

The use of the same tool in different social practices: Socially supported developmental process?

Evidence that what one person learns in one practice does not always translate to other practices makes it possible to question traditional explanations of transfer in terms of an automatic process (Bliss & Säljö, 1999). It is also the case, however, that people move between practices and that some overlapping between uses of knowledge have been observed. Some researchers have argued that difficulty explaining these movements could be linked to lack of a developmental dimension in the investigations (Abreu, 1998a; Saxe, 1991; Van Oers, 1998b). For instance, Saxe (1991) noted that the investigations of the 1980s on how adults address mathematical problems linked to their everyday activities

> do not treat cognition from a developmental perspective, a perspective in which cognitive forms are understood as evolving in a complex psychogenetic process, shifting in function over the course of their evolution. For instance, we rarely observe individuals sampled at different ages or at different points in their acquisition of a trade. (p. 12)

Saxe (1991) addressed developmental issues in his investigations among Brazilian candy sellers (boys aged between 6 and 15 years). His study involved ethnographic observations in which the specific mathematics of candy selling was described and demonstrated through structured interviews. One example examined was how children decided the price of candies for retail sale. Saxe observed that younger children were more likely to rely on other people (wholesale store clerks, parents, or colleagues) while the older ones did the calculations themselves. Among the children who did the calculations, he observed that they predominantly made use of practice-linked strategies. He also observed, however, that some children used school-linked strategies, which in fact involved the appropriation of both practice and school based mathematics. This shift from the practice-linked to the school-linked strategies was linked to the extent of the children's schooling. Saxe also analyzed the influence of selling experience on school mathematics. He compared the performance of sellers and non-sellers in school-like arithmetical problems. He found that sellers obtained more correct solutions than non-sellers did, and that the source of success was linked to their specialized strategies.

Saxe's developmental approach provided two new insights into the way transfer may be conceptualized. The first is to regard transfer as a constructive developmental process taking shape over time through a progressive appropriation and specialization of forms of knowledge. The second is to regard transfer as a socially supported process. Viewing the interplay between school and out-of-school practices from this perspective, I pose new questions: If the construction is a socially-guided process, will the appropriation and specialization outside school follow the same pattern as that occurring in school? Differences in social constraints, or indeed in the way learners and teachers make sense and negotiate perceived constraints, may lead to different kinds of appropriation. It is not enough to have the tool in the kit; one must figure out whether it is appropriate to use the tool in particular communities of practice. Through which processes do learners develop an understanding of which tools should be used and which should be left in the tool kit?

Conventional versus unique appropriation of cultural tools

Most of the theoretical advances mentioned above have resulted from studies conducted within traditional out-of-school practices. There was also a tendency in these studies to describe patterns that applied to the whole group. This means that there is still a need to explain how individual diversity can emerge. I explored this field by studying a traditional practice. Between 1995 and 1998, I carried out an investigation into the use and understanding of mathematics by sugarcane farmers in the Northeast of Brazil (Abreu, 1998b, 1999). Sugarcane farming was introduced to the area in the 16th century and has since played an important role in the local economy. My motive to engage in this particular project was to gain an understanding of why the farmers have difficulties in appropriating modern technology. Both theoretically and methodologically, this study was informed by the so-called everyday cognition approach (Rogoff & Lave, 1984), and by a Vygotskian view of mathematics learning as the internalization of sociocultural tools.

Focusing my analysis on farmers' traditional practices enabled me to document the existence of mathematical tools specific to sugarcane farming. The ethnographic approach led me to describe the particular mathematics, used by the farmers, which differed from school mathematics (which is also the basis of modern technology). For instance, they had specific length and area measures, formulas to calculate areas, and a variety of oral strategies to solve sums, involving both additive and multiplicative structures. In addition, the findings from the interviews about the strategies used by the farmers were revealing about the way their experiences with the use of specific tools mediated their cognition. For instance, in problems related to the amount of fertilizer applied by area they were sensitive both to the units of measurement and to the numerical relations. When solving problems that involved uses of mathematics, farmers chose as mediators those forms of representation that were closely linked to their practices. These tools enabled the user to function efficiently and perform meaningful cognitive operations.

When I look back at my data, it seems obvious that my research was a good example of an approach that was looking for diversity between groups within the wider Brazilian culture, but was still marked by a homogeneous bias of the within group type of analysis. By concentrating on the similarities of tool use among farmers, I overlooked evidence that showed unique appropriation of the tools and did not explore the mechanisms behind this (Wertsch, 1995). I did not carry out any analysis that addressed development at the level of the individual, or in Rogoff's (1995) terms of the individual's participatory appropriation. Nevertheless, this latter type of analysis may well clarify the origins of diversity among individuals in similar cultural practices.

Looking retrospectively at the data, I can see two distinct patterns of the farmers' re-construction of cultural knowledge. For instance, re-examining their procedures for calculating the areas of quadrilateral and triangular plots of land, I can see more than one pattern. In the most prevalent pattern, farmers followed the convention. For example, the area of a triangular plot of land was found by multiplying the average of the two opposite sides by one half of the length of the remaining side $\{Area=[(a+b)/2]\times(c/2)\}$. For those following this pattern, it is as if personal knowledge copied the conventional cultural knowledge, a truly Vygotskian account of a re-construction of the social at a psychological level. A less common pattern seems to indicate a more complex process. For instance, instead of following the conventional procedure, one of the farmers multiplied one of the sides by half the length of the smaller side of the triangle {Area= [b × (a/2)]}. When I asked why he had not averaged the "opposite sides," he said that could not be done, and went on to explain that the largest side was discounted because it can be seen as equivalent to the diagonal of a quadrilateral. Thus, using it in the formulae would result in overestimating the area of the triangle (Abreu, 1998b). In this pattern, there was an indication of uniqueness: personal knowledge was grounded in cultural knowledge, but it was not a copy of it. The reference to the diagonal of a triangle also suggests that the unique solution could be a hybrid form that combines pieces of knowledge from farming practices with pieces from school practices. If this is the case, a crucial requirement in understanding the problem of diversity is to try to gain some insight into what motivates certain individuals to produce these new forms of knowledge: Cognitive understanding? Valorization of knowledge (Abreu & Cline, 2003; Abreu & Cline, 2007)? Combination of cognitive and social understandings? In addition, the evidence that different patterns co-exist suggests that it is necessary to explore developmental processes. What type of experiences can lead some people to follow one pattern and others not follow it? This is an area that needs further research. Some new directions, however, have already been pointed out. One direction is Saxe's study of cognition in flux (Saxe & Esmonde, 2005) that examines the interplay between development of new forms of individuals' mathematical cognition and historical changes in practices, such as economic practices and schooling. Another direction is the study of learners' experiences of transitions between contexts of mathematical practices (Abreu, Bishop, & Presmeg, 2002a), including transitions in multicultural classrooms.

Cultural tool mediation in multicultural classrooms

Issues of cultural tool mediation in multicultural classrooms have become noticeable in studies examining how learners from minority ethnic and immigrant origin experience the transitions between their home and school practices (Abreu, Bishop, & Presmeg, 2002a; Abreu & Elbers, 2005). Similar to the studies in out-of-school mathematics, these studies show that often learners use different tools to solve mathematical problems in each practice (Abreu, Cline, & Shamsi, 2002b; Abreu & Cline, 2005). Another similarity between the studies is that only a few participants, generally high achievers at school, are able to provide a detailed account of the differences between a cultural tool used at home compared with the one used at school (e.g., short vs. long division algorithm). When these accounts were provided, it was usually social support outside school that helped the learner to figure out the relationships between the tools (Abreu et al., 2002b).

For many immigrant and minority learners exposure to the home mathematical tools is prior to exposure to similar tools and concepts at school. Taking this into account, research in mathematics learning in multicultural classrooms has explored ways to support the learners to build on their previous knowledge. A central question in this research is: to what extent do learning situations provide opportunities for learners to externalize and negotiate conflicts related to defining boundaries, when applying the same knowledge tools in different contexts? This question can also be formulated in terms of how social support at school can contribute to making visible the tools one selects and the way one makes use of particular tools. Gorgorió and her colleagues (Gorgorió, Planas, Vilella, & Fontdevila, 1998, Gorgorió, Planas, & Vilella, 2002; Gorgorió & Planas, 2005) have been exploring this dimension in their research with immigrant students in Catalonian schools. They observed that if given a problem that bears some resemblance to out-of-school experiences, immigrant students contextualize it within different frames. The researchers used realistic problems in order to structure teaching situations that stimulated students to externalize their frames of interpretation. In doing this, their aim was to give immigrant students the opportunity to externalize and negotiate conflicts related to what counts as appropriate tools in their home culture and in their Catalonian mathematics classrooms. As Noss et al. (1999) suggested, it is more likely that "pieces of knowledge," rather than a whole concept, can be utilized across practices. These pieces will then be connected through a "webbing mechanism," which will enable the learner to put the pieces of knowledge together and create new webs of meaning. Thus, it seems that what Gorgorió's group is doing in the classroom is, in fact, developing strategies to provide support for learners to make visible the pieces of knowledge and cultural tools they bring in and to negotiate ways of integrating (re-contextualizing) these within the context of school mathematical practices. Whether this will then impact on development of unique patterns of appropriation is an issue that so far had not been systematically addressed in research in multicultural classrooms (Chronaki, 2005).

To sum up, the complexities of cultural tool mediation are well illustrated in studies attempting to map the challenges faced by teachers who believe that it is worth actively helping children to bridge the gap between their school and out-of-school mathematics (e.g., Adler, 1997; Atweth, Bleicher, & Cooper, 1998; Civil & Andrade, 2002; Chronaki, 2005; Fraivillig, Murphy, & Fuson, 1999; Gorgorió & Planas, 2005). Future research needs to continue exploring the developmental dimension. It is crucial to gain more understanding of patterns of development in learning and uses of mathematics followed by individuals and groups and how these patterns can be related to the social and cultural structuring of the practices.

FOCUS ON THE SOCIAL SYSTEM

As described above, studies on out-of-school mathematics arose from cross-cultural psychology and this seems to have shaped the way the impact of the social system was initially explored. First, the studies carried out on particular types of apprenticeship pointed to links between the uses of mathematical tools and sociocultural and institutional contexts. Second, the notion that the tools one uses to think are cultural generated interest in a particular type of asymmetric social interaction: between someone who is more knowledgeable and someone who is less knowledgeable, such as parent-child, teacher-pupil, or master-apprentice. For instance, adults structure and guide young people in a way that facilitates the re-construction on a personal (psychological) plane of knowledge that pre existed on a social plane (Rogoff & Morelli 1989; Rogoff, 1990). Kirshner and Whitson (1997) referred to these two foci of research as the situated cognitionists' ways of breaking out with individual accounts of learning. The first focus could be linked to Lave's (1988) critical anthropology agenda. The second reflects an agenda informed by Vygotsky and aimed at exploring interactions in the Zone of Proximal Development, "an interactive system within which people work on a problem which at least one of them could not, alone, work on effectively" (Newman, Griffin, & Cole, 1989, p. 61). I first review some key studies under these two focuses and then explore some emerging issues.

The mediating role of social institutions

In the sequence of out-of-school studies that suggested the use of tools was linked to the context of the practices, researchers explored the applicability of these ideas to formal institutional contexts. Although research on cultural systems has addressed context-specific cognition related to activities (e.g., tailoring, schooling, selling), research on the social system has addressed issues related to social institutions, where these activities took place. Examples of such studies are Säljö and Wyndhamn (1993) and Schubauer-Leoni (1990). Both investigated the applicability of the ideas to different contexts within school, by comparing performance inside and outside the mathematics classrooms. Säljö and Whyndhamn found that Swedish students, when asked to solve postage problems, called on different strategies according to the context in which the task was presented. In the context of a mathematics lesson, 57% of the students attempted some type of calculation. In a social studies lesson, however, only 29% used calculations. In this context most of the students found the solution by reading the postage table. Schubauer-Leoni (1990; Schubauer-Leoni, Perret-Clermont, & Grossen, 1992) found that 8 and 9 year-old Swiss children used different solutions to addition problems according to the context in which they were tested. Only 3 out of 34 pupils used conventional arithmetical notation when tested outside the classroom, compared with 17 out of 39 pupils when tested in the classroom.

Because the cultural artifacts available were similar, one can hypothesize that differences in the solution were related to what Minick, Stone, and Forman (1993) referred to as "multitude of genres." Mathematics, like language, provides different uses that emerge as function of the ways tasks are interpreted. The institutional place, where the activities, were presented to the children, provided different frames for the interpretation of the task and allowed them a choice of tools. But, what are the mechanisms responsible for this type of dynamics? Walkerdine (1988) used the framework of discursive practice to analyze the relationship between signifier and signified in both the home and school mathematical practices of young children. In her view, the person's construction of school mathematical knowledge is regulated in the discourse of the practice—specifically, in the mathematics classroom. The same reasoning could be applied to the relations of signification that children construct in the mathematics lesson as compared with other school practices (e.g., social science lessons, playtime outside the classroom, etc.). According to Walkerdine, transfer of learning is not a question of central processors inside the human mind, but a result of the production of new relations of signification from practice to practice. Her ideas support the framework that has emerged in studies focusing on the cultural system, such as the view that continuities and discontinuities in the use of knowledge across practices cannot be explained by transfer. In addition, she added new elements by suggesting that discursive practices regulate forms of re-contextualization, and consequently, new patterns of development.

The mediating role of social interactions

Although the influence of social interactions in learning is one of the main lines of investigation in sociocultural theory, most of the empirical research in mathematics learning has been conducted either in controlled experimental situations or in classrooms (Cobb, 1995; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Schubauer-Leoni & Perret-Clermont, 1997; Wood, 1999). Thus analyses of social interactions specific to the transmission of mathematical knowledge in out-of-school practices are rare. Saxe (1996) acknowledged this gap in his own research. In a retrospective analysis, he wrote, "While in the Oksapmin fieldwork I had spent time observing social interactions in which arithmetical problems were generated and resolved in the trade store, I had not attempted systematic social interactional analyses" (p. 288). In subsequent research in the United States (Saxe, Guberman, & Gearhart, 1987) and in Brazil (Saxe, 1991), Saxe and his colleagues introduced detailed analysis of social interactions.

He did this by researching number practices in White families living in Brooklyn, New York. He sampled families from both working- and middle-class groups with young children aged 2.5 to 4 years. Previous research has revealed that children of this age have developed some basic mathematical knowledge, but the enculturating processes through which these children acquire this knowledge had not been addressed. By conducting in-depth interviews with the mothers, Saxe et al.'s research revealed the social structuring of early number practices at home. These practices had been organized taking into account age and class differences. For instance, the younger children tended to be engaged in less-complex activities, while the older children engaged in more-complex activities. Furthermore, analyses of mother-child video taped interactions revealed the dialectical nature of the process: "mothers were adjusting their goal-related directives to their children's understandings and task-related accomplishments and children were adjusting their goal-directed activities to their mothers' efforts to organize the task" (Saxe, 1996, p. 292).

Further support that families play a part in the structuring of practices that contribute to the child's development of mathematical cognition outside school was obtained in Guberman's (1996) study with Brazilian families. Children from working-class families in Brazil often engage in commercial transactions. From an early age, children commonly buy food from local shops. Previous research shows that engagement with commercial activities is linked to the development of mathematical skills (Abreu, 1995b; Abreu, Bishop, & Pompeu, 1997; Saxe, 1982). Taking this into consideration, Guberman observed and interviewed parents of Brazilian children aged 4 to 14 years. He explored how parents structured their children's learning of mathematics by varying the degree of responsibility assigned to the child in purchasing tasks. His findings revealed four levels of engagement in purchase tasks that were related to increasing arithmetical complexity. For instance, at Level 1, the child is given the exact amount of money needed for the purchase. At Level 2, the child is not given an exact amount and is told to wait for the change. At Level 3, the child is required to check the change. At Level 4, the child is expected to calculate the cost of the purchase. Identification of currency and the requirement to make calculations varied in complexity from Levels 1 to 4. Guberman concluded that "even in activities distal from direct verbal interaction with parents, children learning often is regulated by parents" (p. 1621).

The types of mathematical competencies studied by Saxe and Guberman have often been referred to as informal mathematics. Furthermore, some authors assert that "much of this informal mathematics can develop in the absence of adult instruction; indeed, many adults are quite surprised to learn how much their young children or students know in this area" (Ginsburg, Choi, Lopez, Netley, & Chao-Yuan, 1997, p. 165). Saxe and Guberman's findings do, however, suggest that the absence of what groups believe is formal instruction in mathematical concepts cannot be confused with lack of socially organized activities that support a child's learning. In fact, in the same way that adults would not often define their uses of mathematics in outside school activities as mathematical, they would likewise not define their children's practices as mathematical. The mediating role of adults and more expert peers in structuring and supporting the new generations in their local practices, however, is revealed when explored in systematic research.

Issues in social mediation

The impact of changes imposed by macro-social structures

Sociocultural approaches to mathematics learning, in particular those based on initial interpretations of Vygotsky and apprenticeships models, are recognized as providing a limited account of the impact of the social system on learning (Duveen, 1997; Forman, Minick, & Stone, 1993; Goodnow, 1990, 1993; Goodnow & Warton, 1992). Study of interactions such as those described above were criticized for providing "descriptions of perfectly orchestrated dyads" (Litowitz, 1993, p. 187) and for lack of consideration of the influences of the wider social structure (Stone, 1993). To consider the impact of macro-social structures on our current understanding of mathematical learning in outside school contexts, I analyze a second scenario from my research with sugarcane farmers (Abreu, 1998b). This scenario illustrates the farmers' struggle to cope with changes imposed by the macro-social structures that involved varying degrees of exposure to new mathematical tools. At the time of my field work in the farming community, the farmers were in the midst of coping with one of these external demands. Changes in the Brazilian economy at a macro-level led the government to impose new criteria for the payment of sugarcane: since 1984 the system of payment for sugarcane according to quality has been imposed by law. In the past, the criterion had only been a function of the weight of the sugarcane produced, independent of quality. This was the only system the farmers had ever experienced, and mathematically it was quite simple. Following the old system, farmers could calculate how much they would receive from the sugar-mill by multiplying the amount (tons of sugarcane) they had produced by the price per ton. With the new system, they need to deal with the variables that define the quality of the sugarcane. In the end, the price could be found by multiplying the index of quality, times the amount produced, times a fixed price per ton. The new system posed some difficulties for the farmers, such as (a) the index of quality was calculated by highly sophisticated computing and laboratory technology located in sugar-mills, and (b) the index of quality did not apply to the total quantity of sugarcane a farmer produced, but the tests were carried out on each specific delivery. The consequence was that the price per ton was variable. For farmers to find out an average price, they needed to go through complex calculations. (These were not straightforward because the information was presented in sophisticated forms with values for the different factors, and farmers had to read complex tables.)

The farmers' ways of dealing with the new system could be analyzed from a Vygotskian-based perspective focusing on the limits on cognition imposed by not mastering the new mediating tools. This would emphasize a particular type of relationship between the cultural system and the person's system. Such an orientation, as seen above, dominated my thinking at that stage, and can be noted in the questions to be covered when interviewing the farmer (Abreu, 1998b):

1. What do you think about the payment for sugarcane by amount of *sacarose* in it?
2. Do you know what *agio* means?
3. Do you know what *desagio* means?
4. Which system do you think is better for you and why?
5. How did you calculate your income before the new system? And how do you calculate it now?
6. In cases where you have an *agio* of x% (5% and 10%), what will it represent in gain?
7. In cases where you have a *desagio* of x% (5% and 10%), what will it represent in loss?

Apart from the first and fourth question, which left some room for the farmer to articulate his experience in general, the other questions were limited to mathematical understandings. Research, however, as a human enterprise, is not a unilateral process shaped by the researcher (Grossen, 1997). My mathematical focus did not prevent farmers from re-interpreting the questions and articulating answers revealing different facets of their experiences with the change.

For the person, changes in macro-social structures can have different types of impact, which can be linked both to mathematical knowledge itself, and to the way knowledge influences identities. The first impact of the change was that it required a type of mathematical knowledge most of farmers did not have. Traditional mathematics (Abreu, 1999) enabled the farmers to grasp some understanding of the new system, such as when comparing whether they were making or losing money, but it was limited when they had to read and interpret tables combining different variables and when they were required to understand concepts such as percentages, decimals, positive and negative numbers. The second type of impact was on the farmers' identities. The changes made them experience loss of control, it brought uncertainty, and threatened their standing. They were not sure where they stood, whether they could contract services, whether they could borrow money from the bank, and perhaps more important, whether they could survive in business. The third impact was that exposure to technological innovation and modern institutions (e.g., schools; banks) over time raised the farmers' awareness that some forms of knowledge are more powerful than others, and also that some are more accepted than others. For instance, a contract in a bank could either be signed or stamped by a fingerprint. The farmer who signed might be functionally as illiterate as the one who stamps his fingerprint, and both may be unable to read the contract. The first method, however, enabled the person to feel part of the literate community (inclusion), the second assigned the person to the category of "illiterate" (exclusion from a high status group). The ability to sign was then highly valued by the group, whereas use of the fingerprint a reason to be ashamed. The same applied to the whole of traditional farming mathematics when compared with school mathematics. I referred to this phenomenon as a group's valorization of knowledge. This seemed to reflect the status of the practices in the wider social structure.

This second scenario shows some of the shortcomings of the explanations about the learning of mathematics in and out-of-school that are based on studies conducted in traditional practices (Duveen, 1997). In current societies, with the unprecedented levels of traveling and migration, it is more likely that both adults and children will frequently experience exposure to change and co-existing practices (development at the plane of the cultural and social system). In these circumstances, their type of experiences might be closer to those of the farmers when new technologies were introduced, that is, knowledge will be experienced both in terms of mastery and in terms of identity by those in the role of transmitter (e.g., teachers, experts, and parents) and those in the role of learner.

Social mediation in multicultural classrooms

As already indicated in the limited, but emerging field of studies of learning mathematics in multicultural classrooms, examining the relationships between macro-social structures, institutional practices, social interactions, and individual learners' experiences is of theoretical and practical relevance (Abreu et al., 2002a; Abreu & Cline, 2003; Abreu & Elbers, 2005). Areas that start to be addressed in empirical studies in multicultural classrooms include the impact of institutions on the selection, use and creation of cultural tools which can then mediate forms of participation in practices (Abreu & Elbers, 2005). The curriculum (Oliveira & César, 2002) and particular types of classroom discourse are examples of institutional tools (Gorgorió & Planas, 2001; Moschkovich, 2002), which contribute to the ways practices are organized in the classrooms. These tools can facilitate or constrain the learners' access to cultural practices of new communities and therefore promote practices that are more or less inclusive. In turn, it seems that practices that are experienced by the learners as inclusive reflect back to their broader social context in the sense that school is legitimating cultural tools and associated identities of the community of learners it serves (Abreu & Elbers, 2005; César & Oliveira, 2005).

Another emerging focus on sociocultural mediation in multicultural classrooms is on the mediating role of the other in face-to-face interactions (Abreu & Elbers, 2005). Elbers and Haan (2005) examined the construction of word meaning by students in a multicultural mathematics classroom at a Dutch primary school. The school where they worked had 80% of the students from a minority background, mainly Moroccan and Turkish, and the teachers followed a pedagogy that promoted peer collaboration. Part of the students' activities in the mathematics classroom investigated required working and talking together and helping each other in specific mathematical tasks. Elbers and Haan recorded and examined the way students deal with language problems in mathematical tasks that originate from minority students' unfamiliarity with specific Dutch words and expressions. Elbers and Haan initial expectation was that clarifying cultural meanings of Dutch words was crucial for the students to overcome their language problems. But, their analysis pointed out something different. Their study showed that minority students: (1) do not ask for clarification of word meaning during teacher fronted classroom instruction; (2) do raise problems with word meanings, when working in groups, but their conversations about word meaning are restricted to their collaborative group activities. The clarification of meaning was made part of the mathematical discourse, even in groups with Dutch children who potentially did have access to the wider cultural discourse. When providing explanations the Dutch students did not elucidate the meaning of the unfamiliar words in broad cultural terms but gave minimal information, which was sufficient for minority students to solve the task. These findings suggest that the institutional practices take for granted the students' access to cultural meanings. Practices, such as peer collaboration, that may have worked in monocultural classrooms do not necessarily address similar issues in multicultural classrooms. Peer collaboration in the classroom investigated by Elbers and Haan required the Dutch students to take on the role of helping classmates from a diversity of cultural and linguistic background. This is a completely new challenge, which demanded them to engage in forms of interaction unfamiliar to them. Future research has to address the development and promotion of new forms of interaction and classroom communication that better support access to the mathematical curriculum cultural meanings (Abreu & Elbers, 2005; Elbers & Haan, 2005; Pastoor, in press).

THE PERSON SYSTEM: MOVING THE FOCUS FROM COGNITION TO IDENTITY

The need for a better account of the interplay between the cultural, the social, and the person systems was shown in the previous sections. For instance, there were issues related to the uniqueness of the individual and patterns of development in the re-construction of the cultural tools at the person level, and also issues related to the social valorization of knowledge, changes in social structures, and the person and the group's sense of identity. Cultural psychologists suggest that these issues should be addressed by referring to the person, instead of referring to cognition. Thus, they reveal their attempt to bring the concept of self, human agency or identity into the psychology of human learning and development (Lucariello, 1995; Shweder, 1990, 1995). Bruner proposed that one of the tenets of a psycho-cultural approach to education is to take into account the existence of perspectives that reflect both the shared culture and individual histories. To him, "Nothing is 'culture free', but neither are individuals mirrors of their culture" (Bruner, 1996, p. 14).

The understanding of this interplay between sociocultural factors and the individual is still an issue (Damon, 1991). As Gauvain (1998) mentioned in her review of a Special Issue in research in mathematics learning in sociocultural contexts, the individual part of the process has been largely overlooked not only in the field of mathematics learning, but in accounts of "intellectual development from a sociocultural vantage." Furthermore, she argued,

> Although many may argue that individual analysis is anathema to a sociocultural approach, and this may be true in a philosophical or theoretical sense, a conceptualisation that allows the investigator who holds a sociocultural view to locate individual thinking and individual contributions within social processes is sorely needed. In practical terms, such linkages are imperative for evaluating the worth of this overall approach to classroom learning. After all, individual children are experiencing these social learning contexts and individual children will be held accountable for and be provided with or excluded from opportunities stemming from these experiences. (p. 564–565)

According to Engeström (1999), advances in sociocultural explanations of learning will require asking "carefully-focused and theoretically-grounded questions." He distinguished agendas of research in situated learning in terms of weak and strong versions. The basis for this distinction was that the "the former speaks of contexts, the latter speaks of practices, and of participation in communities of practice" (p. 250). In his view, the minimal requirements for a strong version of situated learning are:

- that the study is focused on some relative durable socially-important collective practice, and

- that the researcher presents some sort of ethnographic description and analysis of the collective practice in which learning is embedded (p. 210).

The research I described above with the Brazilian farmers falls in this category. This approach enabled me to obtain an insight into the farmers' (a) specific mathematics tools, (b) mediation of cognition by their specific mathematical tools, (c) awareness of the limitations of traditional mathematical tools to cope with innovation, and (d) perceived relative power of their indigenous mathematics (Abreu, 1999). Continuing with the line that explores mathematical learning in terms of participation in communities of practice, which are both cultural and social structures, I explore how the insights obtained from the studies on farmers produced new research questions related to the relationship between the cultural, the social and the person system. The challenge is to develop ways of understanding the emergence of diversity between individuals who come from groups that share cultural and social systems, without attributing these differences to motives exclusively located inside the individual (see also Engeström, 1999).

Children's Participation in Family Mathematical Practices

A key issue that emerged from my study with the community of sugarcane farmers was the nature of the child's participation in family practices and associated mathematical knowledge. In the apprenticeship approach, the enculturation of the children in their community home practices was often taken for granted. One can ask, however, why the farmers would want to transmit to their children knowledge they knew to be marked as low status. This raises questions about how parents structure the participation of their children in their own mathematical practices. Consequently, questions arose about the within-group homogeneity of children from similar backgrounds. I started exploring these issues by interviewing school-children in rural areas of Portugal (Abreu et al., 1997) and Brazil (Abreu, 1995a, 1995b) about their participation in home practices and the support they received from significant members of the family. Then, I extended this research to children and their parents in multiethnic primary schools in England (Abreu & Cline, 2005; Abreu, Cline, & Shamsi, 2002b).

Studies on children's participation in home mathematical practices

Observations from the studies with schoolchildren in Brazil and Portugal (Abreu, 1995a, 1995b; Abreu et. al., 1997) revealed a new angle on how they participated in outside school mathematical practices. Instead of a common pattern of enculturation into home practices, evidence of heterogeneity emerged. Shopping activities related to the everyday life of the family, for example, fetching bread, fruit, or vegetables, were common practices among the children in my studies. Interviews with children engaged in similar practices revealed differences in what the child was in charge of: (a) some of the children just did the shopping, and adults took responsibility for economic exchanges (that is, they fully accepted the vendor's sums and change or their parents took responsibility for that); (b) some of the children shared the responsibility with their parents or with other adults; and (c) some of the children described themselves as being in charge of the economic decisions involved. The manner in which the children experienced this situation seemed to influence the extent to which they used mathematics in the home practices. At a first glance, these categories of engagement can be seen as developmental. Indeed, it is not difficult to see some resemblance with Guberman's (1996) levels of engagement. However, in-depth comparative case study analysis suggested that the diversity could be linked to the way parents supported their children's engagement in out-of-school practices, but also that they might take into account other factors than the children's developmental level. For instance, children's accounts suggested that not all the families engaged their children in the use of traditional home mathematics. These observations motivated our inclusion of parents in subsequent research. It seemed likely that parents were playing a key role in the diversity among children in their re-construction of the cultural systems of knowledge of the home practices. But the dynamics through which their influences operated were unclear. Were their own valorizations of the co-existing mathematical practices the crucial factor? Or was their influence shaped in interactions (and perhaps negotiations) with the children themselves (i.e., a joint construction)? What type of active role did the children play in their own development? To explore these issues, the research strategy adopted with the Brazilian schoolchildren (Abreu, 1995a, 1995b) was expanded to include a parental perspective. The new studies were conducted in multiethnic primary schools in England (Abreu & Cline, 1998; Abreu, Cline, & Shamsi, 2002b; Abreu & Cline, 2005; O'Toole & Abreu, 2005).

Studies with children and parents in multiethnic primary schools

For these studies, schools with representative numbers of children from Bangladeshi (Abreu & Cline, 1998) and Pakistani families living in England were selected (Abreu et al., 2002b; Abreu & Cline, 2005). The patterns of achievement of children in these multiethnic schools had some similarities to the schools in Brazil. Children of Bangladeshi and Pakistani origin, on average, still underachieve in English schools (Ofsted, 1999). Nonetheless, as in Brazil, within any single year group, there was variation in performance among the children from the same home group, which included both high and low achievers. It was also likely that there were differences between children's home and school mathematics because their parents experienced a different culture and a different school system through having gone to school in their country of origin. Therefore, it was possible to follow the original question: "Do the children who succeed [in school mathematics] establish a different relationship with their home knowledge than the ones who fail?" (Abreu, 1995b, p. 124), by incorporating a parental perspective. That is, if children establish a different relationship do parents play a role and, if so, what does that role involves?

For each child selected as case study, data collection included several methods, consisting of classroom observations, interviews with the child, interviews with the classroom teacher, and one interview with parents. The case studies included White-British and Asian-British ethnic backgrounds. One aspect of the findings from these studies which is worth mentioning before moving to the parents is, "What is home mathematics for the child?" In the communities studied in Brazil and Madeira, questions about what the child usually did after school easily led to a conversation about out-of-school mathematical practices. Their engagement in buying in the local shops or after-school paid work made this link easy to follow. With the schoolchildren in England, this was not the case. When asked to recount their after school activities, the White-British children emphasized leisure activities such as watching television, sports, hobbies, and music. The Pakistani children's accounts emphasized a further structured process of education at home or at the Mosque. Although the schools varied in the amount of homework assigned, the children did not have any difficulty in recounting instances when their parents or other relatives (siblings, aunts) helped them with school mathematics. Therefore, relevance of home mathematics to the children emerged in practices where parents and relatives supported them with school mathematics.

The same pattern of findings was obtained from interviewing parents. The more vivid accounts of their engagement in helping the children with mathematics at home were related to school mathematics. It was also in these accounts that both children and parents made explicit differences between the way mathematics was tackled in the child's school and at home (Abreu & Cline, 2005; Abreu et al., 2002b). Accounts from the parents showed that differences between home and school mathematics could be experienced in terms of:

- The content of school mathematics and in the strategies used for calculations (examples included differences in algorithms for subtraction and division);
- The methods of teaching and the tools used in teaching (for example methods for learning times tables, use of calculators);
- The language in which they learned and felt confident doing mathematics.

Apart from the differences related to language, the first two differences applied to both groups. They seem to be linked to the parents' experience of a different school system (immigrant parents) and or to changes in the curriculum over time in England. This meant that in both ethnic groups, if parents (relatives) were to support the child at school properly, they had to figure out the necessary process of transition between their own mathematical practices and the ones the child was experiencing (Abreu et al., 2002b).

Differences within the same ethnic group emerged when we focused the analysis on (a) the influences of the parents' positions in the way they tackled the differences, and (b) a comparison between information obtained from the parent and their child regarding representations, experiences, and negotiations of differences between home and school mathematics (Abreu & Cline, 2005). There was some evidence that the way parents structured their support to their children was likely to be colored by their own positions about which form of knowledge they valued more. For instance, they took positions regarding the language by which mathematics was communicated, or regarding the importance of knowing the times tables by heart. Understanding the basis on which parents construct valorizations and representations of the mathematical learning of their children is an area for further study (O'Toole & Abreu, 2005).

Abreu and her colleagues' (2002b) examination of the patterns of interaction between the child and the parent's experiences offered further insight into the emergence of within group diversity. Comparative case study analysis highlighted some possible links between patterns of interaction and children's performances. Comparison of two Pakistani children, both in year 2, and in the same mathematics class, showed that for the low achieving child, there was a divergence between the child's preferences and the way the parents were trying to support him in learning school mathematics. His parents were not aware that the transition exposed the child to differences between the way they taught mathematics to him at home and the way he was learning at school. The child's accounts revealed that he believed the teacher knew better than his mother did and that he preferred the English language. At home, however, his parents were teaching him in Urdu, and did not show any awareness that the change in language at school could cause him difficulties. This seemed a case in which the parents' representations of primary school mathematics were still associated with their own schooling. Difficulties in communication with the school might have reinforced their representation. By contrast, in the case of the high achiever, there was a convergence in the way differences in methods, language and identities were negotiated. In this case, the parents—in particular the mother—had developed representations of home and school learning, which included a theory of how her child might experience the transition. For this child, the differences between the mother and his teacher's mathematics did not mean that home practices had "inferior status." Like the low achiever, he also preferred the English language, but in this case the mother was prepared to help the child bridge the gap between the two languages.

A basic distinction between the low achiever and the high achiever was seen in the parents' awareness of the existence of differences in their own ways of doing mathematics and those that their child was being taught at school. The parent of the high achiever also showed more awareness of the child's preferences. The success was then achieved through sensitive interactions, in which the mother learned from the child and then adjusted her strategies to fit with his needs and preferences. A case study with a White-British parent, however, showed that being aware of the difference was not enough to support a child. In this case, the mother had developed an acute sense of the differences between her methods and those of the child, but was experiencing it as a burden. Therefore, interactions conducive to success seemed to require awareness of differences, flexible adjustment, and specialized mathematical knowledge. In sum, the fact of including the perspective of parents revealed a common facet in the intersection of the three systems. It showed that parents, as significant actors in the social system, did not just recreate their own mathematical background for their children. They deliberately selected some forms of knowledge from those available in their cultural system as appropriate to transmit to their children and rejected or hid other forms. Finally, the actions of some parents seemed to take into account the active role of the child. Again, this is an area that needs further investigation. We need to ask which dynamics explain how and why some parents gain an awareness of their child's needs and preferred modes of learning, whereas others do not. Could particular dynamics be related to parents' valorization of their home practices in comparison with the child's school mathematical practices? How can valorizations held by parents and children create resistance in parent-child interactions?

ISSUES IN THE INTERPLAY BETWEEN THE CULTURAL, THE SOCIAL, AND THE PERSON SYSTEMS

Valorization of mathematical practices and social identities

I have argued that the interaction between parents and children in situations in which they must negotiate home and school mathematics is not unidirectional. Of course, one of the aspects that can contribute to bidirection is different type of understanding of a mathematical tool. At this point, I focus on another aspect, which is related to the association between practices and identities. At the base of my argument are the assumptions that children develop an understanding of the valorization of co-existing practices and associated social identities, and that they develop personal positions, which make them active agents in the way they participate in the practices (Abreu & Cline, 2003). These assumptions reflect a well-established view on the way people gain an understanding of the social world around them. The notion of understanding social valorization can be seen in terms of Tajfel's (1978) complementary processes of social categorization and social comparison. Both processes are considered to be constructed as part of socialization. Social categorization relates to "the ordering of the social environment in terms of groupings of persons in a manner which makes sense to individuals" (Tajfel, 1978, p. 61). Social comparison will emerge from a human tendency to evaluate the ordering within the system of values of a society. These processes will then contribute to an individual's development of social identities: how a membership in particular community of practice contributes to an understanding of who they are and where they stand.

The findings obtained in the studies, in which my colleagues and I explored children's understanding of social practices in which mathematics is used, pointed to the development of similar processes. It is important to bear in mind that all the studies involved small samples (20 to 24 children). They were, however, conducted in different countries: Portugal (Abreu et al., 1997), Brazil (Abreu, 1995a) and England (Abreu & Cline, 1998; Abreu et al., 2002b).

First, the findings from these studies indicated that children had developed ways of categorizing some practices of their community as involving uses of mathematics and others as not involving them. For instance, in the farming community in Brazil, it was more likely that children would categorize working in an office as a practice in which people use mathematics than working on a sugarcane farm. In England, few children categorized driving a taxi as a practice that requires the use of mathematics, but they could see shopping as one that does.

Second, the children had developed a basis for the comparison of co-existing practices. Justifications for categorization of a practice as being mathematical or non-mathematical included references to the presence or absence of tools (calculators, cashiers), the nature of the job, and the different status of communities of practice. They also associated performances in school mathematics with given social identities. For instance, for tasks in which they had to choose adults who might have been good or bad at school, the children chose more adults in white-collar professions (office workers) as possibly having been the best in school mathematics; conversely children chose those in blue-collar professions and other low social status practices as possibly the worst.

Third, the children had developed positions that could be indicators of the way they were constructing membership of specific social groups or their own social identities. Again, comparative case study analysis has provided insights into the development of positioning. Children who shared knowledge of the social valorization of sugarcane farming activities could assume different positions. For instance, most children interviewed in the Brazilian study shared the view that sugarcane farming was a low-status practice. On this basis, some developed the position that people engaged in this work did not know mathematics, despite realizing that they were proficient in their indigenous form of mathematics. Some of the children who denied the existence of mathematical knowledge in sugarcane farmers were attributing a low self-identity to themselves because they were already actively engaged in that community.

The understanding of the relationship between learning and identity is an area that requires the development of conceptual and methodological tools; however, the findings from the study with the school children in England (Abreu & Cline, 2003; Abreu et al., 2002b) helped to elaborate some ideas of the contributory factors in the ways children develop their positions. When explaining their preferences for doing mathematics at home or at school, they took into account (a) the mathematics that should be learned, (b) the mediating role of other people, and (c) the mediating role of language. Interestingly, for all the three categories, the children mentioned justifications that covered both cognitive understanding and evaluative judgments. In talking about their preferences for the mathematical tools, they encountered at school or at home, some children referred to cognitive factors, such as facility in using a particular strategy. Alternatively, some children mentioned social comparison, in terms of the valorization of the tool. Children who mentioned the mediating role of others analyzed this role in cognitive terms (e.g., commenting on whose explanations the pupil understood more), in affective terms (e.g., commenting on whom the pupil felt more at ease in asking for help), or in social comparison terms (e.g., commenting on who was seen as having knowledge or competence). Finally, the mediating role of language was treated by some Pakistani children at a cognitive level (e.g., commenting on whether they grasped mathematical concepts more easily in one language or the other). For others, however, language preference had an affective-comparative basis (e.g., defined in terms of the language in which they felt more confident). In sum, the salient factors that the children chose to justify their positions varied. Some gave primacy to their feelings of competence with specific tools, others put the emphasis on the quality of the interpersonal relationships (namely, those whom they felt to be less threatening and more patient in providing support), and still others seemed to be guided by the status of the practice in the social structure. Of course, some children mentioned more than one reason. What was common to all justifications was that they involved some type of identification with the form of knowledge (mathematical tools), with the social actors involved in a practice, or with the practices of a certain social group. Thus, we may see the importance of continued research focusing on the person and incorporating the notion of identity construction with that of the cognitive construction by mapping it into the cultural and social systems.

Cultural identities in the multicultural mathematical classroom

As outlined above, research in out-of-school mathematics, in particular, when focused on children's learning mathematics in and out-of-school in Western societies brought to light the association between the status of mathematical practices and social identities. Research in multicultural and multiethnic classrooms is adding a new dimension, which is the salience of cultural identities. Though issues of cultural identity are not totally new in accounts of mathematics learning, they have somehow been neglected to a secondary plane, or conceptualized at a group level. The quantitative studies of ethnic identity and achievement in school mathematics are examples of this approach. Identity in this perspective is basically seen as a "given," "fixed," and "static" individual characteristic associated with a group membership. Studies informed by this view typically explore how people from a certain ethnic group perform in school mathematics when compared with other groups. This particular focus has been useful in exposing differences in the mathematical performance of people from certain backgrounds. As such, one can argue these studies are important, as tools that in a democratic society provide public information of the access to available cultural capital. However, in terms of providing understanding of the processes that promote successful participation in school mathematical practices, a different focus is needed. The information that students from a particular background generally achieve below, or achieve above, other groups is of limited use for the planning of interventions if the reasons for the differences are not addressed. Indeed, this information may have a detrimental and stereotypical impact. It may be used to ascribe identities as "weak" or "poor" mathematical learners on the basis of cultural memberships. Current sociocultural theorizing asks for more fluid and dynamic conceptualizations of processes of identity development. They ask for accounts that consider both some continuity with the past of the cultural groups one is part of, and for the ruptures and profound discontinuities groups and individuals experience in the course of particular life histories. This more dynamic perspective on identity seems to be more useful in accounting for learning in multiethnic classrooms, which potentially involve many sources of discontinuities for both learners and teachers.

Studies with immigrant and minority students have now illustrated that they become aware of the differences between their home and their school practices (see Abreu, Bishop, & Presmeg, 2002a; Abreu & Elbers, 2005). In addition, some studies have also shown that students talked about these differences in relation to how they perceived their home cultural identities as intersecting with their school mathematical learning. Gorgorió, Planas and Vilella (2002) clearly illustrated this intersection when they reported the case of Saima, a 15-year-old Indian girl, who expressed the feeling of being displaced in the Catalonian mathematical classroom. As she said, "Miss, I'm wrong in your class ... I do the same mathematics as boys, but I will not do the same work ... I do not want to be a mechanic. Please, can I do mathematics for girls?" (p. 44). Saima's positioning was constructed at the intersection of her gender identity, her cultural identity and her identity as a mathematical learner.

O'Toole and Abreu (2004) also found evidence of a similar developmental process in her study with learners in multiethnic mathematical classrooms in England. They examined the case of Monifa, a 10-year-old daughter of a Black African family, who developed awareness that the differences between the mathematical practices of her father and her teacher were linked to their cultural identities. As Monifa explained: "Sometimes they just explain it differently. (...) Because my dad would have done it differently and it's where we come from because my dad was taught in Nigeria, and he taught in Nigeria. And Miss Durham has been here. So, they do it in different ways". When recounting an event where the teacher tried to convince her that her father's solution was not appropriate, she said: "I wasn't too keen but I understand my dad's more so I went with my dad. But she's my schoolteacher in school, so...." Monifa's view was that the best way of coping for her would be to stick to each mathematical practice according to the context. But, as she explained, the practices of the school and home often made requests on her that made her feel as if she were "two people": "Its like I'm two people at the same time and its just hard." Case studies, such as Saima's and Monifa's illustrate that some students develop awareness of the significance of their cultural identity in their school mathematical learning. They also suggest that to cope these students have engaged with complex identity work, which requires them to articulate the impact of differences between home and school mathematical practices in relation to identities. The extent to which these processes have a significant impact on the wider population of students from cultural backgrounds distinct from that of their mainstream schools is an issue that needs to be addressed in further research.

CONCLUSIONS

Research since the 1970s on out-of-school mathematics has followed the same direction as that of Vygotsky's theory on the impact of culture in the mind. According to Minick et al. (1993), Vygotskian research of the late 1970s and early 1980s tested the plausibility of the theoretical framework. It was one-dimensional and focused on a discussion around the relationship between cognition and cognitive tools. Following this initial period, Minick et al. suggest that during the 1980s the framework was broadened to pay attention to the following: (1) the way that institutions (e.g., school, families, commercial and financial institutions, etc.) structure contexts both in terms of styles of interactions between people and of the cultural artifacts made available (e.g., books, computers, calculators); (2) "language" as a multitude of distinct speech genres and semiotic devices in opposition to a generalized or abstract semiotic system (this multitude of genres is linked with participation in specific social institutions and specific practices); (3) real people in opposition to "abstract bearers of cognitive structures," for whom appropriation of knowledge passes through identification with communities of practice.

The cultural psychology approach I adopted in this chapter is intended to pay attention to the areas mentioned by Minick et al. (1993). As far as empirical studies are concerned, the earlier research in out-of-school mathematics scarcely investigated the last two areas. The attention to social institutions and social interactions became apparent when research in out-of-school mathematics shifted from cross-cultural comparisons to social practices within the Western societies. This highlighted the need to understand the constitutive role of the social system on learning. As illustrated by this review, however, the study of issues related to how distinct mathematical discourses are articulated, produced and reproduced and also about participation and identification are merely emerging as foci of research, and in this case linked to the research on mathematics learning in multicultural classrooms. One can speculate this is linked to particular traditions of research in the field. Studies on out-of-school learning have drawn attention to the situated nature of mathematical practices, which in its turn pinpointed the need to explore classroom learning from this perspective. The reconceptualization of mathematics classrooms as situated communities of practice has been a first step, but as yet that have not offered a satisfactory account of the interplay between the individual and the sociocultural processes. The fact that we can hardly produce explanations at this level could be linked to ways of conceptualizing and studying learning and development in situated practices (Damon, 1991). This may be due to an association between situated practices and "sociophysical space" (Kirshner & Whitson, 1997) that overlooked the fact that individuals move between practices (Abreu, Bishop, & Presmeg, 2002a). Attention to this movement will involve issues that can be located at the interface of the social and person system, such as that of identification as related to communities of practice, which are part of wider social structures, rather than isolated units. And, also at the interface of the cultural and person system, such as, the construction of hybridized tools, discourses, and identities in the intersection of practices (Abreu & Elbers, 2005). The relevance of continuing research along these lines has been recognized by those concerned with issues of diversity and equity in mathematics education (Nasir & Cobb, 2002; Cobb & Hodge, 2002; Secada, 1988, 1992). This will require substantial theoretical and methodological investment in order to address:

1. Issues related to human development and learning in changing historical circumstances (Cole, 2005). These include work of cultural developmental psychologists studying the interplay between culture and cognition (e.g., Saxe & Esmonde, 2005). It also includes

studies focusing on children and young people, who themselves or their parents learned mathematics in their home country before emigrating (Abreu et al., 2002a; Civil, Planas, & Quintos, 2005; Guberman, 2004; O'Toole & Abreu, 2005).

2. Issues of learning mathematics in multicultural schools and classrooms. Globalization and the unprecedented levels of migration in many countries have substantially changed the composition of the school population. This cultural, ethnic, and linguistic diversity within schools poses major challenges to systems of education and practices in many countries (Abreu & Elbers, 2005).
3. Issues related to the development of a cultural psychological analysis of new forms of social mediation required in schools to respond to their culturally and linguistically heterogeneous school population (Abreu & Elbers (2005).

In theoretical terms, frameworks, such as the one illustrated in this chapter, will require further development to provide specifications of key constructs that would be part of each system or plane of analysis. This is far from being an easy task. For instance, authors vary in their definitions of culture and cultural systems. The same could be said about the social system, not to mention the person system and the conceptualization of its interplay with the other two systems. Despite the early days of research in multicultural classrooms, there is already some indication that (1) the focus needs to be shifted from transmission to innovation and creation of new cultural tools and practices (Abreu & Elbers, 2005), and (2) that issues of identification are central for learners that experience transitions between home and school cultures (Abreu, Bishop, & Presmeg, 2002a; Abreu & Elbers, 2005). If we emphasize the importance of identity in mathematics learning, as illustrated in my review, this will raise a series of new questions. For instance, what are the dynamics of particular constructions of identities? Types of membership? Sense of belonging? Resistance to participation? How can the construction of identities be related to the development of mathematical knowledge?

In methodological terms, it seems there is still much to be learned by focusing on specific cultural groups or in specific socially-important collective mathematical practices in out-of-school contexts. These foci could possibly clarify the uses of out-of-school mathematics (Guberman, 2004; Hoyles et al., 2001), but need to be expanded to consider transitions between communities of practice (Abreu et al., 2002a), the specific issues of learning in multicultural classrooms (Abreu & Elbers, 2005; Gorgorió, Abreu, César, & Valero, 2005), and the impact of changing historical circumstances (O'Toole & Abreu, 2005; Saxe & Esmond, 2005). The use of ethnographic strategies also is necessary to place the individual and the communities of practice within the cultural and social systems. Saxe and Esmonde (2005) illustrate how ethnographic-longitudinal studies can offer insights into the relationships between historical shifts in economic practices and associated shifts in forms of mathematical cognition. Systematic accounts of development of mathematical understanding and uses of mathematical tools require more than that, however. The person plane of analysis requires methodological tools in order to study and analyze specific aspects. There is now some understanding of the design of tools for investigating the understanding of mathematical concepts and strategies. But, this does not apply to tools to investigating the associated processes of construction of identities. Social identity construction traditionally has been studied as part of individual social development, such as gender identification. We need, however, to be careful with transpositions by questioning the extent to which the same principles should be applied to identities in mathematical practices.

Finally, as far as educational policy is concerned, research in out-of-school contexts as well as in mathematical learning in multicultural schools suggests that in situations where home backgrounds differ markedly from school backgrounds children might benefit from an approach that helps them bridge gaps and cross boundaries (Abreu et al. 2002a). This is still far from being the way curricular reforms are implemented. For instance, the numeracy framework in England emphasizes the need for parents and communities to be involved in their children's mathematics education to ensure achievement (Abreu & Cline, 2005; Brown, Askew, Baker, Denvir, & Millet, 1998; DfEE, 1998). The parents' involvement, however, is not seen in terms of helping the children to integrate home and school numeracies (Abreu & Cline, 2005; Baker, Street, & Tomlin, 2000). It is instead portrayed in unidirectional terms in which the parents are expected to support school numeracies, but no attention is given to home numeracy practices. Although these policies are in line with representations of teachers and parents, who view the school mathematics as the relevant one for the child's success in today's society, they might fail to take into account the actual experiences of the developing child (Abreu & Cline, 2005). Research on the development of children from immigration does suggest that successful pathways are associated with transcultural identities, which promote a creative acquisition of competencies from the culture(s) of origin and the host culture (Suárez-Orozco & Suárez-Orozco, 2001). The case studies reported in the previous section do suggest that mathematics education is far from understanding how to design classroom practices that enable immigrant children to see distinct mathematical practices as part of their transcultural identities. The social and cultural basis of representations from teachers, parents, and curriculum planners need to be understood along with an examination of their impact on the way learners experience transitions between their school and out-of-school mathematical practices.

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