3.1 BEHAVIOUR OF A SINGLE RAILWAY WHEELSET

3.1.1 Movement on straight path

The movement of a conventional single (isolated) railway wheelset is known as a sinusoidal or hunting motion (Figure 3.1) and was first studied by Klingel in 1883 (Julien and Rocard, 1935).

Klingel simulated the railway wheelset with a bicone. He assumed that the axle moved at a constant speed \( V \) in the direction of the track axis while at the same time, at a random moment in time, it is laterally displaced by \( y \) and rotated by an angle \( \alpha \).

Klingel proved that the bicone motion is sinusoidal with a difference of phase of the parameters \( y \) and \( \alpha \) equal to \( \pi/2 \) with the following characteristics:

- **Wave amplitude:** \( y_w \)  
  \[
  y_w = \sqrt{\frac{r_o \cdot e_o}{\tan \gamma_o}} \quad (3.1)
  \]

- **Wavelength:** \( L_w \)  
  \[
  L_w = \frac{2\pi \cdot e_o}{\tan \gamma_o} \quad (3.2)
  \]

- **Frequency:** \( f \)  
  \[
  f = \frac{V}{2\pi \sqrt{\tan \gamma_o \cdot e_o}} \quad (3.3)
  \]

- **Maximum lateral acceleration:** \( y''_{\max} \)  
  \[
  y''_{\max} = 4\pi^2 \cdot y_w \cdot \frac{V^2}{L_w^2} \quad (3.4)
  \]

The reduction of conicity \( \gamma_o \) of the wheels, the increase of the rolling radius \( r_o \) and the increase of the length of the railway wheelset \( 2e_o \) increase the wavelength of the sinusoidal motion and reduce the lateral accelerations of the axle.

Klingel presented a pure kinematic analysis of the phenomenon assuming a harmonious motion without damping and without flange contact. In reality, the motion of a railway wheelset and particularly of a whole vehicle (car body + bogies) is much more complex (Esveld, 2001).

3.1.2 Movement in curves

Let us consider the layout of Figure 1.12. Upon entering the track and trying to achieve equilibrium, the axle is displaced by \( y_o \) with regard to the curve’s outer face.

Let us calculate the displacement \( y_o \) for the above position. The rolling radii of the two wheels will be...
Let us consider $S_1$ and $S_2$ as the paths covered by the two wheels during the time interval $t$, we have

$$S_1 = V_1 \cdot t$$

$$\Rightarrow \frac{S_1}{S_2} = \frac{V_1}{V_2} = \frac{\omega_1}{\omega_2}$$

(3.6)

$$S_2 = V_2 \cdot t$$

and

$$\frac{S_1}{S_2} = \frac{V_1}{V_2} = \frac{(R_c + e_o) \cdot \xi}{(R_c - e_o) \cdot \xi} = \frac{\omega \cdot (r_o + \gamma \cdot y_o)}{\omega \cdot (r_o - \gamma \cdot y_o)} \Rightarrow y_o = \frac{e_o \cdot r_o}{\gamma \cdot R_c}$$

(3.7)

According to the mathematical equation (3.7) the displacement $y_o$ is in reverse proportion to the equivalent conicity $\gamma$ and the radius of curvature $R_c$. On the contrary, the increase of track gauge and the increase of the wheel diameter lead to an increase of the displacement $y_o$. For $y_o = \sigma$ we have a contact of the flange with the outer rail, where $\sigma$ is the flange way clearance.

3.2 BEHAVIOUR OF A WHOLE VEHICLE

3.2.1 Operational and technical characteristics of bogies

3.2.1.1 Object and purposes of bogies

The term bogie sometimes simply denotes a construction that supports the car body without including the wheelsets. However, and this is usually the correct definition, the term refers to the total of ‘secondary suspension – bogie frame and primary suspension – wheelsets’. 
The ability of the inscription of a railway vehicle in curves depends directly on the length of the vehicle. Initially, railway trailer vehicles were relatively short and their inscription in curved sections of the horizontal alignment was achieved through two or three single wheelsets linked directly to the car body. The evolution of railway as a means of transport went hand in hand with the increase in the vehicles’ transport capacity, a fact which dictated the increase in the vehicles’ length. Under these circumstances, the inscription of vehicles could no longer be attained using the technique of the single wheelsets.

Using the bogies, the inscription is achieved essentially via the bogies (wheelbase length \(2a < 4.0\) m) while the car body follows the movement of the bogies (Figure 3.2).

The bogies must

- Allow the smooth inscription of the wheelsets in curves
- Assist the optimum transfer of loads from the car body to the rails
- Provide stability of the vehicles on straight path (development of high speeds)
- Provide dynamic comfort to passengers in three directions
- Have relatively low construction and maintenance cost

### 3.2.1.2 Conventional bogies

#### 3.2.1.2.1 Description and operation

Nowadays, ‘conventional’ or ‘classic’ bogies are broadly used in trailer and power vehicles. In this technology, the bogies are fitted with wheelsets, the wheels of which are rigidly linked to the axle resulting in the rotation of the wheels and the axle at the same angular velocity (classical wheelsets). The bogie frame is connected to the car body and the wheelset by means of elastic elements and dampers providing the vehicle with two suspension levels namely primary suspension: wheelset–bogie suspension (usually materialised by coil springs...
and dampers or rubber elements [chevrons]) and secondary suspension: bogie–car body suspension (materialised by air suspension or coil springs and dampers) (Figure 3.3).

There are various types of conventional bogies; the choice and design among them depends directly on the functionality of the vehicles to which they will be mounted on and on the geometrical characteristics of the track they will run on.

Figure 3.4 shows in detail all the individual parts which form a conventional power bogie (Schneider Jeumont Rail, no date).

3.2.1.2.2 Design of the bogies

Good construction of the track superstructure does not guarantee on its own a smooth train ride and the achievement of the desired performances; design and construction of the rolling stock is of equivalent importance. Developing a railway bogie from design to commissioning involves the following main stages:

- Conception of the bogie technology and physical explanation of the bogie behaviour
- Theoretical study and modelling of its dynamic behaviour using simulation models
- Design and construction
- Testing
- Commissioning and entering into operation

The geometrical and technical characteristics of the bogies that substantially affect the dynamic behaviour of the vehicles are (Joly, 1983)

- The longitudinal ($K_x$) and lateral ($K_y$) stiffness of the primary suspension springs
- The bogie wheelbase ($2a$) (Figure 3.3)
The wheel diameter ($2r_o$)
- The mass of the bogie ($M'$) and of the wheelsets ($m$)
- The equivalent conicity of the wheels ($\gamma_e$)

All the above elements are directly related to the lateral behaviour of the bogies that determine the steady motion of the vehicles on straight paths and the good negotiation of curves as well as

- The car body mass ($\overline{M}$)
- The vertical stiffness ($\overline{K}_z$) of the secondary suspension springs
- The damping coefficients ($\overline{C}_x, \overline{C}_y$, and $\overline{C}$) of the secondary suspension dampers

The last three features are directly related to the vertical behaviour of the bogies that characterise the dynamic comfort of the passengers.

Despite the technological advances for the rolling stock and the track equipment, it is not possible to guarantee both high speeds on straight paths and good negotiation of wheelsets in curves.

Indicatively it is noted that (Pyrgidis, 1990)

- The high value of the longitudinal stiffness of the bogie – wheelsets connection ($3 \times 10^7$ N/m $\geq K_c \geq 10^7$ N/m)
- The small value of the equivalent conicity of the wheels ($\gamma_e < 0.12$)
• And the fixing of devices which restrict the horizontal rotation of the bogie and of the car body (bogie yaw dampers) allow a conventional railway vehicle to move in complete safety on a straight path of good quality at a speed \( V > 350 \) km/h. However, for a radius of curvature \( R_c < 6000 \) m these constructional characteristics relate to

• Wheel slip
• Wheel flange contact with the outer rail

leading to fast wearing out of the wheels and development of guidance forces which in the case of curves with a small radius of curvature \( (R_c < 500 \) m) can provoke a lateral displacement of the track.

Table 3.1 shows the influence of the constructional and geometrical parameters of the bogies at the vehicle’s critical speed \( V_{cr} \) (movement along a straight path) as well as on parameters \((y, \alpha)\) which determine the positioning of the wheelsets in curves.

An increase in the critical speed results in an increase in the stability of the vehicle moving on a straight path, hence the possibility of achieving higher speeds. An increase in the lateral displacement \('y'\) and the yaw angle \('\alpha'\) of the bogie’s front wheelset is equivalent to an increase in creep forces, a likely wheel slip and the appearance of guidance forces (flange contact); and in general an expected poor negotiation of curves (wear on the wheels and the rail, lower speeds, risk of derailment and lateral displacement of the track).

The inability of classical bogies to combine the stable motion of vehicles at high speeds on straight paths with a safe and wear-free negotiation of curves has led to a continuous effort to improve the performance of the wheel–rail system. Within the context of this effort, many improvements have been made regarding the way in which bogies are

| Table 3.1 Influence of constructional and geometrical characteristics of bogies |
|-----------------------------------|-----------------------------------|
| **Constructional and geometrical characteristics of bogies** | **Movement along a straight path – change of \( V_{cr} \)** | **Movement in curves – change of \( y, \alpha \)** |
| Reduction in equivalent conicity \( \gamma_e \) | Increase | Increase |
| Increase of the wheelbase \( 2a \) | Increase | Increase |
| Increase in the diameter of the wheels \( 2r_0 \) | Increase | Increase |
| Increase in longitudinal stiffness of the bogie–wheelset linkage \( (K_x) \) | Increase | Increase |
| Increase in lateral stiffness of the bogie–wheelsets linkage \( (K_y) \) | Increase | Unchanged |
| Increase in mass of the bogies and the wheelsets \((M', m)\) | Reduction | Unchanged |
| Placement of yaw dampers between the bogie and the car body | Increase | Increase |


Note: Movement along straight path and in curves.
designed and manufactured (new techniques, new elastic connecting materials, lighter bogies, wheelsets, etc.).

One parameter that restricts the performance of conventional bogies is the equivalent conicity $\gamma$ which characterises the wheel wear. This parameter significantly influences the lateral behaviour of the bogies (Pyrgidis and Bousmalis, 2010). The increase in the number of kilometres run by the bogies translates to increased wheel wear and increased initial equivalent conicity; this results in a decrease in the vehicle’s critical speed for which it was originally designed. To regain the original performance, the profile of the wheel treads needs to be reshaped at frequent intervals.

### 3.2.1.3 Bogies with self-steering wheelsets

The behaviour of a bogie in curves is improved when the bogie’s wheelsets are placed in a radial way within the curved path. This positioning is not possible with conventional bogies, where generally, bogie–wheelset connections are rigid.

The self-steering (or auto-oriented wheelsets or radial wheelsets or steered bogies) technology allows an almost ideal negotiation of bogies in curves of small radius of curvature ($100 \, \text{m} < R_c < 500 \, \text{m}$). In this technology, the two classical wheelsets of a conventional bogie are also connected to each other by means of elastic connections of defined stiffness $K_\varsigma$ and $K_b$ (Figure 3.5), where $K_\varsigma$ is the lateral stiffness and $K_b$ the angular stiffness.

The bogie frame–wheelset connection is achieved in different ways (Scheffel, 1974; Scheffel and Tournay, 1980; Joly, 1988; Pyrgidis, 1990).

In the case of conventional bogies, the value of the lateral stiffness $K_\varsigma$ between the bogie and the wheelsets depends on the value of the longitudinal stiffness $K_\chi$ ($K_\varsigma = \lambda K_\chi$ where $\lambda < 1$). On the contrary, the technology of self-steering wheelsets allows the manufacturing of springs with independent lateral and angular stiffness values (the angular stiffness $K_b$ plays the same role as the $K_\chi$). As the value of the lateral stiffness does not practically influence the negotiation of bogies in curves, it is possible, using this technology, by reducing the angular stiffness and increasing the lateral stiffness to achieve very satisfactory results in small radius of curvature while securing at the same time average values of speed on straight paths ($V = 160$–$220 \, \text{km/h}$).

Moreover, this technology seems to provide the wheelsets with a guidance mechanism that tends, in most cases, to position the wheelset inwardly to the curve in a radial way. This technology was first proposed by H. Scheffel in South Africa and it was afterwards

---

**Figure 3.5** Bogie with self-steering wheelsets.
developed in countries with railway networks comprising a significant percentage of curves with small radius of curvature (Scheffel, 1974). This technology is also applied in the bogies of tilting trains (Pyrgidis and Demiridis, 2006).

### 3.2.1.4 Bogies with independently rotating wheels

In conventional bogies, during a lateral movement of a wheelset on the track, the two wheels of each axle rotate at different rolling radii due to their conical profile. This results in the appearance of longitudinal creep forces of equal value and opposite direction \((X_1 = -X_2)\) on each wheel.

To avoid the sinusoidal movement of a wheelset, it is necessary to dispense of the rigid connection of the wheels with the axle in order for the two wheels to be able to rotate at different angular velocities, thus maintaining during their movement the mathematical equation  (2.32)

\[
\omega_1 \cdot r_1 = \omega_2 \cdot r_2 = V
\]

where
- \(\omega_1, \omega_2\): angular velocities of the two wheels
- \(r_1, r_2\): rolling radii of the two wheels
- \(V\): forward wheelset speed

This mathematical equation guarantees wheel rolling without slip and nullification of longitudinal creep forces. This simple reasoning led researchers to develop the technology of bogies with independently rotating wheels (Figure 3.6) (Panagin, 1978; Frullini et al., 1984; Frederich, 1985, 1988; Pyrgidis, 1990).

Using this technology each bogie has four wheels, which rotate at different angular velocities (freely).

Two techniques of implementation of this technology are distinguished

- Bogies with wheelsets (Figure 3.6a)
- Bogies without wheelsets (Figure 3.6b)

![Figure 3.6](image-url)
Irrespective of the technical implementation, this technology theoretically allows for the development of very high critical speeds on straight paths. However, the wheelset is very vulnerable to lateral displacement since it can only apply the gravitational force on the track, which is activated at each lateral displacement of the wheelset due to the variable conical wheel profile (Pyrgidis and Panagiotopoulos, 2012). Conversely in curves, wheelsets with wheels, which rotate independently, cannot be placed on the track in a radial position.

It is possible to improve the positioning of the wheelsets in curves should a high value of equivalent conicity be used. This choice does not cause problems on straight paths (since sinusoidal wheelset movement is mitigated), while at the same time, the value of stabilising gravitational forces increases.

The technology of independently rotating wheels is extensively implemented in tramways (Pyrgidis, 2004; Pyrgidis and Panagiotopoulos, 2012).

### 3.2.1.5 Bogies with creep-controlled wheelsets

In this technology, each bogie bears four wheels rotating at different angular velocities; hence in this case, the mathematical equation (2.32) does not apply.

A magnetic coupling of the two wheels (Figure 3.7) generates a damping torsional torque, the value of which is proportional to the difference of the angular velocities of the two wheels (Geuenich et al., 1985):

\[
C_\rho = C_\phi (\omega_1 - \omega_2)
\]

where

- \(C_\rho\): damping torsional torque
- \(C_\phi\): damping coefficient

The torque \(C_\rho\) is of the same nature as the one that causes the pair of lateral creep forces, but much smaller.

The bogie wheelset with controlled creeping technology was mainly developed in the United States and Germany. It allows for very high speeds on straight paths without the use of bogie–yaw dampers, thus significantly simplifying the bogie–wheelset connection.

The performance of this technology can be optimised by varying the damping coefficient \(C_\phi\) as a function of the vehicle forward speed \(V\).

In spite of its positive impact on vehicle behaviour, the development of this technology was abandoned due to its increased implementation cost.

3.2.1.6 **Bogies with wheels with mixed behaviour**

These bogies are equipped with a special mechanism that allows them to behave like conventional bogies on straight sections of the track, and to behave like bogies with independently rotating wheels in curves. This technology is applied in tramway vehicles (Pyrgidis, 2004).

3.2.2 **Wheel rolling conditions and bogies inscription behaviour in curves**

In curves the following wheel rolling and bogie–wheelsets positioning cases may be considered:

1. Rolling of all bogie wheels without flange contact, without slip and without development of creep forces (pure rolling). This case is purely theoretical and is considered to be perfect since due to the absence of forces, no wear is noticed either on the rolling stock or the track.

   In this case, the following mathematical expression applies:

   \[ X_{ij} = T_{ij} = 0 \]

   and

   \[ |y_i| < \sigma \quad (F_{ij} = 0) \]

   where

   \( X_{ij}, T_{ij} \): longitudinal and transversal creep forces exerted on the four wheels of a 2-axle bogie (\( i = 1, 2 \) front and rear wheelset, respectively and \( j = 1, 2 \) left and right wheel, respectively, in the direction of movement)

   \( y_i \): lateral displacements of the two wheelsets of a bogie (\( i = 1, 2 \) front and rear wheelset, respectively)

   \( \sigma \): flange way clearance

   \( F_{ij} \): guidance forces exerted from the four wheels of a 2-axle bogie to the rails (\( i = 1, 2 \) front and rear wheelset, respectively and \( j = 1, 2 \) left and right wheel, respectively, in the direction of movement)

2. Rolling of all bogie wheels without flange contact and without slip. The only forces applied on the wheels are the creep forces. This case is considered ideal and may be met under real service conditions where the negative impact on the rolling stock and on the track is considerably minimised (minimal material fatigue and low-level noise emission).

   In this case, the following mathematical expression applies:

   \[ \sqrt{X_{ij}^2 + T_{ij}^2} < \mu Q_0 \] (3.8)
and

\[ |y_i| < \sigma \quad (F_i = 0) \]

where

- \( \mu \): wheel–rail friction coefficient
- \( Q_0 \): wheel load.

3. Rolling featuring contact of the front bogie wheelset with the outer rail (via their outer wheel flange) and, accordingly, no contact of the rear bogie wheelset (the rear wheelset may or may not slip) (Figure 3.8).

This case is frequently encountered in small radii curves. It results in wearing out of the contact wheel and mainly of the outer rail, rolling noise and fatigue of contact materials. This case of inscription is not desirable; however, it is considered acceptable provided that the value of the exerted guidance force \( F_{11} \) is not very high and, obviously, it is lower than the limits set by derailment and track lateral shift criteria.

In this case, the following mathematical expressions pertain:

\[ F_{11} \neq 0, \ F_{21}, \ F_{22}, \ F_{12} = 0, \ y_1 = +\sigma, \ y_2 \neq \pm\sigma \]

4. Rolling of both bogie wheelsets in contact with the outer rail (via their outer wheel flanges) (both wheelsets may or may not slip) (Figure 3.9). This case is more adverse than case 3 as two of the four wheels come in contact (via the flange) with the rail and as a result the adverse impact is increased. However, this case may be acceptable provided that derailment and track lateral shift are within the set limits.

In this case, the following mathematical expressions pertain:

\[ y_1 = +\sigma \quad F_{11} \neq 0, \ F_{12} = 0 \]
\[ y_2 = +\sigma \quad F_{21} \neq 0, \ F_{22} = 0 \]

---

**Figure 3.8** Third special rolling condition. Left-wheel flange of front wheelset–outer rail contact – rolling of rear wheelset without wheel flange–rail contact.
5. Rolling with front bogie wheelset–outer rail contact (via the outer wheel flange) (Figure 3.10). This case, known as ‘crabbing’, is the most averse and should be avoided as, apart from the adverse impact on the track and rolling stock, the bogie is ‘locked’ on the track and its displacement is hindered. This case is observed when the primary suspension is particularly rigid and the radius of curvature is small.

In this case, the following mathematical expressions pertain:

\[ y_1 = +\sigma \quad F_{11} \neq 0, \quad F_{12} = 0 \]
\[ y_2 = -\sigma \quad F_{21} = 0, \quad F_{22} \neq 0 \]
Remark: The wheel slip of a wheelset is always accompanied by contact of the flange of one wheel with the inner rail face (inner or outer rail).

Conditions 3, 4 and 5 are observed during movement along the horizontal curvatures of the track alignment.

### 3.2.3 Lateral behaviour of a whole vehicle

As mentioned in Section 3.1.1, the movement of a whole vehicle (car body + bogies) is much more complex than that of a single railway wheelset.

Dynamic railway engineering, a division of the applied engineering sector, allows the development of mathematical models simulating the lateral behaviour of a railway vehicle on straight paths and in curves. With the aid of these models it is possible to study the influence of the construction characteristics of the bogies on the ‘geometric’ behaviour of the vehicle and to determine, for a given speed and for given bogie construction characteristics, the minimum radius of curvature in the horizontal alignment, which ensures acceptable rolling conditions and inscription of the wheelsets in curved sections of the track (avoidance of slipping, absence of guidance forces).

There are many models available in the market. These models are used both in the industry and in academia, and also in research institutes. Whether these models approach reality and to what extent depends on the assumptions and the hypotheses made for their development as well as their mathematical approach. These models are constantly evolving helping to improve traffic safety at all levels, and to achieve a lower vehicle construction cost and a lower cost for the maintenance of track infrastructure and rolling stock.

The simulation models that can be acquired from the market are: SIMPACK, UMLAB, Vampire Pro, Adams/Rail, NUCARS, GENSY and MEDYNA. All these models take into account the real wheel profile, the track geometric defects and rolling conditions of the wheels. In straight segments these models calculate speeds and accelerations (some models only calculate speeds) and in curves they calculate the applied forces/stresses, the contact surface area and the geometric positioning of the wheelsets on the line (some models only calculate the forces).

Apart from the above models that are available in the market and some other models that are free to use (wheel rail contact calculator) there are some models that have been developed by individual researchers or research groups for their own use, and cannot be found in the market.

A group of such models are described in the literature references (Joly, 1983, 1988; Joly and Pyrgidis, 1990, 1996; Pyrgidis, 1990, 2004; Pyrgidis and Joly, 1993). With these models it is possible to study the following features for five different technologies of bogies (conventional bogies, bogies with self-steering wheelsets, bogies with independently rotating wheels, bogies with creep-controlled wheelsets and bogies with mixed behaviour):

- In the case of tangent track, the lateral vehicle stability and the influence of the main construction features of the bogies on the ‘critical’ vehicle speed.
- In the case of curved segments of the horizontal alignment, the semi-static lateral vehicle behaviour and the effect of the main features of the bogies on the ‘geometric’ vehicle behaviour (displacements and yaw angles of wheelsets) and the wheel rolling conditions (calculation of the wheel–rail contact forces, verification of appearance of slipping).
For these models the vehicle moves at a constant speed \( V \) and its movement occurs on a railway track without the longitudinal gradient and without geometric track defects. To study the geometry of the contact the wheel as well as the contact surface of the rail are both simulated by a circle profile (i.e. circle to circle contact) (Joly, 1983).

In the case of a vehicle equipped with conventional bogies the mechanical system consists of the following seven solid bodies that are considered rigid and undeformable: 1 car body, 2 bogies and 4 wheelsets.

At the level of the primary suspension the following were considered per bogie: 4 springs, 1 lateral damper, 1 longitudinal damper and 2 vertical dampers.

At the level of the secondary suspension the following were considered per bogie: 2 springs, 1 lateral damper, 1 longitudinal damper and 2 vertical dampers.

This mechanical system illustrates in fact the French passenger vehicles of the ‘Corail’ type.

Additionally, the following key assumptions/assumptions are made:

**At straight segments:**

- The creep forces are expressed on the basis of the linear theory of Kalker and the creep coefficients \( C_{ij} \) are considered to be reduced by 33%.
- The rails are not taken into consideration, and as a result the guidance of the wheelsets during the movement is ensured by the combined action of the equivalent conicity of the wheels and the creep forces that are exerted on the wheel–rail contact level. At the same time their lateral stiffness, which is bigger than the stiffness of the vehicle’s elastic links, is ignored.

**In curves:**

- The study of the lateral vehicle behaviour refers to the circular segment of the curve.
- The vertical loads are distributed equally on both wheels of the axles.
- For the calculation of the creep forces the nonlinear theory of Johnson–Vermeulen was adopted (Vermeulen and Johnson, 1964).

### 3.2.3.1 Vehicles with conventional bogies

Figure 3.11 shows the variation in the vehicle critical speed \( V_{cr} \) is given as a function of the longitudinal bogie–wheelsets stiffness \( K_x \) for both values of bogie–yaw dampers longitudinal stiffness \( (K_o = 3 \times 10^6 \) N/m and \( K_o = 0) \).

There is a zone of values of \( K_x \) between \( 7 \times 10^5 \) and \( 1.5 \times 10^7 \) N/m, where the critical speed reaches its highest values \( (V_{cr} = 465–495 \) km/h) (Pyrgidis, 1990). This area of \( K_x \) values is seen as being the greatest ‘safety margin’ as regards the stability of vehicles on straight paths for the constructional characteristics of the vehicle illustrated in Figure 3.11.

For values \( K_x > 3.5 \times 10^7 \) N/m approximately, the critical speed remains roughly equal to \( V_{cr} = 450 \) km/h. The absence of bogie–yaw dampers reduces the critical speed by about 20%.

The value of \( K_x = 8 \times 10^6 \) N/m is considered as the optimum value for longitudinal stiffness. On the one hand, this specific value is within the \( 7 \times 10^6–1.5 \times 10^7 \) limits and, on the other hand, it is relatively small which facilitates negotiation of curves.

Table 3.2 shows the performances of vehicles with conventional bogies on straight paths and in curves.
Indicatively, it is noted that

- The high value of the bogie–wheelset longitudinal stiffness \(3.5 \times 10^7 \text{ N/m} \geq K_x \geq 7 \times 10^6 \text{ N/m}\)
- The small value of the wheel equivalent conicity \(\gamma_e \leq 0.10\)
- The fixing of devices that limit the horizontal rotation of bogies and car body (bogie–yaw dampers)

allow, in theory, a classical railway vehicle to run on a straight path of good ride quality at speeds \(V > 600 \text{ km/h}\). However, with such properties, in case of radii \(R_c < 4,800 \text{ m}\) (for \(\gamma_e = 0.10\) and \(K_x = 8 \times 10^6 \text{ N/m}\)), we observe (Pyrgidis, 1990)

<table>
<thead>
<tr>
<th>(K_x) (N/m)</th>
<th>(2a = 3.0 \text{ m})</th>
<th>(2e_o = 1.50 \text{ m})</th>
<th>(\sigma = \pm 10 \text{ mm})</th>
<th>(\gamma_e = 0.02 \text{ g})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_y = 10^7 \text{ N/m})</td>
<td>(2r_c = 0.90 \text{ m})</td>
<td>(Q_o = 7.03 \text{ t})</td>
<td>(F_{11} = 35.1 \text{ kN})</td>
<td>(658 \text{ km/h})</td>
</tr>
<tr>
<td>(K_o = 3 \times 10^6 \text{ N/m})</td>
<td>(\gamma_e = 0.05)</td>
<td>(R_c = 6,300 \text{ m}) (contact)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(K_o = 3 \times 10^6 \text{ N/m})</td>
<td>(\gamma_e = 0.10)</td>
<td>(R_c = 4,800 \text{ m})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(K_o = 3 \times 10^6 \text{ N/m})</td>
<td>(\gamma_e = 0.20)</td>
<td>(R_c = 4,300 \text{ m})</td>
<td>(F_{11} = 20.7 \text{ kN})</td>
<td></td>
</tr>
<tr>
<td>(K_o = 7 \times 10^6 \text{ N/m})</td>
<td>(\gamma_e = 0.20)</td>
<td>(R_c = 4,300 \text{ m})</td>
<td>(F_{11} = 0 \text{ (slip)})</td>
<td></td>
</tr>
</tbody>
</table>
• Wheel slip
• Contact of the wheel flange with the outer rail

resulting in an increased wheel wear and in the development of guidance forces.

3.2.3.2 Vehicles with bogies with self-steering wheelsets

If we consider the mechanical system in Figure 3.5 where both wheelsets of a bogie are connected to the bogie using springs of stiffness $K_x$ and $K_y$ and connected to each other using springs of stiffness $K_b$ (angular) $K_{aK}$ (lateral), then the following relations apply (Pyrgidis, 1990):

$$K_{st} = K_s + \frac{d^2 \cdot K_x \cdot K_y}{d^2 \cdot K_x + a^2 \cdot K_y}$$  \hspace{1cm} (3.9)

$$K_{bt} = K_b + K_x \cdot d^2$$  \hspace{1cm} (3.10)

where
- $K_{st}$: overall lateral stiffness of the mechanical system
- $K_{bt}$: overall longitudinal stiffness of the mechanical system
- $2a$: bogie wheelbase

For $d = 1.0$ m and $2a = 3.0$ m relations (3.9) and (3.10) become

$$K_{st} = K_s + \frac{K_x K_y}{K_x + 2.25K_y}$$  \hspace{1cm} (3.11)

$$K_{bt} = K_b + K_x$$  \hspace{1cm} (3.12)

and for $K_s = K_b = 0$ relations (3.11) and (3.12) become

$$K_{st} = \frac{K_x K_y}{K_x + 2.25K_y}$$  \hspace{1cm} (3.13)

$$K_{bt} = K_x$$  \hspace{1cm} (3.14)

From the above relations, the following may be concluded:

• Stiffnesses $K_s$ and $K_x$ increase the total stiffness (lateral and longitudinal) of the system
• The angular stiffness $K_b$ plays the same role (for $d = 1.0$ m) as the longitudinal stiffness $K_x$ of a conventional bogie
• The total lateral stiffness of the primary suspension of a conventional bogie depends as much on $K_x$ as on $K_y$

Table 3.3 shows the performance of bogies with self-steering wheelsets on straight paths and in curves. Compared with conventional bogies, a smaller value of the total longitudinal
stiffness \((K_{bt} = 2 \times 10^6 \text{ N/m} < 8 \times 10^6 \text{ N/m})\) and a smaller value of the total lateral stiffness \((K_{st} = 1.3 \times 10^6 \text{ N/m} < 2.62 \times 10^6 \text{ N/m})\) are observed.

Indicatively, it is noted that bogies with self-steering wheelsets make it possible to combine very good negotiation of small and very small radius curves with a fair value of speed on straight paths.

### 3.2.3.3 Vehicles with independently rotating wheels

Table 3.4 presents the performances of bogies with independently rotating wheels on straight paths and in curves (Pyrgidis, 1990; Pyrgidis and Joly, 1993; Joly and Pyrgidis, 1996).

Indicatively, it is noted that

- This technology allows theoretically, without the fixing of bogie–yaw dampers, the development of very high critical speeds on straight paths while eliminating the hunting of wheelsets (absence of longitudinal creep forces). However, the wheelset is very sensitive to lateral displacements.
- A great value of equivalent conicity facilitates both the negotiation of bogies in curves and the motion on straight paths as it increases the value of the gravitational force that tends to centre the wheelset on the track.
- For wheelbase and wheel diameter values, which are the same as those of high-speed conventional bogies, a wheel slip is observed in comparatively much smaller curvature radii, while the forces exerted in very small radius curves are much smaller.

#### Table 3.3 Performances of vehicles with bogies with self-steering wheelset – running on straight path and in curves

<table>
<thead>
<tr>
<th>Adjustable characteristics</th>
<th>Straight path ((V_{cr}))</th>
<th>Curves (occurrence of slip)</th>
<th>Curves ((R_c = 500 \text{ m}))</th>
<th>Curves ((R_c = 200 \text{ m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_b = 3 \times 10^6 \text{ N/m})</td>
<td>323.5 km/h</td>
<td>1,200 m</td>
<td>F11 = 15.4 kN</td>
<td>F11 = 59.6 kN</td>
</tr>
<tr>
<td>(\gamma_e = 0.10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(K_b = 0 \text{ N/m})</td>
<td>198 km/h</td>
<td>250 m</td>
<td>F11 = 14.8 kN</td>
<td>F11 = 0</td>
</tr>
<tr>
<td>(\gamma_e = 0.20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


#### Table 3.4 Performances of vehicles with bogies with independently rotating wheels – motion on straight path and in curves

<table>
<thead>
<tr>
<th>(K_b = 10^6 \text{ N/m})</th>
<th>(2e_o = 1.50 \text{ m})</th>
<th>(\sigma = \pm 10 \text{ mm})</th>
<th>(\gamma_e = 0.20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_b = 10^6 \text{ N/m})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2e_o = 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Q_o = 7.03 \text{ t})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma_e = 0.02 \text{ g})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Straight path \((V_{cr})\) | Curves (occurrence of slip) | Curves \((R_c = 500 \text{ m})\) | Curves \((R_c = 100 \text{ m})\)

| \(V_{cr} = \infty\) | 1,400 m | F11 = 1.7 kN | F11 = 15.7 kN |
| | | | |

Vulnerability in lateral displacements

3.2.3.4 Comparative assessment

In Table 3.5, a comparison of the performances of the three bogie technologies under examination on straight paths and in curves is attempted (where $P_{4w}$ is the power that is consumed at the level of the four wheels of the bogie when rolling occurs without contact between the wheel flange and the inner side of the rail).

<table>
<thead>
<tr>
<th>Technology characteristics of bogies</th>
<th>Straight path</th>
<th>Curves</th>
<th>Occurrence of slip</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conventional bogies</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_x = 8 \times 10^4$ N/m</td>
<td>482 km/h</td>
<td>$R_c = 4,800$ m</td>
<td>$F_{11} = 20.7$ kN</td>
</tr>
<tr>
<td>$K_y = 10^7$ N/m</td>
<td></td>
<td>$K_a = 0$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_e = 0.10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_o = 3 \times 10^6$ N/m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2a = 3.0$ m $2r_o = 0.90$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bogies with self-steering wheelsets</strong></td>
<td>226 km/h</td>
<td>$R_c = 250$ m</td>
<td>$F_{11} = 0$</td>
</tr>
<tr>
<td>$K_x = K_y = 10^4$ N/m</td>
<td></td>
<td>$P_{4w} = 1.11$ kW</td>
<td></td>
</tr>
<tr>
<td>$K_o = 3 \times 10^6$ N/m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2a = 3.0$ m $2r_o = 0.90$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bogies with independently rotating wheels</strong></td>
<td>$V_{cr} = \infty$</td>
<td>$R_c = 1,400$ m</td>
<td>$F_{11} = 0$</td>
</tr>
<tr>
<td>$K_x = K_y = 10^6$ N/m</td>
<td></td>
<td>$P_{4w} = 4.3$ kW</td>
<td>$P_{4w} = 12.2$ kW</td>
</tr>
<tr>
<td>$\gamma_e = 0.30$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_o = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2a = 3.0$ m $2r_o = 0.90$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vulnerability in lateral displacements


3.2.4 Selection of bogie design characteristics based on operational aspects of networks

The selection of bogie design characteristics is directly related to the operational characteristics of the network in which the trains will operate. In this section, technical characteristics for vehicle bogies are suggested considering the operational characteristics of the network. Data are obtained from the application of the mathematical models developed by Joly and Pyrgidis (Joly, 1983, 1988; Joly and Pyrgidis, 1990, 1996; Pyrgidis, 1990, 2004; Pyrgidis and Joly, 1993).

3.2.4.1 High-speed networks

These are characterised by

Track design speed: $V_d \geq 200$ km/h

Alignment layout: Small percentage of curved sections out of total track length. Large and very large curve radii (for $V_d = 200$ km/h and $R_{cmin} = 2,000$ m)
The following are proposed for the rolling stock:

**Bogies:** Conventional  
Equivalent conicity: Small (e.g., $\gamma_e = 0.05–0.10$)  
Stiffness of the primary suspension: High (e.g., $K_x = 8 \times 10^6$ N/m and $K_y = 10^7$ N/m)  
Bogie wheelbase: High (e.g., $2a = 3.0$ m)  
Wheel diameter: Big (e.g., $2r_o = 0.90$ m)  
Bogie and wheelset masses: Small

### 3.2.4.2 Conventional speed networks

They are characterised by

- **Track design speed:** $140 \text{ km/h} \leq V_d < 200 \text{ km/h}$  
- **Alignment layout:** Mainly medium curve radii ($R_c = 500–1,500$ m)

Conventional bogies are proposed. The selection of values of the bogie design characteristics depends on the track design speed and the track geometry alignment.

*Remark:* If it is desired to improve the performance on an existing track (assuming the track superstructure is in very good state) tilting trains may be used.

### 3.2.4.3 Mountainous networks

Characterised by

- **Track design speed:** $V_d < 140 \text{ km/h}$  
- **Alignment layout:** Large percentage of curved sections out of the total track length mainly medium and small horizontal alignment radii ($R_c = 250–750$ m)

The following proposals are made for the rolling stock:

**Bogies:** With self-steering wheelsets (or conventional wheelsets)  
Equivalent conicity: Medium (e.g., $\gamma_e = 0.20$)  
Total longitudinal stiffness of primary suspension: Small (e.g., $K_{bt} = 2 \times 10^6$ N/m)  
Total lateral stiffness of primary suspension: Small (e.g., $K_{st} = 1.3 \times 10^6$ N/m)

### 3.2.4.4 Metro networks

These are characterised by

- **Track design speed:** $V_d = 90–100 \text{ km/h}$  
- **Alignment layout:** Very large percentage of curved sections out of the total track length. Mainly small curve radii ($R_c = 150–300$ m)
The following options are proposed for the rolling stock:

Bogies: Conventional
Equivalent conicity: High (e.g., $\gamma = 0.30$)
Bogie wheelbase: Small (e.g., $2a = 2.00–2.40$ m)
Wheel diameter: Small (e.g., $2r_o = 0.70–0.75$ m)
Bogie and wheel masses: Small
Stiffness of primary suspension: Small (e.g., $K_x = 4.106$ N/m and $K_y = 10^6$ N/m)

3.2.4.5 Tramway networks

These are characterised by

Track design speed: $V_d = 80–90$ km/h
Alignment layout: Very large percentage of curved sections out of the total track length.
Curve radii mainly in the range of $R_c = 25–50$ m

The following options are proposed for the rolling stock:

Bogies: With independently rotating wheels
Equivalent conicity: Very high
Bogie wheelbase: Small (e.g., $2a = 1.80$ m)
Wheel diameter: Small (e.g., $2r_o = 0.65$ m)
Bogie and wheelset masses: Small
Longitudinal stiffness in primary suspension: No effect
Lateral stiffness in primary suspension: Small (e.g., $K_y = 10^5–10^6$ N/m)

Apart from the technology of bogies with independently rotating wheels, a mixed system

The derailment of a railway vehicle may occur as a result of

- Lateral displacement (shift) of the track
- Overturning/tilting of the vehicle
- Wheelclimb

The causes of derailment can be internal (high exerted forces, excessive speed, poor condition and design of rolling stock, poor quality and track layout) or external (incorrect adjustment of switches) (Figure 3.14).
Figure 3.12 Derailment of railway vehicles. (Photo: A. Klonos.)

Figure 3.13 Derailment. (Photo: A. Klonos.)

Figure 3.14 Causes of derailment.
3.3.2 Derailment through displacement of track

In this case, the track panel (rails + sleepers) of a track segment is displaced due to the effect of significant lateral forces, resulting in the derailment of one or more of the train’s vehicles. Derailment through displacement of the track occurs when

\[ \Sigma Y > H_R \]  

(3.15)

where

- \( \Sigma Y \): total lateral force, which is transferred from the vehicle to the rail
- \( H_R \): lateral track resistance

This type of derailment is solely due to internal causes and is the most common type of derailment.

3.3.3 Derailment as a result of vehicle overturning

During movement or the immobilisation of a railway vehicle on curved sections of the horizontal alignment, the vehicle may overturn under certain conditions.

Overtopping may occur toward the outside or the inside of the curve.

In the first case (Figure 3.15a), the following reasons may pertain (Esveld, 2001):

- Significant deficiency in relation to the passage speed \( V_p \) and the radius of curvature, which translates to an increased value of the lateral residual centrifugal force \( F_{nc} \)
- Crosswind force \( H_w \) directed toward the outside of the curve
- Unequal load distribution on two wheels with lower loading on the inside wheel \( (Q_1 > Q_2) \)

All the above reasons result in the development of moments, which tend to overturn the vehicle toward the outer rail.

In the second case (Figure 3.15b) the following reasons may pertain, respectively,

- Crosswind force \( H_w \) directed toward the inside of the curve
- Immobilisation of vehicles \( (V_p = 0) \) on a curved track section with a high cant \( U \)
- Small axle load
- Displacement of load toward the inside wheels

![Figure 3.15 Vehicle overturning mechanism in curves (Adapted from Esveld, C. 2001, Modern Railway Track, 2nd edition, MRT-Productions, West Germany.): (a) Toward the outside of the curve and (b) Toward the inside of the curve.](image-url)
Under these circumstances moments which tend to overturn the vehicle toward the inner rail develop resulting ultimately in derailment.

### 3.3.4 Derailment with wheel climb

#### 3.3.4.1 Description of the phenomenon

Assuming the case of rolling in Figures 3.13 and 3.16 (Alias, 1977), for the wheel, which is close to a derailment (wheel 1) and the vertical load $Q_1$ and the force $Y_1$ (total force exerted via the wheel flange on the rail) are applied on the point of contact $I_1$. The reaction $R_1$ of the rail may be analysed in two components:

- A force $N_1$ that is perpendicular to the level of wheel–rail contact $xOy$
- The lateral creep force $T_1$ which acts on the level of wheel–rail contact, and is directed upwards, having a value equal to

$$T_1 = C_{22} \alpha_{at} = N_1 \tan \beta_1$$

(3.16)

where

- $\alpha_{at}$: angle of attack (wheelset yaw angle when flange contact occurs) (Figure 3.16)
- $C_{22}$: lateral creep coefficient

In the case where wheel 1 slips, the force $T_1$ is equal to Coulomb’s friction force.

In practice, derailment through wheel climb occurs when the projection of the combination of all forces applied on the axis $yy$ (derailment force axis) are directed upwards and their application time is long enough for the wheel to climb over the rail.

#### 3.3.4.2 Derailment criteria

For testing against derailment various criteria are being used such as the Nadal criterion, the Chartet criterion, etc. (Alias, 1977). Some of these criteria take into account the yaw angle of the wheelset that is under a derailment while others do not. Figure 3.17 in combination with the mathematical expression (3.17) illustrate the Nadal criterion:

$$Y_1 < Q_1 \frac{\varepsilon \phi \beta - \mu}{1 + \mu \varepsilon \phi \beta}$$

(3.17)

![Figure 3.16 Derailment through wheel climb. (Adapted from Alias, J. 1977, La Voie Ferrée, Eyrolles, Paris.)](image)
where

\[ Y_1: \text{the total transversal force exerted on the rail via the wheel flange of the derailing wheel (wheel 1)} \]
\[ \mu: \text{wheel–rail friction coefficient} \]
\[ \beta: \text{wheel–rail contact flange angle} \]
\[ Q_1: \text{static load of wheel 1} \]

For \( \beta = 70^\circ, \mu = 0.25 \) (dry rail), the mathematical equation (3.17) results \( Y_1/Q_1 = 1.5 \) while as for \( \mu = 0.12 \) (wet rail) it results \( Y_1/Q_1 = 2.0 \).

### 3.3.4.3 Factors affecting derailment

In most occasions, the derailment of a vehicle takes place following a lateral displacement of the track.

Derailment by rail climbing can occur only when there is a significant unloading of the derailed wheel with simultaneous loading of the non-derailed wheel. This phenomenon can be observed in the case of movement at low speeds in curves with a small radius of curvature and high values of cant and twist.

Derailment by rail climbing also occurs through external causes, that is, poor operation and adjustment of switches, etc.

It should be noted that most derailments occur in areas of switches and crossings.

The risk of derailment due to rail climb increases when there is

- An increase of the value of the \( Y_1 \) force
- An increase in the application time of \( Y_1 \) force
- An increase in the value of the wheel–rail friction coefficient \( \mu \)
- A decrease in the value of the wheel–rail contact angle \( \beta \)
- A decrease in the value of the vertical load on the derailed wheel with a simultaneous increase of the vertical load of the non-derailed wheel

Rail climb by the wheel does not occur instantaneously. A certain amount of time is required, and thus the derailing wheel covers some distance on the track, usually some metres. This distance is called ‘flange-climbing distance’ and is defined as the distance covered from the moment when the total value of the guidance force is applied until the moment on which the wheel–rail contact flange angle reaches 26.6° (Dos Sandos et al., 2010). The shorter this distance is, the faster the derailment will become apparent.
The influence of various wheel parameters on the above distance was examined with the aid of simulation modelling. The results have shown that the flange-climbing distance increases (and thus the appearance of derailment slows down) when (Dos Sandos et al., 2010)

- The angle of attack is smaller.
- The height of the wheel flange is increased. When the wheel is worn out the distance required for rail climbing increases. The positive effect of a high flange is significantly limited when the yaw angle is large.
- The value of \( q_r \) increases (see Figure 1.8).

**REFERENCES**


Schneider Jeumont Rail. no date, Bogie CL93 à moteurs asynchrones, *Catalogue pieces de rechange*, Le Creusot, France.