4. Explicit modeling as a research strategy
4.5

Mathematical models of world-system development

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Large-scale models of world-system development

The most well-known models of world-system dynamics were developed in the early 1970s on the initiative of the Club of Rome (e.g., Forrester [1971: World Dynamics]; Meadows et al [1972: The Limits to Growth]; Mesarović and Pestel [1974: Mankind at the Turning Point]). The projects by Forrester and Meadows were the first efforts to develop large-scale models of world-system dynamics. Forrester’s WORLD-2 model does not assume regional or zonal divisions, but rather the world-system is represented as a single whole. Forrester chose the following five systemic aspects: population, capital funds, that part of investment that is made in agriculture, natural resources, and pollution. There are five model blocks corresponding to those five aspects. They are linked by a system of positive and negative feedback loops. On the basis of this model, Forrester and his colleagues made calculations up to the year 2100. Forrester’s model was developed further by the project led by Meadows. The Meadows WORLD-3 model was more complex but it retained many of the features of Forrester’s model.

The project by Mesarović and Pestel (1974) was the second (after Meadows’) in the series of reports to the Club of Rome. With this model, they tried to achieve a higher level of detail by representing the world-system as ten interacting regions. Each region is described with a system of submodels that is the same for all the regions. The regional differences are produced by values of initial conditions and parameters. The relations among regions are modeled as imports, exports, and migration.

At present there is quite a large number of such mathematical models of world-system development (e.g., Hughes 2008; Reuveny 2008); however, the principles of construction of these models continue the traditions of the above-described approaches.

Compact macromodels of world-system growth

In 1960, von Foerster et al published a striking discovery in the journal Science. They showed that between 1 CE and 1958 CE, the world’s population ($N$) dynamics can be described in an extremely accurate way with an astonishingly simple equation:

$$ N_t = \frac{C}{t_0-t} \quad (4.5.1) $$
where \( N_t \) is the world population at time \( t \), and \( C \) and \( t_0 \) are constants, with \( t_0 \) corresponding to an absolute limit ("singularity" point) at which \( N \) would become infinite.

Note that the graphic representation of this equation is nothing but a hyperbola; thus, the growth pattern described is denoted as “hyperbolic.” It was shown later (Korotayev et al 2006) that the world GDP dynamics up to the early 1970s can be described in a similarly accurate way with a quadratic hyperbolic equation:

\[
G_t = \frac{C}{(t_0-t)^2}
\]  

(4.5.2)

where \( G_t \) is the world GDP.

The hyperbolic pattern of the world’s population growth (accompanied rather logically by quadratic-hyperbolic growth of the world-system GDP) has been shown to be accounted for by the following nonlinear second order positive feedback mechanism: the more people—the more potential inventors—the faster technological growth—the faster growth of the Earth’s carrying capacity—the faster population growth. With more people you also have more potential inventors and hence faster technological growth (Kremer 1993; Korotayev 2005, 2008; Korotayev et al 2006).

For the period prior to the 1970s, the world-system economic and demographic macrodynamics have been shown to be described in a rather accurate way with an extremely simple two-differential-equation mathematical model (Kremer 1993; Korotayev 2005, 2008), for example:

\[
\frac{dN}{dt} = aSN 
\]  

(4.5.3)

\[
\frac{dS}{dt} = bNS
\]  

(4.5.4)

where \( N \) is the world population, \( S \) is the surplus produced, per person, over the amount minimally necessary to reproduce the population with a zero growth rate in a Malthusian system; \( a, b \) are constants.

In order to describe the world-system’s demographic dynamics in the last decades (as well as in the near future), it has turned out to be necessary to extend the Equation system (4.5.3)–(4.5.4) by adding to it Equation (4.5.6), and by adding to Equation (4.5.3) the multiplier \((1-l)\), where \( l \) is proportion of literate population, which results in Equation (4.5.5), and produces a mathematical model that describes rather accurately not only the hyperbolic growth of the world-system up to the 1970s, but also its withdrawal from the blow-up regime afterwards (Korotayev et al 2006):

\[
\frac{dN}{dt} = aSN (1-l)
\]  

(4.5.5)

\[
\frac{dl}{dt} = cSl (1-l)
\]  

(4.5.6)

The model takes into account earlier empirical findings identifying female literacy as a major factor of fertility decline. The additional equation introduced for literacy growth (4.5.6) has the following sense: the literacy growth rate is proportional to the proportion of literate people in the whole population \( l \) (potential teachers), to the proportion of illiterate people in the whole
population \((1 - l)\) (potential pupils), and the presence of surplus \(S\), which can be used for educational purposes.

References


