

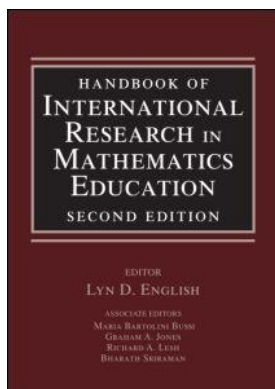
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## **Handbook of International Research in Mathematics Education**

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### **Preservice mathematics teachers' knowledge and development**

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# 11 Preservice mathematics teachers' knowledge and development

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## 1. INTRODUCTION

In order to introduce our topic, we briefly discuss the landscape of teacher education and the specific aspects of it we address in the remainder of the chapter.

### 1.1 Landscape of teacher education

Preservice mathematics teacher education is a complex process which involves many interacting elements. These elements include the kinds of knowledge, competencies, attitudes, and values that teacher candidates should develop, the context in which learning takes place (university, school, and other settings), and the roles, interests, and characteristics of the participants in the process (preservice teachers, university instructors, school mentors, and students). Other elements are program options and conditions such as pedagogical approaches, the relationship of preservice teachers and instructors, access to resources, and use of information and communication technology (ICT). Preservice teacher education also faces issues associated with bridging theory and practice and transforming the identity of preservice teachers from that of student to that of teacher. Other issues include conflicts between what is considered important for preservice teachers to learn and what they actually learn, between university and school contexts, and among the perspectives of the direct participants in the teacher education process and other interested parties such as ministries of education, school administrators, parents, media, and the public. Research also adds a layer of complexity in understanding teacher education in that studies can place emphasis on different aspects of the mathematics curriculum as well as of preservice teachers' learning and related learning opportunities.

Figure 11.1 provides one way of viewing the relationships of key components of this complex landscape of preservice mathematics teacher education. At the center of it are the development of the preservice teachers' knowledge of mathematics and knowledge of mathematics teaching. While these two components can be considered independently, there is an important overlap in which the mathematics and teaching knowledge have inherent connections. Development of the identity of the preservice teacher is shown as embodying both knowledge of mathematics and mathematics teaching. In addition, it deals with factors such as values, habits, norms, dispositions, and in general, ways of being a teacher. The development of identity is nested in, or done in reference to, the group identity of the teachers' professional community, for example, the established values and norms of the profession and the processes of professional interactions. The final component of this landscape consists of several elements that can influence the nature of preservice teacher education programs in different ways. These include:

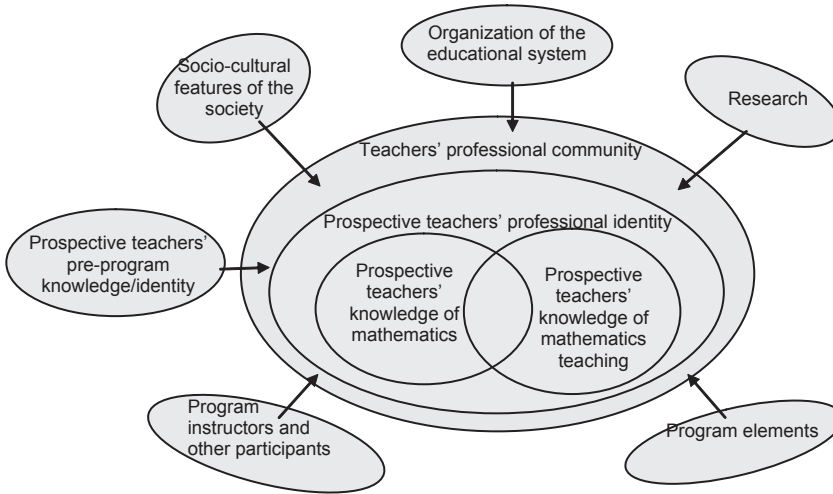


Figure 11.1 Landscape of preservice mathematics teacher education.

- Preservice teachers' characteristics, for example, their motives, interests, knowledge, and conceptions prior to entering the program.
- Program instructors and other participants' characteristics, for example, their roles, motives, interests, knowledge, conceptions, and personal features. Other participants include mentors and students involved in the preservice teachers' field experiences.
- Program elements such as teaching approaches, purposes and objectives, curriculum and materials, assessment instruments and procedures, and the overall organization and pedagogical features of the program (e.g., ways of working/learning emphasized, personal relationships between preservice teachers and other participants, access to resources, and use of ICT).
- Sociocultural features of the society, including the roles and values promoted by Ministries of Education, school administrators, parents, media, and the general public.
- Organization of the educational system, including ways of entering the profession, certification, contracts, career features, and curriculum organization.
- Research, for example, its emphases, values, priorities, and ways of disseminating results.

We offer this landscape of teacher education as a way of thinking about what the field of research could encompass and a way of locating this chapter within the broader boundaries of the field.

## 1.2 Focus of chapter

The comprehensive view of preservice teacher education presented in Figure 11.1 offers many possibilities for framing a paper about this field. However, in keeping with the theme of this section of the handbook, we have chosen to focus on particular aspects of preservice mathematics teachers' knowledge and development. This decision was also influenced by dominant themes from recent research on mathematics teacher education based on studies we reviewed for the period 1998–2005. The journal articles and books we considered cover a broad range of studies about preservice teachers' knowledge of, and attitudes toward, mathematics; knowledge of teaching mathematics, prior to, during, and on exiting preservice teacher education; and professional identity in becoming mathematics teachers. These studies fall in the category

of research undertaken with the intent of better understanding the nature and development of preservice teachers' knowledge and the features and conditions of teacher education that favor or inhibit it. Our focus, then, is on the three innermost components of [Figure 11.1](#), that is, knowledge of mathematics and mathematics teaching and identity. However, we point whenever possible to particular aspects of the other components of [Figure 11.1](#) that are inextricably linked to these three components. This includes factors from teachers' professional community, preservice teachers' characteristics, instructors, program elements, research, educational system, and society.

Our intent is to highlight examples of studies on preservice mathematics teachers' knowledge, identity and development as a way of understanding current trends in the search to establish meaningful and effective preservice teacher education. We have included significant contributions from a wide range of regions and countries, some of which often are not considered in this type of review. These studies suggest that preservice teacher education continues to be a major issue around the world and thus the importance to examine where we are and where we could be headed in order to facilitate the development of competent mathematics teachers.

We organize our discussion of these studies in five sections following this introduction. In [section 2](#), we consider papers dealing with preservice teachers' knowledge of mathematics, paying special attention to the way mathematical knowledge is conceptualized by researchers as well as to the processes through which such knowledge develops. In [section 3](#), we analyze papers dealing with preservice teachers' knowledge of mathematics teaching, again, paying attention to the way this knowledge is conceptualized and to how it is developed. These studies consider preservice teachers' learning in situations other than their practice teaching. In [section 4](#), we analyze papers related to development of a preservice teacher's identity. These studies consider preservice teachers' learning in situations involving their practice teaching. Such studies include how preservice teachers begin to assume a professional identity and, in particular, how they reflect on their practice and on their role as teachers. The studies also explore ways to assist the preservice teachers in developing as beginning professionals. In [section 5](#), we provide an overview of the theoretical frameworks and empirical research methodological features of these studies. Finally, we conclude with [section 6](#) that offers a reflective summary of preservice teachers' learning. We also discuss general issues concerning the state of research on preservice mathematics teachers' knowledge and development.

## **2. PRESERVICE TEACHERS' MATHEMATICS KNOWLEDGE**

Content knowledge is one of the critical attributes of effective teachers (Shulman, 1986). It is the cornerstone of teaching for it affects both what the teachers teach and how they teach it. It is, therefore, no surprise that teachers' knowledge of mathematics continues to be a central theme in research on preservice mathematics teacher education. Ball, Lubienski, and Mewborn (2001) point out that two research approaches have dominated efforts to study the issues surrounding teachers' mathematical knowledge that is required for teaching. The first centers on looking at characteristics of teachers, for example, the amount of mathematics teachers have taken. The second considers teachers' knowledge, in particular, with a qualitative focus on the nature of the knowledge. In this chapter, we focus on the second category of studies. We address these studies in relation to teachers' learning and give special attention to the way mathematical knowledge has been conceptualized or treated by research studies as well as to the processes through which the development of such knowledge has been facilitated in mathematics education courses or programs. We also consider what constitutes mathematics knowledge in relation to teacher education.

## 2.1 Knowledge of mathematics for teaching

While having strong knowledge of mathematics does not guarantee that one will be an effective mathematics teacher, teachers who do not have such knowledge are likely to be limited in their ability to help students develop relational and conceptual understanding (Skemp, 1976) of mathematics. As Ball et al. (2001) argue, quality teaching is directly related to subject matter knowledge. But the nature of this knowledge is a critical factor in this relationship. For example, Chazan, Larriva, and Sandow (1999) in their case study of a preservice secondary teacher's understanding of solving equations found that the participant's substantive knowledge did not provide sufficient support for the development of her students' conceptual understanding. They concluded that conceptual orientation to teaching and conceptual understanding of the topic might not be sufficient subject matter resources for teaching. More generally, Ma (1999) argues that teachers need a profound understanding of fundamental mathematics to be effective teachers. This understanding goes beyond being able to compute correctly and give a rationale for computational algorithms. It is an understanding that is deep, broad, and thorough.

In more specific terms, the National Council of Teachers of Mathematics (NCTM) in its standards for the effective teaching of mathematics describes the knowledge needed for teaching as: "The content and discourse of mathematics, including mathematical concepts and procedures and the connections among them; multiple representations of mathematical concepts and procedures; ways to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality" (NCTM, 1991, p. 132). More recently, Kilpatrick, Swafford, and Findell (2001) describe it as: "Knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and the knowledge of mathematics as a discipline—in particular, how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence that guide argument and proof" (p. 371).

Our review of recent studies of preservice teachers' mathematics knowledge suggests that there is some level of consistency between the proposed dimensions of knowledge discussed above and on those aspects of teacher knowledge that researchers have explored. Specific examples of these dimensions of mathematics knowledge studies of preservice teachers have examined are summarized in [Table 11.1](#).

These studies suggest a trend of viewing the preservice teachers' mathematics knowledge in terms of particular concepts, procedures, representations, and reasoning processes associ-

*Table 11.1* Examples of preservice teachers' mathematics knowledge studied

Decimal numbers	Stacey, Helme, Steinle, Baturo, Irwin, & Bana, 2001
Rational numbers	Tirosh, 2000
Ratio and proportion	Ilany, Keret, & Ben-Chaim, 2004 Lo, 2004
Solving equations	Chazan, Larriva, & Sandow, 1999
Functions	Sánchez & Llinares, 2003
Exponential relationships	Presmeg & Nenduardu, 2005
Points of inflection	Tsamir & Ovodenko, 2005
Mathematics of change	Bowers & Doerr, 2001
Statistical investigations	Heaton & Mickelson, 2002
Word problems	van Dooren, Verschaffel, & Onghena, 2001, 2003 Chapman, 2004
Problem solving	Chapman, 2005
Argumentation	Peled & Herschkovitz, 2004

ated with the school curriculum. They touch on a wide variety of topics such as those involving arithmetic, algebra, geometry, statistics and probability, problems and problem solving, and argumentation. Some studies also consider the preservice teachers' views of mathematics, ability to use mathematics and ability to reflect on the uses of mathematics. A common theme of these studies is that there are serious issues with preservice teachers' mathematics knowledge that teacher education programs ought to address. The nature of these issues is well documented in the literature; thus, we next provide only a brief overview of the current situation, focusing on the studies identified in [Table 11.1](#).

## 2.2 Studies of knowledge of mathematics for teaching

Over the last three decades, studies have highlighted several aspects of preservice teachers' knowledge as being problematic in relation to what is considered to be adequate to teach mathematics with depth. Llinares and Krainer (2006) reference studies over this period that have identified student teachers' misconceptions for different topics of school mathematics: arithmetic and number theory; geometry; logic and proof; functions and calculus; set theory; measurement, area; problem posing and problem solving strategies; probability; algebra; proportions and ratio. In Ponte and Chapman (2006), we summarize examples of several of these studies that consistently show the preservice teachers' mathematics knowledge to be generally of concern both in terms of what they know and how they hold this knowledge. Most of these studies deal with preservice elementary teachers and [Table 11.2](#) provides examples of issues that have been identified. However, a key example of issues with the knowledge of preservice secondary mathematics teachers that has been highlighted is the lack of a good understanding of functions.

Other recent studies continue to reflect this trend of identifying limitations with, or raising concerns about, preservice teachers' mathematics knowledge. The following examples indicate issues with three areas of mathematics that have received significant attention by researchers: functions, rational numbers and word problems.

### *Functions*

Focusing on a specific aspect of functions, Tsamir and Ovodenko (2005) investigated the concept images and concept definitions of points of inflection of 56 prospective teachers in a teacher preparation course in Israel. For the first 20 minutes of class on each of 2 days, participants completed three tasks addressing their understanding of points of inflection. The findings indicated that most of their definitions were the "personal type" and not the "concept type." Two sources of the preservice teachers' image emerged: one rooted in their previous mathematical studies and another rooted in their daily life examples. Nevertheless, the preservice teachers showed erroneous understanding of points of inflection as tangent

*Table 11.2* Issues with preservice elementary teachers' knowledge of mathematics

- 
- Procedural attachments that inhibit development of a deeper understanding of concepts related to the multiplicative structure of whole numbers
  - Influence of primitive, behavioral models for multiplication and division
  - Adequate procedural knowledge but inadequate conceptual knowledge of division and sparse connections between the two
  - Incomplete representations and narrow understanding of fractions
  - Distorted definitions and images of rational numbers
  - Lack of ability to connect real-world situations and symbolic computations
  - Serious difficulties with algebra
  - Difficulty in processing geometrical information and lack of basic geometrical knowledge, skills and analytical thinking ability
  - Inadequate logical reasoning
-

equal to zero. In another study that focused on the concept of function, Sánchez and Llinares (2003) investigated the pedagogical reasoning about functions of four preservice teachers in Spain. At the beginning of a teacher education course, data were collected on the preservice teachers' ways of knowing the concept of function and their images. A key finding was that all of the participants saw the concept of function as a correspondence between sets but, for the purpose of teaching, they emphasized different aspects of the mathematical concept. They also thought about the modes of representation of functions (graphic, algebraic, and real situation) in different ways. A related study, involving modes of representation of functions, was conducted by Presmeg and Nenduardu (2005) in the United States. This case study investigated a preservice teacher's use of representations (tabular/numerical, algebraic, and graphical) in solving algebraic problems involving exponential functions. The participant showed only instrumental understanding of the concept. The researchers concluded that fluency of conversion between modes of representation cannot be used as a sufficient criterion for inferring relational understanding or "grasp of concept" (p. 112), which requires integration of multiple modes of representation and flexibility in shifting between them.

### *Rational numbers*

Studies involving preservice teachers' knowledge of rational numbers tend to focus on fractions. However, Stacey et al. (2001) focused on decimals. In their investigation of elementary teachers' understanding of decimals, they concluded that 1 in 5 of the participants lacked good content knowledge of decimals and could pass on their poor knowledge to students. The participants in this study were 553 preservice teachers in four universities in Australia and New Zealand and 25 teachers returning for an upgrade course. Classification of patterns of teacher thinking about decimals included length of decimal, truncation of decimal, and location of a zero in the decimal.

### *Word problems*

Van Dooren et al. (2001), in their study of preservice teachers' preferred strategies for solving word problems, showed that the secondary teachers clearly preferred algebra, even for solving very easy problems for which arithmetic would be appropriate. About half of the primary teachers adaptively switched between arithmetic and algebra, and the other half experienced serious difficulties with algebra. The participants were 97 primary and secondary preservice teachers of a Belgium teacher education institute. The researchers doubted that the primary preservice teachers who experienced great difficulty with algebra would have the disposition to prepare their students for the transition to algebra. They also wondered if the future secondary teachers would be empathic with students coming directly from primary school and bringing only an arithmetic background.

The implication of these studies of preservice teachers' mathematics knowledge is that intervention through teacher education is necessary to correct the highlighted deficiencies. Some studies explicitly suggest aspects of mathematics knowledge to which teacher education should attend. For example, Lo (2004), based on an investigation of prospective teachers' solution strategies for proportional problems, suggested that pre-service courses would benefit from giving prospective teachers tasks rich in context and encouraging them to represent these tasks with pictures and diagrams in order to convey meaning to their solutions. Peled and Herschkovitz (2004), based on their study of non-standard issues in solving standard problems, suggested that teacher education programs should make teachers aware of the existing tensions between applying a mathematical model and using situational considerations, and of the dangers of applying a mathematical model without fully understanding why it fits. Van Dooren et al. (2003), based on their investigation of preservice teachers' knowledge of arithmetic and algebra, suggested that it seems valuable that students' transition from an

arithmetical to an algebraic way of thinking be treated explicitly in the mathematics education courses of preservice primary school teachers. Finally, Tirosh (2000), based on her study of prospective teachers' knowledge of children's conceptions of division of fractions, suggested that teacher education programs should familiarize prospective teachers with common, sometimes erroneous, cognitive processes used by students in dividing fractions and the effects of the use of such processes.

Collectively, these studies provide an important but partial picture of preservice teachers' knowledge of mathematics in terms of the coverage of the mathematics curriculum. They also take for granted the basis on which the knowledge is considered inadequate for teaching, in particular, reform-based teaching. The underlying assumption suggests that we have a clear understanding of not only what knowledge is meaningful but also how the teachers need to hold and use that knowledge in order for it to be meaningful in their teaching. But as Ball et al. (2001) point out, "What becomes clearer across those studies [about the nature of teachers knowledge] is that studying what teachers know, is insufficient to solving the problem of understanding the knowledge that is needed for teaching. What is missing with all the focus on teachers is a view of mathematical knowledge in the context of teaching" (p. 450). They suggest that there is an important distinction between knowing how to do mathematics and knowing mathematics in ways that enable its use in teaching practice. It is not only what mathematics teachers know but also how they know it and what they are able to mobilize mathematically in the course of teaching, that is, "What mathematical knowledge is actually entailed in teaching? How is it used?" (p. 452). To address such questions, Ball, Thames, and Phelps (2005) offer a set of hypotheses about knowledge of mathematics for teaching from the perspective of practice that includes:

- *Common content knowledge*, that is, knowledge associated with teachers having to recognize wrong answers, spot inaccurate definitions in textbooks, use notation correctly and doing the work assigned to students.
- *Specialized content knowledge*, that is, knowledge associated with teachers having to analyze errors and evaluate alternative ideas, give mathematical explanations, use mathematical representations, and be explicit about mathematical language and practices.

There is, thus, room to further explore the nature of the preservice teachers' mathematics knowledge on entering, during and at the end of teacher education programs as a basis of further informing and understanding the nature of effective programs. However, we have gained significant insights about preservice teachers' knowledge from the large number of studies already conducted in this area. These studies suggest that preservice teachers need to be involved in doing meaningful mathematics as a way to develop their knowledge of mathematics for teaching. In the next section we consider what has actually been done to facilitate this development.

### 2.3 Studies of the development of knowledge of mathematics for teaching

In addition to identifying issues with preservice teachers' knowledge of mathematics, recent studies also address ways to enhance this knowledge or remedy specific issues with it. In this section, we focus on some of these studies to examine the approaches that have been investigated. These studies represent alternative ways for developing more appropriate mathematics knowledge for teaching. In general, they embody features that have been proposed as important to teachers' learning. For example, Cooney and Wiegel (2003) addressing the kinds of mathematical experiences that promote an open and process-oriented approach to teaching proposed three principles for teaching teachers, suggesting that preservice teachers should: (1) experience mathematics as a pluralistic subject; (2) explicitly study and reflect on school mathematics; and (3) experience mathematics in ways that foster the development of process-



oriented teaching styles. In terms of more specific elements, Cramer (2004), influenced by NCTM standards, identified the following pedagogical model to frame mathematics courses for teachers:

- Mathematics content is embedded in problem settings; learners collect data, generate hypotheses, and verify conjectures.
- Learners work in small groups to optimize the opportunity for discourse.
- Questions are posed to help learners construct mathematical knowledge.
- Learners' language (oral and written) plays an important role in facilitating the transition from problem solving and exploration to formal mathematical abstractions.
- Connections within and among mathematical topics are emphasized.
- Technology use is integrated into the daily activities of the course. (p. 181)

These ways of thinking about how, and in what, preservice teachers should be engaged in learning or relearning the mathematics they will be teaching seem to make sense because they mirror the reform recommendations for mathematics education. The studies we describe next provide some evidence for the effectiveness of such approaches in teacher education programs. The underlying premise is not simply to provide the preservice teachers with more mathematics, but more importantly, to allow them to understand and reconstruct what they know with more depth and meaning. We highlight the type of learning activities being investigated and suggested as promising or effective ways to facilitate the development of mathematics knowledge for teaching.

The importance of integrating K–12 mathematics content and materials in college-level mathematics courses for preservice teachers formed the basis of a study by Beckmann et al. (2004). They reported on a project to enhance six core courses for prospective elementary and secondary school teachers by using exemplary mathematics tasks to introduce important concepts. The authors focused on reasoning and proof to discuss the nature of those courses in which preservice teachers were expected to conjecture, reason, communicate logically, and write valid proofs. One activity developed to promote conjecturing, reasoning, and proof was a probability problem that required participants to reason informally by making and testing conjectures; another was a patterning and recursion activity in which participants began with informal reasoning and later formalized their arguments; and a third was a geometry activity using formal proof methods. The authors concluded that inclusion of grade K–12 mathematics activities in core college-level mathematics courses enabled the preservice teachers to acquire a deeper and more connected understanding of the mathematical content. In a related study, Blomm (2004) reported on her institution's use of a sequence of three courses (a content and two methods courses) for preservice secondary mathematics teachers designed to let them experience the structure of mathematics and the relations of various content domains to each other and to the real world. She elaborated on the content course, which focused on having the participants solve problems from a wide variety of secondary school mathematics resources including textbooks, teaching journals, and contests. Homework was designed to include a set of problems selected from different content areas to allow the participants to first determine the mathematical facts and concepts they needed to proceed. In the author's view, as the semester progressed, the barriers that kept content areas compartmentalized started to break down and the preservice teachers began to see mathematics as a network of interrelated concepts, rather than a set of isolated skills and formulas.

In contrast to the Beckmann et al. (2004) and Blomm (2004) studies that dealt with several content areas, most of the studies focused on a specific topic or concept from the school curriculum. But they used different ways to engage preservice teachers in them. Five examples of these methods are as follows.

**Approach 1:** This is centered on the use of, what the authors describe as, authentic investigative activities. It took the form of a comprehensive model for teaching ratio and proportion

topics in the Ilany et al. (2004) study. This model consisted of four components. The first and core component included authentic investigative activities dealing with ratio, rate, scaling, and indirect proportion. Simultaneously, participants are referred to articles dealing with ratio and proportion, mathematically and pedagogically. The second component included the structure of the activity, that is, a context familiar to participants and content involving missing value, numerical comparison, and qualitative prediction and comparison problems. These problems required comparisons that were not dependent on specific numerical values. The third component involved the structure of the didactical unit, including elements such as working in groups. The fourth component dealt with the evaluation of the participants' knowledge. Findings indicated that this method was successful in producing changes in the preservice teachers' understanding of ratio and proportion.

In contrast, in the Heaton and Mickelson's (2002) study, the approach took the form of two assignments. This study investigated a way to help prospective elementary teachers develop statistical knowledge, in particular, knowledge about statistical investigations. The study was carried out in an undergraduate mathematics education course situated in the final semester of coursework of a field-based teacher education program. Participants were given two assignments. In the first, they learned to conduct a statistical investigation to address a question they posed regarding some aspect of mathematics teaching and learning at a practicum site. They were required to pose questions, identify variables, plan and carry out data collection, summarize data, report findings, and recommend changes in teaching practice. In the second assignment, they were required to develop and teach a statistical investigation unit with children in a practicum site. The aim of both assignments was the same: to engage the participants in authentic learning of statistical process and content through investigation as a way to develop insight, some technical skill, and enthusiasm for a realistic statistical investigation. Findings indicated that formulating a question that can be addressed quantitatively was problematic for the participants. In reflecting on the project, some of the preservice teachers mentioned learning statistical content and process from the assignments, but showed very little progress on the more ambitious aims of the unit.

**Approach 2:** This focuses on instructional explanations, for example, devising and debating different explanations for a mathematics concept and was used by Kinach (2002). The researcher conducted a teaching experiment in a secondary mathematics methods course that involved engaging the participants in instructional explanations of integer addition and subtraction tasks. Version A of the process invited prospective teachers to devise and thoroughly debate different explanations for integer addition, using a number line or algebra tile unit squares, before they attempted to explain integer subtraction. It was anticipated that the representations for integer addition would transfer to explanations for integer subtraction. Version B invited prospective teachers to explain integer addition and subtraction in anyway they chose. Following this, preservice teachers were invited to explain both integer addition and subtraction using representations on the number line. The author reported on Version A and indicated that it was effective in deepening the participants' knowledge of these secondary mathematics concepts.

**Approach 3:** This is centered on self-reflection and inquiry of mathematical concepts and processes. In one study by Chapman (2005), she investigated the knowledge 28 preservice secondary mathematics teachers held of problem solving and the role of reflection and inquiry activities in enhancing their knowledge of problem solving. The activities included self-reflection of the preservice teachers' knowledge of problems and problem solving, comparing problems, solving problems and reflecting on the process by writing and analyzing narratives of the problem-solving experience, representing the process with flow charts, interacting with peers in role-playing, and creating a problem solving model. The findings showed that most of the participants initially made sense of problems in terms of the routine problems they had previously experienced and that they understood the problem-solving process in a way consistent with the traditional classroom practices. Following the inquiry-reflective activities, the

preservice teachers thought of problems more from the perspective of the solver, for example, in terms of the thinking required and the level of challenge. Their understanding and model of problem solving now reflected a process in which one moves back and forth as opposed to taking a linear path to a solution.

In another study by Chapman (2004) involving problem solving and arithmetic word problems, the process engaged 20 preservice elementary teachers in analyzing and representing the problems in a variety of ways in order to enhance their understanding of them. The tasks included creating and comparing word problems of similar structure, analyzing and representing word problems for situations involving arithmetical operations in a variety of ways, focusing on the structure of the problems and the meaning of the operations, and writing journals on the meaning of the experience for them. Findings indicated that after the intervention the preservice teachers showed more depth in their understanding of the arithmetic operations and word problems. They were now able to represent different word problem situations in different modes to show the meaning of the operations.

**Approach 4:** This is centered on the use of technology to explore the mathematics concepts. In the Bowers and Doerr (2001) study, they engaged prospective secondary teachers in activities involving the use of technology as a way of developing their understanding of the mathematics of change. The objective was to introduce perturbations in their knowledge of the mathematics of change and use technology to assist in resolving them. Three core instructional sequences were designed to engage participants in experiential and graphical ways of challenging their formal knowledge of the mathematics of change. The first two sequences, which involved investigations of relative motion and parabolic motion, were specifically designed to provide opportunities to explore the richness of the fundamental theorem of calculus by examining the relationship between a velocity graph and its linked position graph. The third sequence involved having the participants design, implement, and reflect on a set of technology-based lessons to help younger students interpret various concepts of the mathematics of change. The findings indicated that the participants' efforts to resolve perturbations ultimately led them to develop a deeper understanding of the underlying quantities represented in velocity and position graphs. They also devised a more meaningful interpretation of the mean value theorem based on a graphical interpretation of rate and gained insights of the importance of differentiating between local and global interpretations of graphs and the importance of appropriate contexts.

In contrast to Bowers and Doerr (2001), where the focus of the approach was to use the technology to create and resolve perturbations, Zbiek's (1998) study used technology for developing and validating a mathematics concept. In this study, preservice secondary teachers engaged in using computing tools to develop and validate functions as mathematical models of real-world situations. Activities for exploring these functions involved using a variety of technology, such as graphing tools, symbolic manipulators, and spreadsheets. Findings indicated four categories of modeling that emerged from the activity of the participants: potential function generator that relied on the interpretation of graphical features; function-fitted selector which used only numerical comparisons to determine the optimal goodness-of-fit value; scatter plot/graphing tool that required linking algebraic forms with graph behaviors; and unneeded/unused tool that required the recall and understanding of ratios and formulas in addition to numerical comparisons of model values to data values.

**Approach 5:** This is centered on the use of concept maps and was used by Bolte (1999). This study focused on enhancing and assessing preservice teachers' integration and expression of mathematical knowledge. Concept maps and interpretive essays were used at the beginning of a lesson as a measure or review of past learning, during instruction to develop understanding, or at the end of a lesson as a summative activity. After completing a draft of their concept maps, the participants wrote an accompanying interpretive essay in which they had to clarify and expand on the relationships expressed in the maps. Findings indicated that the participants felt that the construction of the concept map and the writing of the corresponding

interpretive essay encouraged them to reflect on their knowledge and enhanced their ability to make mathematical connections. Some participants were uncomfortable with the open-ended nature of the task. The author concluded that the use of concept maps and interpretive essays provided the preservice teachers with an opportunity to mature mathematically and to experience an alternative method to instruction and assessment.

These studies, involving the preceding approaches, show a positive relationship between mathematics education coursework and preservice teachers' understanding of mathematics when the courses incorporate principles like those suggested by Cooney and Wiegel (2003) and Cramer (2004) as previously discussed. The studies suggest that a variety of approaches may lead to positive learning outcomes for preservice teachers. Thus, while the preservice teachers' understanding of school mathematics may lack depth or be deficient, with appropriate experiences they can make the transition to developing deeper and richer understanding of at least some topics. The studies also suggest a current trend of having preservice teachers probe more deeply fundamental mathematical ideas from the school curriculum. Teachers are asked to revisit familiar content and to examine it in ways unfamiliar to them in order to get to the underlying meanings of the mathematics concepts or procedures. A central goal of such studies, then, is considering approaches for helping preservice teachers to understand the mathematics they teach. But, in general, more research is needed on how specific approaches fit different aims and needs of different groups of preservice teachers, primary and secondary, with different backgrounds and in different educational systems.

### 3. PRESERVICE TEACHERS' KNOWLEDGE OF MATHEMATICS TEACHING

In order to teach mathematics, teachers need to know not only mathematics but also about mathematics teaching. If we take *knowledge* to refer to a wide network of concepts, images, and intelligent abilities possessed by human beings, including beliefs and conceptions, then knowledge of mathematics and knowledge of mathematics teaching have something in common, as indicated in [Figure 11.1](#). However, knowledge of mathematics has a referent in the academic discipline of mathematics—one of the most formalized and sophisticated fields of human thought—whereas knowledge of mathematics teaching is in the realm of professional knowledge—a field highly dependent on evolving social and educational conditions and values, curriculum orientations, and technological resources. In this section, we discuss studies related to knowledge of mathematics teaching in preservice teacher education, focusing on the nature of this knowledge and on approaches used to facilitate its development by preservice mathematics teachers.

#### 3.1 Knowledge of mathematics teaching

Knowledge of mathematics teaching has long been the focus of “mathematics methods” courses in teacher education, usually organized under the topics of “curriculum” and “instruction.” It involves the general goals of mathematics teaching, the nature of tasks and materials to use in the classroom, lesson planning, ways of organizing students, classroom communication, and assessment. Developments in research on mathematics education since the 1970s indicated the need for teachers to take into account students' thinking and learning processes. In recent years, the growth and consolidation of curriculum reform ideas in many countries has led many preservice teacher education programs to use such perspectives and materials as a major source for their organization and activities.

The nature and status of knowledge of mathematics teaching, however, is a controversial matter. Is it an outgrowth of the “wisdom of practice”? Is it a direct application of results of research in mathematics education? Is it something more special, as Shulman (1986) suggested

when he coined the term “pedagogical content knowledge” (PCK) to mean a special blend of mathematical and pedagogical knowledge? Shulman’s notion of PCK gave special emphasis to “the most useful forms of representation (...) the most powerful analogies, illustrations, examples, explanations and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others” (1986, p. 9). Associated with this notion, the model of Ball et al. (2005) to describe mathematics knowledge for teaching also describes knowledge of mathematics teaching as:

- *Knowledge of content and students*, that is, knowledge associated with teachers having to anticipate student errors and common misconceptions, interpret students’ incomplete thinking, and predict what students are likely to do with specific tasks and what they will find interesting or challenging.
- *Knowledge of content and teaching*, that is, knowledge associated with teachers having to sequence content for instruction, recognize instructional pros and cons of difficult representations, and size up mathematical issues in responding to students’ novel approaches.

Similarly, Kilpatrick et al. (2001) suggest two categories of this knowledge. First, *knowledge of students*, that is, knowing who they are, what they know, and how they view learning, mathematics and themselves; the mathematical skills, abilities, and dispositions that students bring to the lesson; the unique ways of learning, thinking about, and doing mathematics that students have developed; their common conceptions and misconceptions, and the likely sources of those ideas. Second, *knowledge of practice*, that is, knowing what is to be taught and how to plan, conduct, and assess effective lessons on that mathematical content, organizing one’s class to create a community of learners and in managing classroom discourse and learning activities to engage students in substantive mathematical work.

Our review of recent studies of preservice mathematics teachers suggests that there is some level of consistency between these proposed dimensions of knowledge of mathematics teaching and those that researchers have explored in their studies. Table 11.3 provides specific examples of this knowledge as used in such studies.

Table 11.3 Examples of preservice teachers’ knowledge of mathematics teaching studied

Beliefs about the means and purposes of mathematics teaching	Cotti & Schiro, 2004
Nature of tasks for functions	Sánchez & Llinares, 2003
Nature of tasks for statistical investigations	Heaton & Mickelson, 2002
Working with different representations, for example, fractions	Llinares & Sánchez, 1998
Algebraic and graphical representations of functions	Sánchez & Llinares, 2003
Using teaching materials, especially ICT	Bowers & Doerr, 2001 Gorev, Gurevich, & Barabash, 2004 Kurz, Middleton, & Yanik, 2004
Knowledge of students’ processes	Nicol & Crespo, 2004 Tirosh, 2000
Forms of communication	Brendefur & Frykholm, 2000
Instructional explanations	Kinach, 2002
Orchestration of class discussions	Blanton, Berenson, & Norwood 2001
Assessing students	Azcárate, Serradó, & Cardeñoso, 2005 Doig & Groves, 2004 Herrington, Herrington, Sparov, & Oliver, 1998 Santos, 2005 Zevenbergen, 2001

Shulman's notion of PCK is rather appealing to mathematics educators because it points to important issues of professional practice and offers the perspective of combining knowledge of content and pedagogy. However, it has been the subject of serious criticisms, especially regarding the epistemological status of such knowledge (e.g., Fenstermacher, 1994). Is it formal and declarative knowledge that may be learnt and assessed in a verbal way? Is it practical knowledge that only can be seen implicitly in teaching? Another important issue is how such knowledge develops. Does it only develop in contexts of practice, or may it also develop during university mathematics methods courses? What are the conditions regarding contexts of practice that support its development? As we point out in Ponte and Chapman (2006), Shulman himself has become a critic of his model. In his view, it should have more emphasis on the level of action; should consider issues of affect, motivation or passion; should pay attention to role of the community of teachers and not only the individual teacher; and its starting point should include students, community, and curriculum and not only content knowledge (see Boaler, 2003). But despite its shortcomings, PCK has helped mathematics educators make sense of important aspects of mathematics teaching practice. This practice, however, is changing led by many factors including the emergence of curriculum reform movements. As Lampert and Ball (1998) explain, preparing preservice teachers to fit within existing school practices is by itself rather problematic and teacher education programs have been long criticized for their poor performance. Preparing preservice teachers to enact innovative curriculum practices is likely to be even more problematic and to present a serious challenge to mathematics educators.

### 3.2 Studies of knowledge of mathematics teaching

As we noted above, several recent studies have included various aspects of teachers' knowledge of mathematics teaching as a focus of their investigations. These studies provide information on the nature of this knowledge prior to and/or during interventions. We next summarize a sample of such studies, some of which were previously cited in [section 2](#) in relation to the preservice teachers' knowledge of mathematics. These studies deal with three categories of preservice teachers' knowledge associated with children's mathematics knowledge, classroom communication, and reform mathematics curriculum/pedagogy.

#### *Children's mathematics knowledge*

This group of four studies addressed preservice teachers' knowledge of children's mathematics knowledge in different ways. First, Tirosh (2000) conducted a study to enhance prospective elementary teachers' knowledge of children's conceptions of division of fractions. She found that, before the course, the participants mentioned only the children's algorithmic or reading comprehension errors. After the course, they discussed the children's attempt to apply the properties of whole numbers directly to rational numbers. Most of the preservice teachers had naive beliefs about teaching and learning.

Second, Stacey et al. (2001), in their study of preservice elementary teachers' understanding of decimals, also included a focus on the participants' pedagogical knowledge in relation to students' errors. They found that the preservice teachers had only moderate understanding of their errors as being consistent with that of the students. Those who made errors on the test were more aware of potential student errors than the other participants. The preservice teachers were good at identifying features of decimal comparisons that led to students' errors but were not good at explaining why.

For the third study in this category, Klein and Tirosh (1997) evaluated preservice and inservice teachers' knowledge of children's common difficulties with multiplication and division word problems involving rational numbers and the possible sources of the difficulties. Findings of the study indicated that almost all the inservice teachers provided correct expressions

for the multiplication and division word problems (93% of correct responses), but this did not happen with the preservice teachers (only 69%). In addition, most of the preservice teachers exhibited little knowledge of the difficulties children experienced with word problems involving rational numbers and possible sources of these difficulties. The researchers suggested that direct instruction related to students' common ways of thinking could enhance the teachers' PCK.

Finally, Crespo (2004), using mathematics letter exchanges with school students, investigated how preservice teachers interpreted students' work. She found that the preservice teachers initially tended to focus on the correctness of their students' answers. However, later, they focused on the students' mathematical abilities and attitudes. They also became more analytical of the mathematics involved in the students' responses and exhibited greater attention to the meaning of student's mathematical thinking rather than to the surface features of the students' work.

### *Communication*

This group of four studies addressed situations dealing with preservice teachers' knowledge of mathematical communication with children. First, Nicol (1999) in her study of a course involving learning to teach mathematics in terms of questioning, listening, and responding, highlighted tensions prospective elementary teachers experienced in their efforts to engage students in mathematical thinking and communication. Findings of the study suggested that the participants seemed to be inclined to asking questions that focused on getting students to an answer and this seemed to be in tension with posing questions that might also elicit student thinking. However, as the course progressed, they began to pose questions, listen, and respond to their students differently.

Second, a study by Moyer and Milewicz (2002), examined the questioning strategies used by preservice elementary teachers during one-on-one diagnostic mathematics interviews with children. They identified the following six questioning strategies: (1) check-listing, with no recognition of students' responses; (2) teaching and telling, as the interviewers moved from questioning children to teaching; (3) probing and follow-up, which demonstrated the interviewer's greater attention to the child's thinking; (4) questioning only the incorrect response; (5) non-specific questioning, when interviewers consistently followed up children's answers but did so with questions that lacked specificity; and (6) competent questioning, when interviewers listened to children and used their responses to construct a specific probe to get more information about their thinking. The authors also concluded that it is important for preservice teachers to recognize that there are various types of questioning that can be used to assess and understand children's thinking in mathematics.

A third study that focused on communication is by Brendefur and Frykholm (2000). They investigated the conceptions and practices of communication in the classroom of two preservice teachers. The researchers provided a framework of four constructs to analyze forms of classroom communication—unidirectional, contributive, reflective, and instructive—and used these constructs to consider the two participants' concepts of communication and their classroom practices. They found that the two teachers had different teaching approaches and their classrooms depicted different forms of communication.

Finally, Blanton, Westbrook, and Carter (2005) used classroom discourse to identify what two preservice teachers allowed (zone of free movement) and promoted (zone of promoted action) as a way to know their potential for development. The researchers found that in some cases the participants promoted actions or events that they did not allow pupils to experience. They concluded that these preservice teachers had a limited capacity to carry out active sense making with their students. They also suggested that active listening to students' thinking could help the preservice teachers to move toward inquiry-based forms of practice.

*Reform curriculum/pedagogy*

This group of two studies deals with preservice teachers' knowledge of mathematics teaching linked to reform curriculum or pedagogical orientations. First, Frykholm (1999) conducted a 3-year study of 6 cohorts of 63 preservice secondary mathematics teachers, examining the ways in which their knowledge of the NCTM (1989) *Standards* contrasted with their teaching practices as beginning teachers. He found that most of the participants reported detailed knowledge of the reform movement, recognized what reform-based instruction should look like, and valued the *Standards* as an orientation document. He indicated, however, that such instruction "was seldom evidenced in their teaching practices" (p. 88) as beginning teachers.

Second, Steele (2001) conducted a 4-year longitudinal study that followed four participants from the time they were preservice primary teachers in a program incorporating a reform-based mathematics methods course to the end of their second year of teaching. Steele found that only two of the four teachers sustained their cognitively based conceptions about mathematics teaching and learning, and implemented these conceptions into practice.

The preceding discussion about preservice teachers' knowledge of mathematics teaching suggests that there are limitations in this knowledge that teacher education ought to address. In particular, we highlighted knowledge of children's mathematical thinking, aspects of classroom communication and reform-oriented curriculum/pedagogy. Without intervention, this knowledge seems to be mainly on a common sense level or on a theoretical level in the case of reform-oriented pedagogy. However, learning to plan and conduct teaching according to reform recommendations is likely to be rather challenging for the preservice teachers since it requires a high level of integration of knowledge of aims, tasks, materials, and students' thinking, background and interests, often in non-supportive environments.

The number of studies we were able to assign to this section of the chapter, in contrast to those on teachers' knowledge of mathematics, suggests that studies on preservice teachers' knowledge of mathematics teaching are not as well represented in the research literature. This may be the case because, as an independent component of the landscape of teacher education, this is a less developed domain of research. Alternatively, many of the studies may have integrated knowledge of mathematics teaching with other components of the landscape. This is reflected in some of the studies we assigned to identity that we discuss later in [section 4](#). It is also reflected in studies of development of knowledge of mathematics teaching, which we discuss next in order to consider what has actually been done to facilitate the preservice teachers' development in this area.

### 3.3 Studies of the development of knowledge of mathematics teaching

How should preservice teachers be supported to develop meaningful knowledge of mathematics teaching? Recent studies address this question by investigating ways of helping preservice teachers to enhance this knowledge of mathematics teaching or remedy specific issues with it. In this section we focus on some of these studies in order to examine the methods that have been investigated. There are two themes that are central to these approaches: reflection and integrating content and pedagogy.

According to Dewey (1962), teacher education should aim at "making the professional student thoughtful about his [or her] work in the light of principles, rather than to induce in him [or her] a recognition that certain special methods are good, and certain other special methods are bad" (p. 22). While this goal is conveyed in the intent of current perspectives on mathematics education, one of the challenges for teacher educators is dealing with preservice teachers who have developed their own sense of what teaching methods are good or bad and use this to frame their learning. Jaworski and Gellert (2003) also discuss this issue of preservice mathematics teachers' preconceptions. They explain that when preservice teachers enter



initial mathematics teacher education they already have extensive knowledge about mathematics teaching and have views on the nature of mathematics. But this knowledge is limited because it is based mainly on their experience as students.

The implication, then, is that it is important for programs to engage preservice teachers in learning opportunities that will allow them to re-construct their initial knowledge and understanding of mathematics teaching. However, many studies about preservice teachers, prior and during teacher education, have shown that the knowledge preservice teachers developed about mathematics teaching and learning before teacher education tends to resist change (Brown & Borko, 1992; Lampert & Ball, 1998). Thus, as Jaworski and Gellert (2003) suggest, without scrutiny of this previous knowledge, purposeful and ambitious teacher preparation is difficult. For preservice teachers, an essential part of their education is to become aware of their personal theories and preconceptions in order to make those theories explicit, and to confront, clarify and extend them by the challenge of others or alternative theories. Reflection is a key process for creating awareness of this knowledge. However, achieving effective reflection can be problematic depending on the way it is conceptualised and carried out. As Lerman (1997) notes, “Reflection on one’s own actions presumes a dialogical interaction in which a second voice observes and criticizes. In order to lead to learning it would seem that this must be more than the ongoing observation of one’s own actions” (p. 201). This suggests the need for a social context, for example, group opportunities, in which agreement and disagreement, or comparing and contrasting thinking and behaviours, can help one to become aware of taken-for-granted behaviours and notice alternative possibilities to pursue.

While the theme of reflection focuses on self-understanding, the theme of integrating content and pedagogy focuses on ways to develop new understandings of mathematics teaching. This may be achieved in a single course, focusing on content and teaching issues side-by-side, or in combining different kinds of experiences that run in parallel, sometimes even in different settings (university, school), and that draw from one another. A further way of integrating content and pedagogy is through the “isomorphism” principle, that is, the idea that preservice teachers must be taught the same way that they are expect to teach later as teachers. Jaworski (2001) also details the nature of the teacher educator action as facilitating the connection between theory and practice by developing effective activities that, in turn, promote teachers’ ability to create effective mathematical activities for their own students.

In different ways, these two ideas of reflecting on personal theories and conceptions and integrating content and pedagogy are evident in many of the studies we reviewed. For these studies, we highlight three groups of pedagogical approaches that are related to these two ideas: reflection, reflection plus content and pedagogy, and content and pedagogy.

### *Reflection*

In relation to reflection, we encountered one study that explicitly included a focus on it by testing a way to facilitate it. In this study, Cotti and Schiro (2004) were concerned with foundational aspects of mathematics education, and addressed teachers’ beliefs about the purposes and means of mathematics teaching. The participants included 109 preservice primary school teachers from two USA universities. The authors designed an instructional tool to highlight the different ways in which children’s literature may be used to teach mathematics and analyzed if this tool could stimulate teacher discussion and reflection. They indicated that most of the preservice teachers (83%) identified the child study ideology as their primary position, a view that was consistent with the orientation of the education faculty at both institutions. The instructional tool stimulated the participants’ reflection about their own beliefs and the ideological nature of educational environments. The authors argued that if knowledge about teaching mathematics is not just technical knowledge but involves value choices and moral issues, preservice teachers need to deal with these in their professional preparation.

*Reflection plus content and pedagogy*

This approach integrates the learning of content and pedagogy through special emphasis on problem solving. It implies that developing preservice teachers' understanding of problem solving could positively affect their knowledge of mathematics teaching about, and through, problem solving and other related issues. Reflection also plays an important role in it as a way of promoting learning, as in a study by Roddick, Becker, and Pence (2000). In this study, preservice secondary teachers were provided with rich and varied problem solving experiences. The authors organized two courses aimed at improving the prospective teachers' problem solving abilities, their learning of ways to assess problem solving, broadening their views of problem solving and mathematics, and enhancing their understanding of equity issues in teaching mathematics. The ultimate purpose was to promote teaching practice consistent with the NCTM *Standards*. The first course (24 participants) focused on problem posing and modeling and the second (20 participants) provided a model for reflecting on one's problem solving and concentrated on specializing, generalizing, and justifying their work. Both courses devoted substantial time to group work on problems and giving presentations and justifications to the class. The results showed that participants fell on a continuum ranging from not much discernible implementation of problem solving to substantial integration of it in their teaching. One participant, described in detail, experienced considerable growth in her views of problem solving and its role in instruction and she incorporated such learning into her classroom practices.

Another study that exemplifies this approach, but used a different way to facilitate reflection on one's problem solving was conducted by Chapman (1998). This study investigated the effect of using metaphor as a tool for facilitating preservice teachers' understanding of problem solving and its teaching. Two groups of preservice elementary teachers in Canada were studied. The study was carried out during a one-term, post-practicum year with a sample of eight who took the course following an "uncued-metaphor process" (not requiring an explicit determination of a metaphor) and the other of seven using a "cued-metaphor process" (requiring an explicit determination of a metaphor). For each problem posed, participants reflected first on an individual level, then on a group level, then again on an individual level, and wrote journals. Based on the participants' written work, the author found that "cued metaphors" provided a meaningful way of helping the participants to enhance their interpretations of problem solving and its teaching. The uncued-metaphor group regarded problem solving as "a sequential process in which a successful solution depended on a clear, logical choice among alternative strategies [and] the teaching of problem solving was viewed as guiding students through these steps" (p. 180). The other group developed a more flexible view of problem solving and its teaching that reflected a learner-centered approach.

*Content and pedagogy*

This group of studies that integrated learning of content and pedagogy highlighted two methods in engaging the preservice teachers. The first is centered on engaging the preservice teachers in teaching strategies similar to those recommended for their own teaching. It was used in the Amato's (2004) action research study aimed at improving the understanding of, and attitudes toward, mathematics and mathematics teaching of two cohorts of 42 and 44 primary school preservice teachers enrolled in a mathematics education course in Brazil. Amato explained that the teaching strategies used in the course were similar to those suggested for the preservice teachers' future use in teaching children. Theory related to the teaching of mathematics and strategies for teaching the content in the primary school curriculum were discussed in the course. However, the findings reported do not deal with knowledge of mathematics teaching but with the participants' attitude towards mathematics, which showed mixed results.

The Presmeg's (1998) study also used this approach focusing on the use of projects. Presmeg described a graduate course on ethnomathematics for prospective and practicing mathematics teachers that had the potential of indirectly modeling the use of cultural mathematics projects in their teaching. This course stressed as a key element the participants' ownership or their individual and personally meaningful cultural mathematics projects. The participants personally chose their projects and reported on them orally to the class and in writing. They also prepared activities suitable for school students. Presmeg provided details of two projects to show that such activities had a strong meaning to the participants. One involved the investigation of stick charts used in traditional North Pacific navigation. The other involved the investigation of the mathematical and historical features of the South Korean national flag.

The second method for this group of studies integrated the learning of content and pedagogy through the use of technology. It is exemplified in a study by Ponte, Oliveira, and Varandas (2002) that focused explicitly on the use of computers in a one semester ICT course in a preservice program for secondary school mathematics teachers. This course was designed to help preservice teachers develop a positive attitude toward ICT and to use it confidently. It was based on project-work pedagogy and focused on the exploration of educational software and of the Internet's potential as a means of researching and publishing educational material. The authors concluded that the preservice teachers changed from an initial attitude of fear and suspicion of ICT to a positive relationship with this technology, which they were able to learn to use confidently. They were also able to grasp more connections among mathematics topics, the historical development of topics, applications, and aspects of classroom learning processes. The participants also developed a general perspective about the uses of this technology in mathematics education. The authors suggested that teachers' future professional identity will involve this kind of relationship with ICT, in which they are not only consumers of Internet contents but also producers and co-producers of web pages with their pupils, sharing their explorations of mathematics themes and their teaching-learning experiences.

Another example of this technology-based approach involves a study carried out by Gorev et al. (2004). This study investigated the effects of using computers in mathematics courses associated with their teacher education program at an Israeli university. Based on written responses by 70 primary and secondary preservice teachers at different stages of the program, findings indicated that most of the participants who studied various mathematical courses and experienced intensive work with the computer used it to find solutions (93%) and better understand problems (69%). On the other hand, only a few of those who were only briefly acquainted with the computer indicated that they used it for the better understanding of problems (12%) and even fewer to find problem solutions (3%). The participants indicated that mastering several mathematical packages was essential for their success and supported embedding computers in their learning process. The authors argued that all computerized tools are to be learned and taught in meaningful contexts and that preservice teachers need to be able to lead enlightened mathematical discussions about the capabilities of the tool.

These three categories of studies suggest a current trend of making reflection and content and pedagogy key aspects of framing learning experiences to facilitate the development of preservice teachers' knowledge of mathematics teaching. They point to important dimensions of professional practice that teacher education programs need to address and exemplify in a variety of approaches. The value of integrating content and pedagogy, teaching in a way consistent with the proposed curriculum, and promoting reflection and ownership in learning seem to be clearly established. But it is not clear how much such reflection addresses the preservice teachers' preconceptions and how much empowerment they are able to develop for teaching in their own classrooms. As we will discuss in [section 4.3](#), reflection also plays an important role in studies on identity and we will revisit this issue then. Finally, the studies in this section tend to provide a global picture of success in the specific dimensions of knowledge to which they attend. Mathematics teaching, however, is a holistic activity and teacher education programs have to consider how the teacher, as a person, gets involved in it. We consider this in our next section.

## 4. PRESERVICE TEACHERS' IDENTITY

In order to be able to carry out a successful professional practice, preservice teachers need to learn about mathematics and mathematics teaching. However, they also need to develop several competencies and to develop as individuals, assuming the values and norms of the profession. In the previous sections, we considered preservice teachers' knowledge of mathematics and of mathematics teaching primarily from a subject matter and a curriculum perspective. We discussed what mathematics the mathematics teacher needs to know in order to teach this subject and what current mathematics curriculum orientations indicate that teachers need to know for planning, conducting classes, and assessing students. However, there is more to professional practice. Teachers are engaged in practice not just with their knowledge but also with all their being. This perspective informs studies that consider not only what teachers know but also who they are, how they see themselves as teachers, relate to students, deal with problems, reflect on issues, and identify themselves with the profession. That is, besides a subject and a curriculum perspective there is also a professional perspective. Teachers' professional practice provides an entry point for looking at teachers' identity—seen at the individual and the collective level—and teachers' learning processes—seen as a movement between theory and practice. In this section, we discuss studies related to preservice mathematics teachers' development of a professional identity with special attention to their views about their roles and the ways they reflect on themselves and on their teaching. The section highlights the nature of the construct of identity and discusses research on its development in preservice mathematics teachers through practice-based teacher education approaches.

### 4.1 Teachers' professional identity

In recent years, identity has become an important construct in the field of teacher education. This importance is associated with two key ideas that link it to teaching and learning. First, as Palmer (1997) argues, good teaching cannot be reduced to technique, as it comes from the identity and integrity of the teacher. He maintains that the complexities of teaching have three important sources: the first two, subject (content) and students, are commonplace, but the third and most fundamental, we teach who we are, is rarely given its due. As we teach, we project the condition of who we are onto our students, the subject, and our way of being together. Thus, knowing oneself is as crucial to good teaching as knowing the students and the subject. In fact, knowing the students and the subject depends heavily on self-knowledge. Second, as Lave and Wenger (1991) indicate, learning may be seen as involving the construction of identities: "The development of identity is central to the careers of newcomers in communities of practice... Learning and a sense of identity are inseparable: They are the same phenomenon" (p. 115).

The notion of identity is used in the social sciences with many different meanings (Baker, 1999). In anthropology one speaks of the identity of a group, what makes that group unique and distinct from other groups—an idea closely related to the notion of culture. In psychology and psychoanalysis, fields that focus essentially on the subject, identity becomes close to notions of self, autoconcept, and personal representation. Social psychology—particularly symbolic interactionism—provides another view, striving to connect the individual and the group, through an identification process. Identifying with a group, the individual assumes its culture, values and norms, but also has the possibility of transforming these factors and of contributing towards the transformation of the group.

This is the view of Wenger (1998), for whom identity, as the who-we-are, includes our experiences and knowledge, our perceptions of ourselves, others' perceptions of us, and our perceptions of others and of others' perceptions of us. Such perceptions develop as we interact with others. Therefore, identities exist both in us and in our relations with others. They are constructed as we interact with others and regulate our participation according to the

reactions of others to us. Identity, then, develops through “negotiated experiences of self” (Wenger, 1998, p. 150). Such experiences lead us to develop beliefs, commitments, and intentions adjusted to a particular community (in form and content). They give us a sense of who we are in relation to the community and its aims, how we must participate, and where we belong and what we are becoming in the community. Similarly, Berger and Luckmann (1966) refer to the professional identity as a process of socialization in the profession, through which the individual assumes the roles, values and norms of the professional group. In a related view of identity, Tajfel (1981) defines social identity as the elements of the individual’s self-concept that originates in his or her knowledge of belonging to a social group as well as with the value and emotional meaning of such belonging.

These ways of thinking about identity are all relevant in defining a teacher’s professional identity. For example, it may be seen as the teachers’ “professional self” or an instance of a social identity. It includes the way teachers see themselves as professionals, their relation with authority, and their professional autonomy. Also important are the processes through which teachers participate in the life of the professional group, interacting with other teachers, collaborating, and reflecting on their own activity and about themselves as teachers. In the case of preservice teachers’ professional identity, it may be considered to be about the professional self they construct and reconstruct in becoming and being teachers. It includes their appropriation of the values and norms of the profession; their core beliefs about teaching and about themselves as teachers; a vision of what it means to be an “excellent teacher” and of the kind of teacher they want to be; a sense of self as a learner and a capacity to reflect on experience. Professional identity, then, is a complex notion, as we conveyed in [Figure 11.1](#), presenting it at two levels, individual and community.

We offered the preceding brief discussion of identity to highlight key features of it that will provide a basis of identifying examples of some of these features in studies of the preservice mathematics teacher. We thus consider preservice mathematics teachers’ identity in a broad sense, that is, it is not only about what it means for one to know, do, learn, and teach mathematics but what it means to view oneself as a professional teacher and how one sees one’s ongoing development as a teacher of mathematics. This allowed us to classify several studies as dealing with identity. Following are examples of what these studies explicitly or implicitly suggested as features of, or ways of thinking about, identity.

Only three of the studies we reviewed explicitly identified, or referred to, identity as a key construct, but they differed in how they used it. First, there is Walshaw (2004), who discussed the constitution of preservice primary teachers’ identity during the teaching practicum. The author stressed “the material and embodied relations of discourse and practice” of schools and refused the view of “identity as synonymous with the teacher’s role and function” (p. 65). In her view, instead, identity “is always contingent and precarious” and may be explored “as a ‘technology of the self’, directing our attention to the political and institutional processes central to its constitution” (p. 66). Second, there is Goos (2005), who investigated the pedagogical practices and beliefs of preservice and beginning teachers in integrating technology into the teaching of secondary school mathematics. The author viewed teachers’ learning as increasing participation in sociocultural practices, leading to identity formation. Finally, Oliveira (2004) studied the impact of a teacher education program on the construction of two beginning secondary mathematics teachers’ professional identity. She highlighted the importance of biography and suggested that identity is an idiosyncratic, complex, and multi-dimensional process.

In contrast to these three studies, many others implicitly addressed particular aspects of identity. Examples of aspects of preservice mathematics teachers’ professional identity studies have examined are summarized in [Table 11.4](#). These aspects deal with the way the preservice teachers see themselves in the profession. We elaborate on some of these studies later in this section of the chapter.

There are also some studies that implicitly suggest specific aspects of identity in relation to different theoretical perspectives. For example, from a cognitive perspective, identity can be

Table 11.4 Examples of aspects of preservice teachers' professional identity studied

Sense-making of roles of teachers and students	Gellert, 2000
Sense making of teaching and themselves as teachers	Ambrose, 2004
Understanding oneself as pedagogical problem solver	Taplin & Chan, 2001
Understanding oneself and reflection in practicum context	Blanton et al. 2001 Ponte, Oliveira, Varandas, Oliveira, & Fonseca, 2005
Autonomy as related to locus of authority	Cooney, Shealy, & Arvold, 1998 Mewborn, 1999
Ability to reflect on and about practice or to investigate practice	Artzt, 1999 Cooney et al., 1998 Ebby, 1999 McDuffie, 2004 Mewborn, 1999 Nicol & Crespo, 2004
Ability to connect theory and practice, in conventional or virtual contexts	Doig & Groves, 2004 Ebby, 1999 Oonk, 2001

associated with the view of teachers as active decision makers who have to deal with difficult problems and define their priorities rather than just implement standard routines following external directions (Sullivan & Mousley, 2001). A cognitive perspective may also regard teachers as professional problem solvers with several dimensions of competence (Bergsten & Grevholm, 2004). From a humanistic perspective, identity can be associated with a view of teacher as engaging in a special kind of artistic activity, in which different forms of reflection—such as reflection on self (Palmer, 1997) and reflection on practice, about practice, and about reflection on practice (Schön, 1983)—underlie the process of professional growth. Making such reflection systematic, disciplined, thorough, and continued leads to a form of inquiry on practice or even of investigating practice (Ebby, 2000; Mewborn 2000) and suggests the need of connecting in new ways practice and theory (Jaworski & Gellert, 2003; Skott, 2005). Finally, from a sociocultural perspective, the development of identity is not a purely individual process but occurs in the context of interactions with other teachers and educational actors, especially students, school administrators, and teacher educators (Goos, 2005).

As the preceding discussion suggests, the notion of teacher identity is becoming prominent in studies of preservice mathematics teacher education. It provides a way to connect cognitive, affective, social and cultural issues and offers teacher education another new perspective from which to consider preservice teachers' knowledge, practices, and development.

#### 4.2 Studies of preservice teachers' identity

What do we know from research about preservice mathematics teacher professional identity? In section 4.1, we showed that studies that concentrate explicitly on this topic are very few and recent. However, as we noted, many studies have implicitly addressed closely related issues. These studies provide us with important elements of the identity preservice teachers develop within teacher education programs and thus can promote teacher educators' awareness of this process. In this section, we summarize a sample of such studies to highlight the elements of identity they identified in their findings.

The studies by Walshaw (2004), Goos (2005), and Oliveira (2004) that explicitly address identity provide different ways in which the preservice teachers' identity can be manifested. In the case of Walshaw (2004), she discussed the constitution of identity of preservice primary teachers during their teaching practicum in New Zealand. The participants were 72 second-year preservice primary teachers. Data were gathered through a questionnaire exploring

instances of production of teaching knowledge, as interpreted by the preservice teachers. Walshaw found that the preservice teacher accounts “reveal that teaching mathematics in primary school involves processes of normalization and surveillance, in which the spoken and the unspoken becomes intricately linked both to the production of teaching knowledge and to the subjectivity of teachers” (p. 80). In her view, “issues of power and privilege” concerning the relationships of preservice and cooperating teachers are prominent, “contributing in no small way to the shaping of teacher identification” (p. 80).

In the study by Goos (2005), the focus was on the pedagogical practices and beliefs of preservice and beginning teachers in integrating technology into secondary school mathematics classrooms in Australia. The participants were four secondary school preservice teachers who were selected because of their high interest in technology. One case is presented in detail and documents how this teacher’s modes of working with technology changed over time and across different school contexts. The author identified relationships between several personal and contextual factors that influenced the development of this participant’s identity as a teacher and showed that he moved from a perspective of using technology as a “servant,” during the practicum, to using technology as a “partner” and as an “extension of the self” during the first year of teaching. For Goos, this teacher constituted an example of an active agent in his own development, “not simply reproducing the practices he observed nor yielding to environmental constraints, but instead re-interpreting these social conditions in the light of his own professional goals and beliefs” (p. 55).

Finally, Oliveira (2004) reported on the impact a teacher education program had on the professional identity of two beginning secondary mathematics teachers in Portugal. Using data collected through biographical interviews, the author characterized the dynamics of the identity construction of one teacher as “being,” that is, her professional identity was an extension of her personal identity. Her form of being in the profession was largely her form of being in life. The other teacher’s dynamics of identity construction was characterized as “having.” She felt that she had a profession, a means of survival and financial stability, and separated her personal and professional lives. Interestingly, both teachers reported similar experiences in teacher education, noting, for example, that their mathematics courses were not a positive experience and they valued innovative approaches in mathematics teaching. However, the first teacher had an opportunity to develop a well-grounded set of perspectives about mathematics teaching, whereas the second had little involvement in the course’s activities. The first teacher also had a very positive experience of collaboration with the school mentor during the practicum, but the second did not, in addition to having to face difficult, unmotivated pupils. This study suggested that identity construction is an idiosyncratic, complex and multidimensional process, and highlighted the importance of biography in the teacher’s construction of professional identity. Oliveira indicated that the two teachers attending the same teacher education program developed distinct professional identities largely because they interpreted and experienced it in different ways according to their personal trajectories.

In contrast to these three studies, the others discussed next do not explicitly address identity. But they offer elements related to identity that add to our understanding of the preservice teachers during the process of becoming a teacher of mathematics. These studies highlight two themes: the way preservice teachers view teaching and learning and their use of reflection.

#### *View of teaching and learning*

This theme is addressed in a study by Gellert (2000). Assuming that teachers’ sense-making can be explained in terms of pedagogical content knowledge, beliefs and conceptions, Gellert studied the views of mathematics teaching and learning of 42 prospective elementary teachers in a German university. The results showed that most of the preservice teachers drew a distinction between unimportant and important mathematics and between abstract and reality-oriented mathematics. They considered that mathematics has a cultural value in our society

and valued its use in everyday situations but often equated it with arithmetic. Whereas they valued the mathematics that they know, the preservice teachers wanted to avoid allowing children to suffer negative experiences. They held the idea “that mathematics classes have to be changed from frightening and subject-matter-oriented lessons to friendly and child-centered safe spaces for learning” (p. 265). To achieve this goal, they introduced child-centered media for instruction and reduced the mathematical content. Gellert concluded that the preservice elementary teachers were more concerned with protecting pupils than with imparting in them mathematical knowledge and noted that this issue of child orientation versus subject orientation of elementary school teachers has a long history in education. This study suggested that the reason for this behavior is not simply a matter of beliefs and knowledge, but also a matter of the roles teachers want to assume in their personal relation with students.

### *Reflection*

Another element of importance to professional identity is the preservice teachers' ability to reflect on their professional activities. This element is implied in the study by Cooney et al. (1998). They combined the notions of belief and reflection in studying four preservice secondary mathematics teachers as they progressed through a four-quarter sequence in mathematics education that included student teaching. The authors selected four participants who represented a wide range of positions on mathematics teaching and learning. Reflective activity was woven throughout the program as students reflected not only on experiences but also on their beliefs about mathematics and its teaching and learning. The findings indicated that for some preservice teachers, reflection yielded epistemological crises on their existing belief structures as they examined critical features of their thinking and experiences in light of what they valued; for others, it meant very little or no change at all. The authors posited three perspectives to describe the teachers' reflective thinking (naive idealist, isolationist, and connectionist). They suggested that inculcating doubt and posing perplexing situations is central to promoting movement from naive idealist to isolationist to connectionist.

A similar view of reflection is also prominent in a study by McDuffie (2004). She examined the reflective practices of two primary preservice teachers during their practicum. The author suggested that the use of research-based resources in the teacher education program prepared the preservice teachers to approach instruction as a problem solving activity, focusing on facilitating students' understanding and anticipating problems in teaching and learning. The preservice teachers used their PCK in foreseeing problematic events and in reflecting on them. However, the author found that such knowledge was limited, since it had not been used in practice. The preservice teachers also exhibited some lack of confidence that inhibited their reflection during teaching. Even so, they reflected on their practices in a long-term way. One of them was engaged in carrying out an action research project, which proved to be a powerful setting to promote reflection.

A different treatment of reflection, that is, as problem solving, is central in the study by Mewborn (1999). She studied four preservice primary teachers during a field-based mathematics methods course in order to investigate the elements of mathematics teaching and learning they found problematic and how they resolved them. The author concluded that the preservice teachers were able to generate solutions to problems, exhibiting greater intellectual curiosity as the field experience went on. Initially, they generated hypotheses solely based on their past experiences but later they were able to take into account more information. They also progressed from making suggestions that dealt with superficial aspects of a problem to making more grounded suggestions. The preservice teachers gradually moved from using only logic to support their hypotheses about teaching to using both logic and evidence they had gained from their observations of children's mathematical thinking. Their concerns were grouped into four categories of decreasing importance: (1) classroom context and management apart from mathematics teaching and learning, (2) pedagogy of mathematics teaching,



(3) children's mathematical thinking, and (4) mathematics content and curriculum. Mewborn explained, "the decisive issue that determined whether the preservice teachers were inclined to think reflectively was the locus of authority for pedagogical ideas" (p. 335).

These studies that focused on reflection provide important insights about engaging preservice teachers in this process and how it relates to the way they assume their professional roles. For example, Cooney et al. (1998) described three main levels of preservice teachers' reflective thinking of increasing consistency. McDuffie (2004) showed that personal confidence grounded in the use of knowledge in classroom practice is important to enable reflection during teaching. Mewborn (1999) described changes in the way preservice teachers reflect, using more resources, including logic and observational evidence. Her study also showed that preservice teachers have stronger concerns with the issues that directly question their role regarding students (as classroom management) than with other elements that bear on this role (as knowledge of children, mathematical pedagogy, content and curriculum).

Taken together, the studies reviewed in this section suggest three important factors that play an important role in the development of preservice mathematics teachers' identity. The first is the nature of the relationships that they develop with experienced professionals. This is an underlying theme in most papers and is explicitly argued by Walshaw (2004) who indicated that issues of power and privilege concerning the relationships of preservice and cooperating teachers prominently contributed to shaping teacher identity. A second factor is the preservice teachers' agency. In this regard, Oliveira (2004) suggested that the preservice teachers developed distinct professional identities even though they attended the same teacher education program because they interpreted and experienced it in different ways. Cooney et al. (1998) and Mewborn (1999) proposed that the teacher's relationship with authority is of major importance to understand his or her evolution within a teacher education program. Goos (2005) posited that the preservice teacher should be an active agent in his or her own development "not simply reproducing the practices he observed nor yielding to environmental constraints, but instead re-interpreting these social conditions in the light of his own professional goals and beliefs" (p. 55). Finally, in relation to this second factor, McDuffie (2004) suggested that there is a visible difference when preservice teachers have a strong agency in their own development. A third factor is the development of personal confidence in assuming a professional role. In this respect, Gellert (2000) and McDuffie (2004) underlined the importance of preservice teachers' confidence in their own mathematical and pedagogical knowledge, which may require more appropriate experiences and time than teacher education programs often provide. Collectively, then, these studies show that institutional conditions and program features may have a strong influence on preservice teachers' learning but preservice teachers themselves are important agents in such processes.

### **4.3 Studies of the development of preservice teachers' identity**

The development of a teacher's identity is a continuing and dynamic process. Multiple influences, located in the educational, social, historical, and cultural contexts in which a teacher learns and works, shape a teacher's identity. The teacher's life course before education can be considered the first stage in the development of her or his professional identity. That is, identity begins to form a long time before entering an educational program, and formal teacher education is only one stage in its development. Teacher education programs have their cultural stories about what it means to be a teacher. These stories become one's own when a student teacher populates it with her or his own intentions and emphases, adopting her or his own expressive intention (Wertsch, 1991).

The development of a teacher's identity, then, is shaped by multiple influences. However, usually, such development is seen as a by-product of teacher education programs rather than as a targeted outcome. A focus on development of identity in these programs could include features such as the images, values, and norms of preservice teachers concerning their profes-

sional roles (about students, other teachers, and the community) and concerning themselves as teachers. We considered such features as the basis for identifying studies that deal with their development. We also considered the view that the visibility of the development of the preservice teachers' identity increases as they undertake activities and assume roles closer to those of the practicing teacher. Therefore, in this section our focus is on development activities carried out in the context of practice during field experiences. Such activities may center on observing the practice of experienced teachers or reflecting on one's own practice as student teacher.

Two key approaches that emerged from the studies in promoting the development of the preservice teacher's identity are related to aspects of reflection and investigation of practice. We use these to organize the sample of studies we discuss next and to highlight how they were used to facilitate preservice mathematics teachers' learning in different ways.

### *Reflection*

In this case, there are two common themes that are explicit or implicit in these studies: reflection on practice and on self before, during, and after the practicum and other field experiences. The first theme recognizes Schön's (1983) view, that reflecting on one's own practice is the key feature of accomplished professionals, as being important to teacher education. Reflecting on practice includes considering the appropriateness of the materials and strategies used in class, the dialogue and atmosphere established, instances of students' understandings or difficulties in grasping the concepts and specific episodes that surprise the teacher (either positively or negatively). On the other hand, regarding the second theme, Palmer (1997) suggests that self-understanding plays an important role in fostering teacher identity. Reflection on self may include the preservice teachers attending to their on-going development as teachers of mathematics; exploring the relationships between self, personal experiences, and pedagogy; evaluating their ongoing learning and development without being dependent on formal, external feedback mechanisms; and constructing and critiquing their own mathematical and educational identities. In both cases, the development of preservice teachers' reflective skills is as important as the development of new knowledge and views. The following three studies exemplify how these two focuses of reflection appear in research of preservice mathematics teacher.

First, a study with an explicit focus on reflection on practice is by Artzt (1999). She investigated the involvement of preservice teachers in structured reflection on their own practice based on the notion of developmental stages of teaching and reflection. The two participants were engaged in both prelesson and postlesson reflective activities. They submitted a lesson plan and a written account of their prelesson thoughts, indicating the goals and knowledge that drove their lessons. During the postlesson conference, they reflected on their instructional practice, thoughts and decision making. Following the conference, they did a written analysis of their lesson and a description of their thinking about the lesson using the tasks, learning environment, and discourse as starting points. One preservice teacher was able to assess her lesson carefully and critically, to generate constructive ideas for revision, and to evaluate insightfully her own competence as a teacher. The other, who learned mathematics in a highly teacher-centered environment, was led through the reflective activities to question his approach to instruction. Artzt indicated that the requirement to probe, express, examine, and question their own thinking processes led the participants to understand better and improve their instructional practices. She also noted that accessing their thinking in a structured way helped her to provide better support.

In another study, Blanton et al. (2001) explored pedagogy for supervision through a case study of one prospective middle school mathematics teacher during her practicum. The participant's models of teaching were challenged by classroom observations by the university supervisor, teaching episode interviews, and focused journal reflections about her pupils,

mathematics and mathematics teaching. The supervision episodes were conducted with (1) open-ended questions that led the participant in the process of sense making; (2) an indirect and non-authoritative supervision style avoiding evaluation of the participant's practice; (3) a sustained focus on the participant's practice; and (4) an effort to be sensitive to the participant's zone of proximal development. The supervision included prolonged conversations between supervisor and preservice teacher and supervisor visits to the school. Those visits began with an observation of the participant teaching a general mathematics class, followed by collaboration in a 1-hour teaching episode led by the supervisor and ended with a classroom observation of another class of the participant on the same topic. The authors found that their approach emerged from the participant's conflicts in practice, provided for successive transformations of a concept, allowed the participant's ownership of solutions and encouraged risk-taking in his or her practice.

The third example of reflection is a study by Ponte et al. (2005) who explored the role of ICT mediated interactions in preservice mathematics teacher education, analyzing the activity of a virtual supervision setting, including e-mail and a discussion forum. These resources were used as a complement to university supervision during the 1-year practicum. The participants were eight preservice teachers, their school mentors and university supervisors. The results showed that the forum enabled fruitful reflections and subsequent discussions but the e-mail was mostly used for organizational matters. The setting was attractive and significant for preservice teachers who, from the beginning, had a more reflexive attitude but the setting did not serve this role for the others. The authors concluded that virtual supervision may be a helpful complement to face-to-face discussions but requires the establishment of a culture of participation in the forum and a fluent use of e-mail.

These three studies suggest that reflection is more powerful when carried out both orally and in writing and could be supported by taking advantage of new communication media. It could also benefit from having some underlying structure to frame it. For example, it could have more depth if it is part of a larger framework that includes advanced planning and discussions about curriculum issues as well as short term and long term reflection about classroom events. Social interactions between the field supervisor and the preservice teacher also play an important role in all these studies. One of them suggests that using ICT may provide new opportunities for preservice teachers to reflect in a social space with their supervisors and peers. These studies, however, are more explicitly about reflection on practice. While reflection on self may have also occurred as a by-product, it is not explicitly addressed. This is similar to our findings for studies involving reflection in the development of knowledge of mathematics teaching in [section 3.3](#). However, as we noted before, it may be argued that the extent to which preservice teachers critically examine conceptions of self as mathematical can have an important effect on the pedagogies they will enact into their classrooms. For example, as Jaworski and Gellert (2003) also point out, preservice teachers tend to hold knowledge about mathematics teaching and have tacit views on the nature of mathematics that are limited and superficial. In their view, one of the main challenges to initial mathematics teacher education is to make preconceptions and tacit knowledge explicit, thus, an essential part of preservice teachers' identity development is for them to engage in reflection to become aware of their preconceptions.

### *Investigating practice*

Another theme of studies that implicitly addresses identity development of preservice mathematics teachers is related to inquiry, investigation, or research (often used synonymously) of practice. This theme extends the notions of observation and reflection by assuming an inquiry attitude that could include a more systematic process of questioning practice, collecting and analyzing data and reporting results. The underlying premise is that investigating practice may be a powerful way for preservice teachers to develop as teachers. As Ponte (2001) sug-

gested, investigation may be regarded as a thread that links the classroom, the school and the university and is a way for preservice mathematics teachers to learn how to teach by addressing problems of their own practice. Similarly, Ruthven (2001) considered learning to teach as a continual process of hypothesis testing based on the detailed analysis of the values and practical constraints of teaching, which he called “practical theorizing.” Peter-Koop (2001) also argued that viewing preservice teachers as researchers should be given more attention in teacher education. She sketched the results of ongoing work at several German universities in which primary school preservice teachers were involved in “interpretative classroom research” and indicated that with this approach the student teachers learn both about pupils and listening to them and about themselves as teachers.

The literature depicts a variety of approaches that engage the preservice teacher in inquiry of practice-oriented experiences as a basis of understanding and developing themselves as competent teachers of mathematics. We provide four examples of these approaches to highlight the nature of each and the effect on development.

**Approach 1** deals with problem-based learning as in the Hong Kong study by Taplin and Chan (2001). They considered the development of preservice primary school mathematics teachers’ skills and understanding of themselves as pedagogical problem solvers. The 11-week program was implemented with two groups, each with 14 preservice teachers. The participants were presented with a series of content-pedagogical problems, typical of the kind that classroom teachers encounter in their planning and daily activity. They discussed the problems in groups striving to reach a practical solution. Their discussions were recorded and they were asked to write journals reflecting on themselves as problem solvers and on the ways they changed their thinking about mathematics teaching. The results indicated that the participants maintained or improved their attitude towards problem-based learning and most were positive about the knowledge and skills they developed. The authors suggested that this approach can be effective for facilitating teachers’ development, provided that the tasks have classroom relevance and applicability; the teachers have some early experience of success, to build their confidence; there is plenty of opportunity for collegial discussions; and support is given when they experience negative emotions in their attempts to implement new ideas.

**Approach 2** involves analyzing self-created video clips as in the study by Nicol and Crespo (2004). They studied a process in which preservice teachers made and analyzed video clips of their own teaching in order to develop their understanding of classroom practice. The approach included an emphasis on noticing—the ability to notice particular events when useful and not only in retrospective reflection. It involved the participants in recording, editing, and sharing video excerpts of their teaching as a medium for individual and collective analysis of practice. The case of one preservice teacher showed that shifting from noticing what to record in retrospect to noticing and recording what was happening in the moment was key to helping her see her own practice as a learning site. Framing a question, analyzing her students’ thinking, and considering implications for practice involved her in taking risks. Her interest in understanding her students’ stance towards risk taking in mathematics led her to examine her own risk taking in her teaching. The authors suggested that what prompted her inquiry was the opportunity to investigate her own question related to her own teaching.

**Approach 3** involves analyzing realistic mathematics education experiences, the focus of Wubbels et al. (1997), in the Netherlands. They described a preservice program with an inquiry-oriented structure designed to prepare prospective secondary school teachers to teach in a manner valued by realistic mathematics education. The program emphasized the teachers’ capacity to analyze their own teaching and direct their own development. The aim was for teachers ultimately to be capable of tracing a five-stage process, the ALACT model, from confronting a real situation and acting upon it, to looking back and becoming aware of its essential aspects, to creating and trailing alternative solutions. The authors suggested that the program was successful in changing the prospective teachers’ views of mathematics education towards an inquiry-oriented approach. Their practice changed in a way that promoted more

effective behavior in the classroom. The authors indicated that most of the preservice teachers realized that pupils have different preferences for learning and that a variety of possible explanations for problems should be offered. However, they also indicated that only a small number of the prospective teachers reached the stage of recognizing the principle of building on pupils' own constructions, an important feature of realistic mathematics education.

**Approach 4** engages the preservice teachers in integrated coursework and fieldwork as in Ebby (2000). In this study, the approach was used with preservice primary school teachers as they learned to teach. The study was conducted in a masters-level methods course with a parallel field experience. The teacher education program strived to prepare prospective teachers to be researchers of their own practices and act as change agents in their schools. The field experience was regarded as a site for inquiry. The author found that the coursework helped the participants to think about children's learning of mathematics in new ways and that observing children's learning in the fieldwork helped them to clarify their thinking about what they were learning in the coursework. They were not simply translating theory they had learned from the methods course into action in the fieldwork classroom, but they were developing a new form of thinking, "the intellectual methods required of teaching [including] the disposition one takes towards the classroom" (p. 94). The author concluded that the goals of a methods course should include developing and nurturing particular habits of mind that help preservice teachers learn from their own teaching.

Another study that relates to Approach 4 is by Mewborn (2000) who discussed a process in which the field experience and the methods course were interwoven in order to provide the preservice teachers with an integrated learning situation. The methods class addressed issues regarding mathematics pedagogy, classroom materials and students' diversity that were observed in the school. Similarly, the preservice teachers' observations in the field were taken into account in the class discussions and readings. The author found that the locus of authority to generate and test hypothesis on teaching and learning was crucial to lead the preservice teachers to think reflectively. She suggested that three aspects were particularly important to promoting a shift in locus of authority: (1) an inquiry perspective that involved the analysis and reflection of what was going on in the field in a supportive and challenging environment, leading the preservice teachers to identify problematic issues and explore them with their colleagues, mentor, and supervisor; (2) a cohort group related to the need of student teachers to have a community with their peers with whom they could transcend the orientations that they formulated by themselves and become more oriented towards the others identifying themselves with a teaching culture; and (3) school-university collaboration, critical because the field experience and the methods course were interwoven.

In general, these studies suggest that investigating one's own practice could be a rather powerful way of constructing knowledge. It is an intensive activity that may lead teachers to reflect on themselves (Taplin & Chan, 2001), to learn about themselves (Peter-Koop, 2001), and to direct their own development (Wubbels et al., 1997). However, it is not a simple idea to implement in preservice teacher education. To investigate requires support and an extended learning process, that is, time and resources that often do not abound in teacher education programs. Furthermore, when considered as research, it raises the issue of the willingness of preservice teachers to embark wholeheartedly in such a process. Doing research involves a more demanding commitment than carrying out structured reflection, and what appear to be interesting research activities may turn out to be just a mimicking of research procedures. If the aims or the intensity of the proposed work are not well tuned, many participants may drop out as in the Wubbels et al.'s (1997) study. On another level, the studies show that different teacher education settings are successful in promoting reflection and inquiry. For example, group activities with specific tasks (as in Taplin & Chan, 2001) or extended experience in a community of preservice teachers supported by school/university collaboration (as in Mewborn, 2000) are prominent features in some studies whereas in others the focus is only on the supervisor/student teacher relationship. We, however, still have much to learn about the conditions at the personal, collective and program levels that may foster these activities.

## 5. THEORETICAL PERSPECTIVES AND METHODS IN STUDIES OF PRESERVICE TEACHERS

In the preceding sections, we focused on discussing relevant studies on preservice teachers' education in terms of the nature of the preservice teachers' knowledge and approaches to their learning, with little attention to the methods and perspectives used to frame the studies. In this section, we shift our attention to these methods and perspectives as a way of understanding the underlying landscape and boundaries of these studies. This landscape, more generally, can be tied to studies of the activity of the teacher in the classroom, which have a long tradition in education. In the 1970s, it was common to address questions on teacher classroom activity through process-product research focused on teachers' behaviors. There was close attention to what the teacher did in the classroom, considering variables such as "wait time" and "opportunity for practice," which were correlated with students' achievement to determine optimal teaching behaviors. In the 1980s, cognitive approaches became popular. Major attention was placed on teachers' beliefs and conceptions as overriding frameworks that explained teachers' activity, as well as on teachers' problem solving and decision making processes in dealing with practical problems. The notion of reflecting on practice as a way to improve it also became prominent. Finally, the 1990s saw the emergence of sociocultural perspectives, stressing the importance of viewing teachers in the classroom and social contexts and as members of professional communities. The perspectives and methods that have been used in recent studies of preservice teachers reflect this evolution. We discuss examples of these studies, most of which have been considered in the previous sections of this chapter. Since these sections are interrelated, we combine them and organize this discussion in terms of the common perspectives and methods.

### 5.1 Theoretical perspectives of studies

Many of the studies we reviewed did not explicitly identify their theoretical perspectives. However, for studies involving the preservice teachers' development of content knowledge, a cognitive perspective can be implied. For a few studies, this connection was clearly reflected by references to works of particular theorists or researchers. For example, Bowers and Doerr (2001) referenced Von Glasersfeld's (1987) constructivist perspective of experiencing dissonance and resolving perturbations in their study of prospective teachers' understanding of the mathematics of change. Stacey et al. (2001), in their study of preservice elementary teachers' understanding of decimals, and Tirosh (2000), in her study of enhancing prospective teachers' knowledge of children's conceptions of division of fractions, referenced Shulman's PCK. Chazan et al. (1999), in their study to describe preservice teachers' understanding of mathematics topics, and Presmeg and Nenduardu (2005) in their study of a preservice teacher's use of representations in solving algebraic problems involving exponential relationships, referenced Skemp's instrumental and relational understanding. Tsamir and Ovodenko (2005), in their study of preservice teachers' images and definitions of inflection points, used Tall and Vinner's (1981) notions of concept image and concept definition. Some studies such as Lo (2004), who studied prospective teachers' solution strategies for proportional problems, Kinach (2002), who studied preservice teachers understanding of representing mathematics, and Stacey et al. (2001) referenced Ma (1999) in terms of the nature of mathematical understanding elementary teachers need in order to teach for understanding. Such cognitive approaches have an important place in the landscape of theoretical perspectives. They seem to make sense for considering the preservice teachers' content knowledge and development of that knowledge. However, this cognitive focus is also implicitly reflected in many of the studies of the preservice teachers' knowledge of mathematics teaching and development of identity. These include studies dealing with PCK, addressing beliefs and conceptions (directly or indirectly), and using reflection as a major way of learning.

Other studies used theoretical frameworks based on sociocultural perspectives. For example, Walshaw (2004) considered political and institutional processes, in a Foucaultian perspective, in a study on identity. Oliveira (2004) used a framework of teaching as a moral activity in studying the impact that a teacher education program had on the construction of professional identity. Blanton et al. (2001) drew on a Vygotskian framework of discourse analysis. In a later study, Blanton, Westbrook, and Carter (2005) used Valsiner's zone theory to interpret teaching practices in mathematics and science classrooms. The authors used classroom discourse to identify what two preservice mathematics teachers allowed (zone of free movement) and promoted (zone of promoted action) as a way to know their potential for development. Goos (2005) drew on sociocultural perspectives to study the pedagogical practices and beliefs of preservice and beginning teachers in integrating technology into their teaching. The author viewed teachers' learning as increasing participation in sociocultural practices and used Valsiner's concepts to regard teacher learning as identity formation. Gómez and Rico (2005) used the framework of communities of practice to study the development and the interactions within a group of Spanish preservice secondary school mathematics teachers as they were learning to analyze the topics of equations and functions, their conceptual structure and phenomenology. Finally, based on a sociocultural framework, Ensor (2001) drew on notions of sites of practice and discursive theory to address why preservice teachers often do not implement in their practice what they learn in teacher education courses.

A few of the studies constructed their own frameworks, usually combining pieces from previous research. For example, Brendefur and Frykholm (2000) in their study of the conceptions and practices of two preservice mathematics teachers regarding communication in the classroom used a framework of four constructs to analyze forms of classroom communication—unidirectional, contributive, reflective, and instructive. Droujkova, Berenson, Slaten, and Tombers (2005), focusing on the dynamics of a mathematics methods class, developed a framework for analyzing metaphoric mechanisms of the growth of collective understanding among prospective mathematics teachers. The framework stands on the notions of PCK, growth of understanding, metaphor, and collective understanding and is illustrated with an example from a teaching experiment on the notion of multiple instructional representations in a mathematics methods class. This model has a strong cognitive emphasis but takes into account what happens in the classroom as a collective entity and may be helpful in mapping the process of development of preservice teachers' knowledge in an early stage of their preparation. Cooney et al. (1998), taking into account teachers' beliefs in relation to the voices of significant others or to what the participants valued, concluded that the locus of authority is a key variable in understanding how teachers reflect and change their beliefs.

The preceding overview of theoretical perspectives indicates that research on preservice mathematics teacher education has been using diverse and powerful theoretical frameworks or constructs. An issue underlying this is that research perspectives and topics in this field seem to follow "waves of interest," often generated by new (and sometimes not so new) concepts that then gain strong visibility. This happened, in successive periods, with the constructs of beliefs and conceptions, PCK, reflection, inquiry, and, more recently, communities of practice. But the complexity of the process of preservice teacher education is hard to comprehend through a restricted set of constructs even if those constructs are parts of powerful theories. Therefore, more than merely discrete constructs, the study of preservice teacher education requires a mobilization and integration of different fields and theories.

## 5.2 Research methods of studies

Many of the studies we discussed in the preceding sections of the chapter are qualitative case studies or small-scale studies. They used a variety of instruments and processes of data collection such as interviews, observations, classroom productions and journal writing of the preservice teachers, researchers' field notes, audio and video recording, questionnaires, and

tests. We next provide a brief overview of how these techniques appeared in a sample of studies for each section.

The studies in [section 2](#) (knowledge of mathematics) covered the range of the data collection techniques combined in a variety of ways. A few used mainly one technique, such as tasks, tests or questionnaires. For example, van Dooren et al. (2003), in their study of student teachers' content-specific knowledge concerning arithmetic and algebra, described their method as involving a paper-and-pencil test consisting of six arithmetic and six algebra word problems offered in randomized order to be solved individually by the participants within one hour. The problems were divided equally over the three different semantic categories labeled "unequal partition," "transformation," and "relation between quantities." The participants were instructed to write not only the answer, but also the underlying reasoning and solution process. Their solutions were scored as correct or incorrect and in terms of the strategies used. Ilany et al. (2004) used authentic investigative activities for teaching ratio and proportion to pre-service teachers and described their instrument as a proportional reasoning questionnaire. The questionnaire included five rate and density, five ratio, and six scaling problems. The 11 participants responded to it at the beginning and end of a course and were asked to give reasons for their answers. Similarly, Tirosh (2000), in her study of prospective elementary teachers' development of knowledge of division of fractions and sources of related common misconceptions held by children, used a diagnostic questionnaire at the beginning of the 1-year mathematics methods course. The instrument asked the participants to answer questions, indicate possible student errors for the same questions and describe possible sources of those errors. At the end of the course, the participants were then involved in class discussions with a final in-class test on concepts. Finally, Lo (2004) investigated prospective elementary teachers' solution strategies and reasoning for proportional problems using a missing value proportion task, which was presented half way through the course after instruction of ratio and proportion.

Other studies combined a variety of techniques. For example, Zbiek (1998) investigated prospective teachers' use of computing tools to develop and validate functions as mathematical models using interviews, participants' written work from individual interviews, and observations while participants worked on modeling activities in computer laboratories. The interviews at the beginning of the course probed the preservice teachers' perceptions of their mathematical backgrounds. An open-ended modeling task was used in the final interview. The data were used to generate hypotheses about how the participants developed and validated function models based on categories and patterns within the data. Bowers and Doerr (2001), who investigated prospective teachers' understanding of the mathematics of change when using an exploratory microworld, used copies of the participants' written work on the two required motion assignments, written reflections on their teaching, and the instructors' daily teaching journals to measure their growth. Data analysis involved identifying the most striking trends in the participants' mathematical and pedagogical thinking. Heaton and Mickelson (2002) investigated the opportunities preservice elementary teachers had to learn statistical investigation processes using data sources that included artifacts from classroom teaching, products of two inquiry projects, interviews with students and cooperating teachers, and the researchers' observation notes. Interviews included open-ended questions to elicit information about prior experiences with inquiry, reflection on the two assignments, and views on the value of statistical investigations in the elementary classroom. Finally, Kinach (2002) investigated preservice secondary mathematics teachers' understanding and learning to explain by representing mathematics relationships. Data were taken from 5 hours of video recordings from two of her secondary mathematics methods course classes; from her preservice teachers' written instructional explanations for integer addition and subtraction; their written reflections assessing their learning at the end of the course; and course instructor field notes consisting of the plan of learning experiences for each class and post-class reflections on the actual learning path taken during class discussions.



Some studies discussed in [sections 3](#) (knowledge of mathematics teaching) and [4](#) (identity) used mostly one qualitative technique for data collection. For example, in her study of preservice primary teachers' identity, Walshaw (2004) gathered data through a questionnaire exploring instances of teaching knowledge in production, as interpreted by the participants. In contrast, Gellert (2000) for her study of teachers' sense-making in terms of their views of mathematics teaching and learning asked preservice teachers to keep journals during a period of self-analysis and stimulated their reflection on their mathematical biographies, beliefs about mathematics and mathematics education, conceptions for future teaching, and beliefs regarding the importance of reflection. Similarly, Oliveira (2004) studied the construction of the professional identity of two beginning mathematics teachers by collecting data mainly through biographical interviews. Other studies used a variety of techniques. For example, Mewborn (1999, 2000) interviewed individually the preservice teachers, analyzed their personal journals, audiotaped and videotaped their lessons, conducted group discussions, interviewed the mentor teacher and kept her own field notes. Nicol (1999) audiotaped and videotaped classes, preservice teachers interactions with students, and debriefing meetings, interviewed preservice teachers and analyzed their journals and coursework and also her own journal.

The preceding overview of research methods indicates that studies of preservice teacher education tend to privilege small-scale designs and qualitative techniques. This is understandable since the variables and issues addressed are intimately related to personal meaning, institutional practices and traditions that do not lend themselves to quantitative processes. The overview and the other studies we discussed also indicate that much of the research on programs and interventions have been conducted by researchers closely related to those activities. This type of research is important in that it may take advantage of the insiders' views and its results may be directly used to improve practice. However, there is also a need for research in which an external view adds new perspectives and questioning power.

## 6. CONCLUDING REFLECTIONS

The preservice education of teachers who teach mathematics, either as specialists in the discipline or along with other curriculum subjects, constitutes one of the main fields of activity of mathematics educators. As we noted in [Figure 11.1](#), it is a complex field with many intermingled issues. To recap some of its specific features, it covers the purposes and objectives of teacher education, the curriculum and materials used with preservice teachers as well as the assessment instruments and procedures, the pedagogical approaches, the motives, interests, previous knowledge, and conceptions of prospective teachers, the organization of teacher education programs, the organization of the educational systems, and sociocultural features of the society. In this chapter, we sought to analyze recent international research about preservice mathematics teacher education with special focus on the most important knowledge and development domains and the pedagogical approaches used. We gave special attention to research studies carried out in the last 8 years in three main domains: (1) the development of the mathematics knowledge of prospective teachers; (2) the development of their knowledge of mathematic teaching; and (3) the development of their professional identity. We do not claim that any of these domains is more important than the others; in fact, they complement each other and only seen together do they make sense. In this section, we offer some conclusions about these domains and draw attention to the fact that there are many other levels of problems related to the institutional aspects of preservice teacher education that have a decisive role in this process.

The studies we reviewed on the preservice mathematics teachers' knowledge, identity, and development offer some key insights about teacher learning. For example, studies continue to show that there are deficiencies in preservice teachers' mathematics knowledge that require

special attention in teacher education. There seems to be consensus that the focus should be on school mathematics—the mathematics they will have to teach—and that the curriculum should be a determining factor in deciding on the nature and quality of this knowledge. However, it is still not clear how the preservice teachers ought to know and hold this knowledge in order for it to play a meaningful role in students' learning. The issue is not about developing content knowledge, that is, “standard” mathematics, but knowledge of content that is situated in the context of teaching. Although research about the mathematics knowledge of preservice teachers has a long tradition in mathematics education, a key problem is still to know from which perspective this knowledge should be analyzed and how to adopt a positive attitude to support the preservice teachers in developing such knowledge. The trend of placing emphasis on mathematics geared for their educational activity is an important step.

The studies also suggest that a variety of instructional approaches could make a difference to the quality of the preservice teachers' mathematics knowledge. Generally, interesting results arise when teacher candidates engage in (at least to some degree) exploratory approaches that offer broad opportunities for discussing, arguing, conjecturing, testing, and validating results. These situations support their activity as agents in mathematical sense-making. Preservice teachers need to be involved not only in doing meaningful mathematics, but also in reflecting, communicating, and discussing their mathematical ideas with their colleagues and instructors. School mathematics must be the focus with emphasis on the connections between mathematical ideas and their use in extramathematical contexts. Special attention must be paid to the work of teachers in planning units and tasks and understanding students' thinking. Activities must be authentic, from a double point of view: that of the mathematics challenge and that of the work of the mathematics teacher in the classroom. However, there are many open questions related to putting this into practice: Which mathematics topics must receive more attention? How to combine the different mathematical emphases? How to integrate mathematics and mathematics for teaching into mathematics education courses and into teaching practice? The responses may have to vary according to the grade level of the teacher candidates, their previous mathematics preparation, and the characteristics of the school systems.

The studies about the development of the knowledge of mathematics teaching and professional identity have shown that, as a whole, it is possible to build confidence in teacher candidates with respect to an approach based on current mathematics curriculum orientations. Some of these studies show cases of success, in which preservice teachers are beginning to develop an identity and assume practices aligned with current curriculum orientations, but the general picture is one of results falling behind expectations. Some studies underscore the view that preparing preservice teachers to teach according to reform curriculum orientations is a demanding task that requires teacher education programs to pay close attention to a wide variety of issues. They also agree on the importance of the mathematics preparation and an intensive interaction between university supervisors and student teachers during field experiences of mathematics teaching, which may have strong implications at the institutional level—how teacher education programs organize its different components and what relationship they develop with cooperating schools and educational systems.

The studies we reviewed dealing with the concept of identity are those in which these relationships play a significant role. In this context, preservice teachers get opportunities to experience what it means to be a mathematics teacher in a realistic way that (1) allows their identity to develop naturally, (2) exposes their true identity and level of competence, and (3) provides a basis for understanding their socialization into the profession. The significant number of studies related to identity suggests the growing importance of these notions as a way of promoting and understanding teacher learning. However, it is very complex to provide preservice teachers with effective competence for teaching according to the reform orientations. In a sense, what is asked of preservice teacher education is an impossible task. In a short time, it must prepare a young, perhaps rather immature person to assume a highly complex

professional function. We should not overlook the fact that teaching mathematics involves conducting mathematical activity with large groups of students (20, 30, 40, and more), who often have little interest in mathematics, have serious affective and social needs, and come from increasingly mixed cultural settings. Further, we ask new teachers to engage in practices that are compatible with innovative curriculum orientations but which generally are not the established school approach.

Given such features of teaching, teacher education should strive to provide prospective teachers with opportunities that will allow them to understand, appreciate, and embrace the complexity of their practice as a basis for ongoing inquiry. Such inquiry would facilitate developing more holistic, grounded notions of teaching and learning mathematics. A key challenge in achieving this is helping preservice teachers to integrate the knowledge of mathematical content and processes, the specific students that are to be taught, and the curriculum guidelines and orientations. This integrated knowledge must then be applied to identify and integrate resources in a practical way so as to design appropriate tasks, select materials, and create a stimulating classroom environment. Research studies suggest a range of strategies that provide some direction for meeting this challenge, in particular, the notions of integrating content and pedagogy and teaching preservice teachers the same way that they are expected to teach their students. Again, this requires that the prospective teachers have the opportunity to engage in authentic activities in which they carry out mathematics tasks, reflect, and discuss their ideas with their colleagues and instructors. Some of these authentic activities may be carried out in a simplified way at the university, while other activities must take place in school classrooms to allow the prospective teachers to develop, not only awareness and declarative knowledge about mathematics teaching, but also a capacity to carry out teachers' tasks in the usual school settings. Studies, on a local scale, suggest that with highly motivated teacher candidates, the results are often encouraging. However, true progress in this field will require a relationship among the teacher education institution, the schools and the educational authorities that provides not only a strong synergy between the work carried out at the university and at the schools that participate in teacher education, but also supports novice teachers in the initial phase of their career in education, integrating them into professional networks, helping them to reflect on their difficulties, and involving them in innovative and stimulating projects.

Another learning perspective that has been proposed is reflecting on personal theories. Reflection has been an important focus of many studies but often is only addressed in an implicit way. It seems that a general approach to preservice teachers' learning is helping them to learn how to learn, for example, engaging them in a cycle of experience/exploration, reflection, and knowledge creation as a way of equipping them to take charge of their future growth. However, many other questions remain about the nature and content of the knowledge of mathematics teaching necessary for future teachers. For example, what are the structuring elements of that knowledge and with how much depth must they be approached? How should teacher education combine the practices of daily classroom life with notions that structure the teachers' activity, such as curriculum management and assessment practices? What is the role of cross sectional issues such as multiculturalism, equity, educating for citizenship, and so on?

We conclude this section by drawing attention to some of the many other levels of problems related to political and institutional aspects of preservice teacher education. For example, in some countries the mandating of high-stakes testing to measure student performance and to hold individual schools and school systems accountable for that performance may lead many schools and many teachers to commit too much attention and time to the mathematics that appears on tests. Contextual situations, such as this, adds to the challenge of educating preservice mathematics teachers in that it becomes difficult to obtain field experiences that exemplify reform teaching and a professional community that will allow them to experience growth consistent with the philosophy of their teacher education programs. There are also the

requirements for admission into teacher education programs, the way graduates are certified at the end of the program or at a later stage, and the expectations graduates have with respect to obtaining a professional appointment that have a strong influence on preservice teacher education. Also important are aspects of the duration and the organization of the programs, the nature of their curricular units, the way the different curriculum components (mathematical, pedagogical, and practical) are interrelated, the assessment culture and *ethos* of the institution, and the role of mathematics education in the program. These are important issues concerning the interface of mathematics education with educational policy, school administration, and program evaluation that have received little attention in systematic investigations by mathematics educators. In fact, they have much in common with problems faced by preservice teacher education in other content areas such as the sciences and language teaching.

Currently, there is much controversy around preservice teacher education. Some views advance proposals that implicitly or explicitly imply deprofessionalization of teaching. Such views deny the value of the specific professional education (including learning mathematics for teaching, learning mathematics education, and developing a teacher identity). Other perspectives argue the need for a strong professionalization, with specific courses for teachers, carried out at specific teacher education institutions. Many intermediate positions are possible, allotting some of the studies to the mathematics departments, others to education departments, and yet others in schools. Each country has different arrangements, arguably more adapted to its traditions and conditions. However, we do not see how quality teacher education is possible without the organized contribution of diverse teacher education components based on the knowledge and wisdom of distinct academic and professional groups such as mathematicians, generalist educators, experienced mathematics teachers, and mathematics educators.

Finally, the work of the mathematics teacher is not just to teach mathematics (notwithstanding all its importance). In fact, many teachers, especially in the elementary school, teach many subjects other than mathematics. Besides, every mathematics teacher has a fundamental role in helping the students to grow and develop as socially integrated, active and critical human beings. Teachers are also responsible for the construction and development of the educational project of their institutions, the development of their profession and of education in general, and for their own development. Those roles emerge from the general expectations that society holds for schools, but they also result from the specific needs of mathematics learning and teaching, and of the work of the school as an institution. It would be too narrow to consider only the practical aspects of the work of the mathematics teacher, ignoring that, the teacher is also deeply involved as a person.

Two interconnected factors that contribute to this aspect of the teachers' work and of teacher education have acquired strong visibility: (1) the growing value attributed to the role of practice in the teachers' activity and in his or her development processes, and (2) the emergence of teacher development approaches that value the reflective dimension, inquiry, work in collaborative groups and communities of practice. After mathematics has been (justifiably) valued, after the curriculum and professional knowledge has been (also justifiably) valued, the time is ripe to value the developing person and the professional. Thus, rather than seeing the teacher candidate as a *tabula rasa*, into whom a pile of knowledge is poured or who needs to be trained in some techniques and skills, we may see him or her as a person, with his or her professional and personal dimensions, within institutional, social, and professional contexts. In this process of professional development the capacities of reflection, problem solving, and inquiry of practice become particularly important. The strategies used to develop them—reflecting, problem-based learning, inquiry, and connecting theory and practice—share the common features of deeply involving teachers in dealing with complex issues, framing problems, seeking and evaluating solutions. However, the strategies differ in the emphasis they put on structure, theory, social values, and professional culture. More research is needed to better understand the possibilities and implications of each of them in different social and institutional settings as well as the conditions at the personal and

collective level that may support the development of teachers' professional identity as active agents in the educational process.

Although preservice mathematics teacher education has gained a better understanding of the processes by which one learns how to teach mathematics and to carry out one's professional role as teacher, many questions still remain in this field. As we learn more and more, many of these questions may remain unanswered because local conditions vary widely. Preservice teacher education is not a closed system, but a subsystem that depends on other larger social systems that also evolve in relation to the larger social changes. Such inherent incompleteness is a major challenge and source of interest in preservice mathematics teacher education.

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