3 SAR Data and Signal

3.1 INTRODUCTION

The signal property of synthetic aperture radar (SAR) is one of the core issues to grasp in order to process the raw data into an image. The mapping between the object domain and image domain is persistently bridged and thus controlled by the signal domain. This chapter deals with this aspect in depth. Although in SA, various waveforms, like continuous wave (CW) and phase-coded [1–3], may be used, we will focus our discussion on linear frequency modulation (LFM) (chirp) and frequency-modulated continuous wave (FMCW) for their common use in modern SAR systems.

3.2 CHIRP SIGNAL

3.2.1 ECHO SIGNAL IN TWO DIMENSIONS

In Equation 1.54, we indicated that the pulse repetition frequency \( f_p \) must follow the Nyquist criteria to avoid the range ambiguity within the maximum probing range \( R_{\text{max}} \). Also, there is a minimum range \( R_{\text{min}} \) corresponding to the near range. The antenna elevation beamwidth \( \beta_e \) subtends from the near to the far range (swath) (Figure 3.1). To avoid the air bubbles, SAR is not pointing its boresight at the nadir. The delay time of returns from the near range and far range, \( \tau_{\text{near}}, \tau_{\text{far}} \), must be confined within a period. Mathematically, the following two relations should be obeyed [4,5]:

\[
T_p < \frac{2R_{\text{min}}}{c} \tag{3.1}
\]

\[
T = \frac{1}{f_p} > \frac{2R_{\text{max}}}{c} + T_p \tag{3.2}
\]

Remember that for spaceborne SAR, the maximum range is limited by the earth curvature [6,7], depending on the satellite height and elevation angle. If the transmitted signal is of the form given by Equation 1.54, the received signal as a function of fast time is the output of the matched filter:

\[
s_x(\tau) = A_0 \otimes s_i(\tau) = \int_{t-T_p/2}^{t+T_p/2} A_0(t)s_i(\tau - t)\,dt
\]

\[
= A_0p_r\left(\tau - \frac{2R}{c}\right)\cos\left(2\pi\frac{f_c}{c}\left(\tau - \frac{2R}{c}\right) + \pi a_r\left(\tau - \frac{2R}{c}\right)^2\right) \tag{3.3}
\]
where $A_0$ is the scatter amplitude [4] and, without loss of generality, is assumed constant. Equation 3.3 is indeed also dependent on the slow time because the slant range $R$ is varying with the sensor position within the target exposure time. Referring to Figure 1.6, the slow time-dependent slant range can be expanded about $R(\eta_c)$, with the beam center crossing time, $\eta_c$:

$$R(\eta) = R(\eta_c) + \frac{u^2 \eta_c}{R(\eta_c)} (\eta - \eta_c) + \frac{1}{2} \frac{u^2 \cos^2 \theta_{ic}}{R(\eta_c)} (\eta - \eta_c)^2 + \ldots$$  (3.4)

where $\theta_c$ is look angle to the scene center and

$$\eta_c = \frac{R_0 \tan \theta_c}{u} = \frac{R(\eta_c) \sin \theta_c}{u}.$$  (3.5)

Due to the time variation of the range, a point target response will continuously appear along the path according to Equation 3.4 within the synthetic aperture length. A coherent sum of these responses during the course of the target exposure time will be out of focus if no range-induced phase variation is corrected. Figure 3.2

![FIGURE 3.1 Observation geometry defined by pulse and antenna beamwidth.](image)

![FIGURE 3.2 Mapping from the object domain to the data domain.](image)
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schematically illustrates data collection and mapping from the target (object) to the SAR data domain.

In practice, we should take the antenna pattern into the signal reception. The received signal may be written as

\[ s_r(\tau, \eta) = A_0 p_r(\tau - \frac{2R}{c}, T_p) g_a(\eta) \cos \left( 2\pi f_c \left( \tau - \frac{2R(\eta)}{c} \right) + \pi a_r(\tau - \frac{2R(\eta)}{c}) \right) \]  

(3.6)

A typical two-way antenna pattern in the azimuthal direction may be of the form [4]

\[ g_a(\eta) \equiv \text{sinc}^2 \left( \frac{0.886\theta_{\text{diff}}(\eta)}{\beta_{az}} \right) \]  

(3.7)

where \( \theta_{\text{diff}} = \theta_{sq} - \theta_{sq,c} \), with \( \theta_{sq,c} = \theta_{sq}(\eta_c) \) being the squint angle at the scene center. For small squint approximation, Equation 3.7 may be written as

\[ g_a(\eta) = \text{sinc}^2 \left( \frac{0.886[\theta_{sq}(\eta) - \theta_{sq,c}]}{\beta_{az}} \right) \cong \text{sinc}^2 \left( \frac{0.886}{\beta_{az}} \tan^{-1} \left[ \frac{\mu(\eta - \eta_c)}{R_0} \right] \right) \]  

(3.8)

Figure 3.3 plots a right side-looking SAR (left) to show antenna pattern illuminating on a point target with gain variations (middle) along the azimuth direction; on the right is the intensity of the point target within the cell. The antenna gain pattern effect along the azimuthal direction is clearly observed.

**FIGURE 3.3** A right side-looking SAR (left) to show antenna beam pattern (middle) on a point target response (right).
3.2.2 Demodulated Echo Signal

The demodulation is to remove the carrier frequency, which bears no target information. Following [4], a typical demodulation scheme is illustrated in Figure 3.4, where the echo signal in ① is downconverted by mixing with a reference signal, resulting in ② and ③. By filtering out the upper frequency component and going through an analog-to-digital conversion (ADC) if necessary, the echo signal for postprocessing ⑥ is obtained.

\[
\begin{align*}
①: & \quad \cos \left( 2\pi f_c \tau - \frac{4\pi f_c R(\eta)}{c} + \pi \alpha_r \left( \tau - \frac{2R(\eta)}{c} \right)^2 \right) = \cos[2\pi f_c \tau + \phi(\tau)] \\
②: & \quad \frac{1}{2} \cos[\phi(\tau)] + \frac{1}{2} \cos[4\pi f_c \tau + \phi(\tau)] \\
③: & \quad \frac{1}{2} \sin[\phi(\tau)] + \frac{1}{2} \sin[4\pi f_c \pi \tau + \phi(\tau)] \\
④: & \quad \frac{1}{2} \cos[\phi(\tau)] \\
⑤: & \quad \frac{1}{2} \sin[\phi(\tau)] \\
⑥: & \quad \frac{1}{2} \exp\{j\phi(\tau)\} = \frac{1}{2} \exp\left\{ -j \frac{4\pi f_c R(\eta)}{c} + j\pi \alpha_r \left( t - \frac{2R(\eta)}{c} \right)^2 \right\}
\end{align*}
\]

The final demodulated two-dimensional echo signal is of the form

\[
s_o(\tau, \eta) = A_0 p_r \left( \tau - \frac{2R}{c}, T_r \right) g_a(\eta) \exp \left\{ -j \frac{4\pi f_c R(\eta)}{c} + j\pi \alpha_r \left( t - \frac{2R(\eta)}{c} \right)^2 \right\}
\]

(3.9)

Figure 3.5 plots a demodulated echo signal from a point target, showing both amplitude and phase. Again, the antenna gain pattern effect can be seen on the amplitude response.

FIGURE 3.4 Demodulation of the echo signal.
When the imaging taken from the object domain to the data domain is within the synthetic aperture, as illustrated in Figure 3.6, range cell migration (RCM) occurs [4,5,8–10]. This is the energy from a point target response, supposed to be confined at $R_0$, spreading along the azimuth direction when SAR is traveling. This is easily understood from the slow time-varying range $R(\eta)$, as already illustrated in Figure 3.2. The phase variations associated with slant range, which changes with SAR moving, consist of constant phase, linear phase, quadratic phase, and higher-order terms. The RCM is contributed mainly from the quadratic phase term. This quadratic phase term also determines the depth of focus. For a given synthetic aperture size, the maximum quadratic phase was given in Equations 1.65 and 1.66. Detailed discussion on the phase error and its impact can be found in [4,8,10]. Due to the energy spreading, the effects of the quadratic phase, if not properly compensated, cause image defocusing, lower gain, and higher side lobes in the point target response; that is, in image quality, spatial resolution is degraded with higher fuzziness. Notice that the phase variations, up to the second order, due to the slow-time-dependent range are low-frequency components compared to the phase error in pulse-to-pulse, modulation error and nonlinearity of frequency modulation, which are high frequency since they are induced in the fast-time domain.

The total range migration during the course of synthetic aperture exposure time $T_a$ can be estimated more explicitly from the geometry relation for the cases of low-squint and high-squint angles, as shown in Figure 3.7.

In the low-squint case, the total range of migrations is

$$\Delta R_{\text{total}} = R \left( \eta_c + \frac{T_a}{2} \right) - R(\eta_c) \equiv \frac{u^2}{2R_0} \left[ \left( \eta_c + \frac{T_a}{2} \right)^2 - \eta_c^2 \right]$$ (3.10)
FIGURE 3.6 Illustration of the object domain and data domain showing the range of cell migration.

while for high squint we have

\[ \Delta R_{\text{total}} = \frac{u^2}{2R_0} T_a \eta_c \]  

(3.11)

where the exposure time was given previously as

\[ T_a = 0.886 \frac{\lambda R(\eta_c)}{L_a u \cos \theta_{\eta_c}} \]  

(3.12)

FIGURE 3.7 Total range migrations during the course of synthetic aperture.
The Doppler frequency is obviously also a function of the slow time. Measured at the scene center, the Doppler centroid takes the expression

$$f_{\eta_c} = -\frac{2}{\lambda} \left. \frac{\partial R(\eta)}{\partial \eta} \right|_{\eta=\eta_c} = -\frac{2 u \eta_c}{\lambda R(\eta_c)} = \frac{2 u \sin \theta_{sq,c}}{\lambda} \quad (3.13)$$

and the Doppler rate is

$$a_a = \frac{2}{\lambda} \left. \frac{\partial^2 R(\eta)}{\partial \eta^2} \right|_{\eta=\eta_c} = \frac{2 u^2 \cos^2 \theta_{sq,c}}{\lambda R(\eta_c)} \quad (3.14)$$

The Doppler rate as defined is sometimes also called the azimuth FM rate [4] for a stationary target. Note that the Doppler change is only caused by SAR moving along the azimuthal direction. According to Equation 1.61, the total Doppler bandwidth is

$$B_{df} = \left| a_a \right| T_a = 0.886 \frac{2 u \cos \theta_{sq,c}}{\ell_{ra}} \quad (3.15)$$

With the total Doppler bandwidth in mind, it is noted that the pulse repetition frequency (PRF) must be selected to meet the Nyquist criteria [11–13], namely,

$$f_p > B_{df} = 0.886 \frac{2 u \cos \theta_{sq,c}}{f_{ra}} \quad (3.16)$$

Together with Equations 3.1 and 3.2, Equation 3.16 poses constraints on the selection. Under this condition, the selection of PRF must usually be compromised to meet certain requirements [1,2,14,15]. For example, in order to avoid interference from repeated pulse eclipsing, PRF must be such that

$$\frac{(n-1)}{\left(\frac{2}{c} R_{near} - T_p\right)} \text{< PRF < } \frac{n}{\left(\frac{2}{c} R_{far} + T_p\right)} \quad (3.17)$$

where $n$ is the $n$th pulse, while to avoid the overlapping (indifferentiable) nadir returns, PRF must be chosen by

$$\frac{(n-1)}{\left(\frac{2}{c} R_{near} - T_p - \frac{2}{c} h\right)} \text{< PRF < } \frac{n}{\left(\frac{2}{c} R_{far} + T_p - \frac{2}{c} h\right)} \quad (3.18)$$

where $h$ is the sensor height. Equations 3.17 and 3.18 together implicitly state that there is an available zone of PRF to be selected for a given antenna size, imaging swath, incident angle, and sensor height. Also, bear in mind that the PRF determines the azimuth ambiguity-to-signal ratio for a give Doppler bandwidth.
Other important parameters associated with the two-dimensional raw data domain, such as the range bandwidth, azimuth resolution, and range resolution, can be reexpressed accordingly by taking the antenna gain pattern and squint angle effects into account. Inclined readers are referred to [4,5] for excellent treatment.

### 3.3 PROPERTIES OF DOPPLER FREQUENCY

#### 3.3.1 SQUINT EFFECTS

For a side-looking SAR system, there always exists a squint angle that is the angle between the antenna boresight and flight path minus 90°. The squint angle induces residual Doppler and causes the shift of the Doppler centroid, as shown in Figure 3.8, where for numerical illustration, we set $f_p = 1000$ kHz, $T = 4 \times 10^{-6}$ s, and $\eta_c = 10^{-6}$ s = $T/4$. Accordingly, the shift of the Doppler centroid is $f_{dc} = f_p/4$ Hz.

#### 3.3.2 FM SIGNAL ALIASING

If the sampling frequency is lower than or equal to the PRF, signal aliasing results [11,12]. This aliasing will cause a “ghost image” after azimuth compression. Beyond $\Delta \eta_{PRF}$, there exist FM signal aliased regions.

$$\Delta \eta_{PRF} = \frac{\eta_c}{a_p}$$

\[(3.19)\]

**FIGURE 3.8** Squint effects on the Doppler shift.
For better inspection and to reveal the aliasing problem, we duplicate Figure 5.4 in [4] in Figure 3.9, showing the signal spectrum (real part) of a point target, a two-way antenna beam pattern as a function of $\theta_{\text{diff}}$, and the aliases associated with the main compressed target. The decreasing power of the aliased targets (ghosts) is obvious for the weaker gain of the antenna side lobes. The spacing between the true target and those ghosts is about equal to the pulse period (pulse repetition interval [PRI]).

Figure 3.10 is an example of aliasing where target features along the coast repeatedly appear but with decreasing intensity along the azimuthal direction (indicated by arrows). This is exactly due to azimuth ambiguity.

When there are multiple targets randomly presented in the scene, the Doppler centroid is more uncertain in terms of its width and position. Figure 3.11 shows the resulting Doppler spreading effect, with one target at $t = 0$ s, five targets at $t = 1 \times 10^{-6}$ s, and two targets at $t = 1.5 \times 10^{-6}$ s, including a small squint effect. The presence of frequency spreading is caused by the mixing of the individual center frequencies corresponding to each target within a Doppler bandwidth. This poses a challenge to focusing a scene with a global Doppler centroid, as illustrated in Figure 3.12. Treatment of Doppler centroid estimation is discussed in Chapter 5.

![Figure 3.9 FM signal aliasing.](image-url)
3.3.3 Platform Velocity

The platform velocity $u$ plays an essential role in SAR system operation. From the radar equation, the slant range as a function of slow time is dependent on the radar velocity, Equation 3.4. As discussed earlier, the Doppler centroid and Doppler rate are both proportional to the squared velocity. Finally, the range cell migration that

![Image of an SAR image with arrows indicating azimuth ambiguity phenomena caused by signal aliasing.](FIGURE 3.10 Azimuth ambiguity phenomena caused by signal aliasing (indicated by arrows).)

![Graphs of real part and spectrum with time and frequency axes.](FIGURE 3.11 Effect of Doppler spreading.)
needs be corrected is inversely proportional to the squared velocity. Knowing the precise information of $u$ is extremely critical in image focusing. However, it must differentiate the platform velocity and its ground velocity when the antenna beam sweeps across the target within its swath. As explained in [8], for the airborne system, because of the low flying altitude, the platform velocity is equal to the speed of the antenna beam sweeping across the target on the ground. In the spaceborne system, due to the earth’s curvature effect and rotation and noncircular satellite orbit, the relative velocity is generally not constant and nonlinear. By taking these factors into account, the effective velocity is approximated as [4]

$$u \equiv \sqrt{v_s v_g}$$

(3.20)
where \( v_s \) is the platform velocity and \( v_g \) is the ground velocity; both vary with orbital vector, including the position and range (Figure 3.13). Consequently, the squint angle must be scaled from the physical squint angle according to [4]

\[
\theta_r = \frac{u}{v_g} \theta_{sq} = \frac{v_s}{u} \theta_{sq}
\] (3.21)

### 3.4 FM CONTINUOUS WAVE

#### 3.4.1 Signal Parameters

Referring to Figure 3.14, the transmitted signal is of the form [16,17]

\[
s_t(t) = \exp\left\{ j2\pi \left( f_c t + \frac{1}{2}at^2 \right) \right\}
\] (3.22)

where \( a \) is the frequency change rate within the transmitted bandwidth \( B \). With \( a \), the intermediate frequency (IF) bandwidth \( B_{if} \) determines the range of the minimum and maximum delay times, \( \tau_{min}, \tau_{max} \).

The maximum unambiguous range is determined by

\[
R_{max} = \frac{f_{max}c}{2a}
\] (3.23)

where \( f_{max} \) is the corresponding maximum sampling frequency.

The received signal in one dimension is given by

\[
s_r(\tau) = \exp\left\{ j2\pi \left[ f_c (t - \tau) + \frac{1}{2}a(t - \tau)^2 \right] \right\}
\] (3.24)

FIGURE 3.14  FMCW signal parameters.
The beat signal (intermediate frequency) for a single target takes the form

\[ s_g(t) = s_i(t)s_i^*(t) = \exp \left\{ j2\pi \left[ -f_i(t - \tau) - \frac{a}{2} (t^2 - 2t\tau + \tau^2) + f_d t + \frac{1}{2} at^2 \right] \right\} \]

\[ = \exp \left\{ j2\pi \left[ f_d t + at\tau - \frac{1}{2} at^2 \right] \right\} \quad (3.25) \]

The delay time is range dependent; that is, it is subsequently a function of slow time:

\[ \tau = \frac{2R(t, \eta)}{c} \quad (3.26) \]

\[ R(t, \eta) = \sqrt{R_0^2 + u^2(t + \eta)^2} \quad (3.27) \]

For short range, the slant range may be approximated to

\[ R(t, \eta) = \sqrt{R_0^2 + \nu^2(t + \eta)^2} \equiv R_\eta + \frac{u^2}{R_\eta} t \quad (3.28) \]

with

\[ R_\eta = \sqrt{R_0^2 + (u\eta)^2} \quad (3.29) \]

Now Equation 3.25 may be written more explicitly, leading to

\[ s_g(t, \eta) = \exp \left\{ j2\pi \left[ f_c \left( \frac{2R_\eta}{c} - \frac{f_d \lambda}{c} t \right) + at \left( \frac{2R_\eta}{c} - \frac{f_d \lambda}{c} t \right) - \frac{2a}{c^2} R_\eta^2 \right] \]

\[ + \frac{2a}{c^2} R_\eta f_d \lambda t - \frac{af_d^2 \lambda^2}{2c^2} t^2 \right\} \right\} \quad (3.30) \]

where the range–Doppler frequency is induced by the time dependence in Equation 3.27 with

\[ f_d = -\frac{2}{\lambda} \frac{\partial R(t, \eta)}{\partial t} = -\frac{2}{\lambda} \frac{u^2 \eta}{R_\eta} \quad (3.31) \]
In focusing, it is essential to remove the Doppler frequency term by designing a proper filter [17–21]. The technical aspect is treated in Chapters 5 and 7.

### 3.4.2 Object to Data Mapping

The mapping from the object domain to the data domain is illustrated in Figure 3.15; $t_0$ is the near-range time. The range sampling frequency is confined in

$$f_r = \left( f_c - \frac{\alpha}{2}, f_c + \frac{\alpha}{2} \right),$$

while the chirp rate is

$$a_r = \frac{4\pi}{c} \left( f_c - \frac{\alpha}{2}, f_c + \frac{\alpha}{2} \right).$$

Note that $f_r = a_{\text{osr}} / \text{PRI}$. A simple simulation is carried out using the parameters given in Table 3.1. Two targets are located 750 m apart. The phase and amplitude of the IF signal is plotted in Figure 3.16, where the Fourier transform was performed on the range. Relevant focusing issues are discussed in Chapter 5.

![Mapping of the object domain and data domain.](image)

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**TABLE 3.1**

FMCW Signal Simulation Parameters for Point Targets

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitted bandwidth</td>
<td>$B$</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>$f_c$</td>
<td>24 GHz</td>
</tr>
<tr>
<td>Pulse repeat interval</td>
<td>PRI</td>
<td>4.0 ms</td>
</tr>
<tr>
<td>Target location</td>
<td>$R$</td>
<td>250, 1000 m</td>
</tr>
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<td>Slant range</td>
<td>$R_{\text{range}}$</td>
<td>750–2000 m</td>
</tr>
</tbody>
</table>
REFERENCES


