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Electromagnetic, Optical, Radiation, Chemical, and Biomedical Measurement
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Q Factor Measurement

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Q factor is a method of characterizing the rate of dissipation of energy from an oscillating system [1]. Q is defined as \(2\pi\) times the energy stored in a resonant system divided by the energy dissipated per cycle. The term system used in this context refers to any type of resonance: mechanical, electric, nuclear, etc. For the purposes of this handbook, \(Q\) will be that of an electric circuit. Also, for the discussion of \(Q\), very low values of \(Q\), typically less than 10, will not be considered as these low values of \(Q\) produce highly damped oscillations that are more exponential than oscillatory and the concept of \(Q\) does not fit.

A common interpretation of \(Q\) is quality factor that explains the use of the letter, \(Q\), but this is misleading. A component that has a high \(Q\) is not always beneficial and may not be of high quality. In many circuits, a component requires a specific value of \(Q\) rather than "higher is better." In other cases, a high \(Q\) is an advantage.

The \(Q\) factors encountered in common circuit components range from a low of about 50 for many inductors to nearly 1 million found in high-quality quartz crystals. \(Q\) can be applied to a resonant circuit or to capacitors and inductors. When \(Q\) is applied to a component, such as an inductor, the \(Q\) would be that obtained if the inductor were used in a resonant circuit with a capacitor that dissipates no energy. In this case, the value of \(Q\) depends on the frequency.

For most \(LC\) resonant circuits, the losses in the inductor dominate and the \(Q\) of the inductor is, essentially, the \(Q\) of the circuit. It is easy to make very low-loss capacitors even in the ultra high frequency (UHF) region. On the other hand, varactor diodes have considerably more loss than fixed capacitors, and a varactor can play a more significant role in setting the circuit \(Q\).

### 35.1 Basic Calculation of \(Q\)

**Figure 35.1** shows a simple resonant circuit. The capacitor can store energy in the electric field and the inductor in the magnetic field. The circuit oscillates with the energy transferring between the two elements. For the ideal elements shown, this continues forever. Since the definition of \(Q\) has the energy lost per cycle in the denominator—which is zero—the result is an infinite \(Q\).

In practice, circuit elements are not perfect and the energy initially contained within this circuit would be lost by the circuit and the oscillations would decrease in amplitude as the energy diminished.
FIGURE 35.1  A simple resonant circuit with no loss elements.

FIGURE 35.2  Resonant circuits: (a) A simple series resonant circuit with the equivalent resistance and (b) a parallel resonant circuit with a parallel equivalent resistance. For the same Q circuit, the values of the resistors are not the same.

Energy loss in a circuit is represented by that in a resistor, which can be included in one of two ways. The first way is shown in Figure 35.2a, where the resistor is in series with the capacitor and inductor. A second representation is a parallel resistor as shown in Figure 35.2b.

To derive the relationship between the circuit element values and the Q of the circuit, either the current or voltage of the circuit can be used in the equations. Current is the obvious choice for a series circuit, while voltage is the common thread for the parallel circuit. Assume that the amplitude of the current through the series circuit of Figure 35.2a is given by

\[ i(t) = I(t) \cos(2\pi f_0 t) \]  

where \( f_0 \) = resonant frequency

\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \]  

and \( I(t) \) is the amplitude, which is decreasing in some unspecified fashion. The circuit's peak current occurs when the cosine function is equal to 1 and all of the energy is contained in the inductor and is equal to \((1/2)Li(t)^2\).

Assume that a relatively high Q is present in this circuit and that \( I(t) \) changes by only a slight amount during the time of one cycle. During this cycle, the peak current is \( I(t) \), and the rms value of the current is \((0.707)I(t)\). Therefore, the energy dissipated in one cycle is \((0.5)I^2(t)r/f_0\). Substituting these values in the definition of Q yields

\[ Q = 2\pi \frac{(1/2)(Li^2(t))}{(1/2)(rI^2(t)/f_0)} = \frac{2\pi f_0 L}{r} = \frac{X_L}{r} \]  

where \( X_L \) is the reactance of the inductor. The same procedure can be used with the parallel resonant circuit of Figure 35.2b using voltage equations to obtain the relationship between a parallel resistance and Q, which is

\[ Q = \frac{R}{X_L} \]  

It is very important to understand the nature of the circuit resistance in Figure 35.2a and b. The resistance represents all of the losses in the resonant circuit. These losses are from a variety of sources, such as
the resistance of the wire to make an inductor or the leakage current of a capacitor. It can also represent
the deliberate addition of resistors to set the Q of the circuit to a specific value. The resistance shown
in the circuits of Figure 35.2 represents the equivalent resistance of all of the energy losses. The resistance
cannot be measured with an ohmmeter as the value of the equivalent resistor is a function of frequency
and other variables such as signal level. Some of the loss in a resonant circuit is due to radiation, which
is a function of frequency. The resistance of conductors is mostly due to skin effect, which increases with
increasing frequency. The losses in the ferromagnetic materials used for making some inductors are non-
linear; thus, the equivalent resistance is not only a function of frequency but of signal level.

Most resonant circuits are not stand-alone circuits as shown in Figure 35.2 but are a part of other
circuits where there are sources and loads. These additional resistances further remove energy from the
resonant circuit. The Q of a resonant circuit when there are sources and loads is called the loaded Q. In
most applications of resonant circuits, the Q of the resonance is set by the external loads rather than the
capacitor and inductor that form the resonance.

35.2 Bandwidth and Q

The bandwidth of a resonant circuit is a measure of how well a resonant circuit responds to driving
signals of a frequency near the resonant frequency and is a function of Q. The relationship between the
3 dB bandwidth and Q will be derived.

Applying a driving signal to a resonant circuit can overcome the losses of the circuit and cause the
resonant circuit to oscillate indefinitely. As an example of this, consider the voltage generator in the
series resonant circuit shown in Figure 35.3.

When the frequency of the voltage source is equal to the resonant frequency of the circuit, the equiva-
 lent impedance of the series resonant circuit is the resistance of the circuit and the current in the circuit,
simply \( E/r \).

At frequencies higher or lower than the resonant frequency, the impedance of the circuit is greater
because the net reactance is not zero and the circuit current will be less than at resonance.

At what frequency will the circuit current be 3 dB less than at resonance? This frequency is where the
circuit impedance is 1.414 that of the impedance at resonance. This is the frequency where the reactive
part of the impedance is equal to the real part. This situation occurs at two frequencies. Below the reso-
nant frequency, the net reactance is capacitive and is equal to \( r \), while at a second frequency above the
resonant frequency, the reactance is inductive and equal to \( r \). This can be represented by two equations
for the two frequencies:

For \( f_1 > f_0 \), \[ |X_L - X_C| = 2\pi f_1 L - \frac{1}{2\pi f_1 C} = \frac{(f_1 - f_0)^2 - 1}{2\pi f_1 C} = r \] (35.5)

\[ \left( \frac{f_1}{f_0} \right)^2 - \frac{1}{Q} \left( \frac{f_1}{f_0} \right) - 1 = 0 \left( \frac{f_1}{f_0} \right) = \frac{1}{2Q} \pm \frac{1}{4Q^2} + 1 \]
For \( f_1 < f_0 \), \( |X_L - X_C| = \frac{1}{2\pi f_j C} - 2\pi f_j L = \frac{(f_1 - f_0)^2 - 1}{2\pi f_j C} = r \)

\[
\left( \frac{f_1}{f_0} \right)^2 + \frac{1}{Q} \left( \frac{f_1}{f_0} \right) - 1 = 0 \quad \left( \frac{f_1}{f_0} \right) = \frac{1}{2Q} \pm \frac{1}{4Q^2} + 1
\]

\[ (f_1 - f_0) \frac{f_0}{Q} = \text{bandwidth} \]

### 35.2.1 Measuring \( Q \)

There are a number of methods of measuring \( Q \) using a variety of bridges, several of which are described in [2]. One method of measuring a capacitive or inductive \( Q \) is to place the component in a resonant circuit. When the \( Q \) to be measured of a device that is, in itself, a resonant circuit such as quartz crystal, similar techniques are used except the device's own resonance is used for the measurement. Circuit voltages or currents are measured at the resonance frequency and the \( Q \) is determined.

### 35.3 \( Q \)-Meter

One simple and very popular method of measuring \( Q \) is with a device called, appropriately, the \( Q \)-meter. Consider the resonant circuit in Figure 35.4 for measuring the \( Q \) of inductors. The circuit has a very low-loss capacitor of known value and a constant voltage source.

The usual components measured by the \( Q \)-meter are inductors. It was previously mentioned that inductors are the weak link in resonant circuits, and the \( Q \) of a circuit is usually set by the inductor. The \( Q \)-meter can measure capacitance and capacitor \( Q \). In this theoretical circuit, the circuit resistance is the equivalent series resistance (ESR) of the inductor under test. This is due to the fact the variable capacitor is assumed to be lossless, the generator has zero resistance, and the voltmeter does not appreciably load the circuit. In a real circuit, it is not possible to achieve this situation, but the instrument is designed to approach this goal.

To measure the \( Q \) of an inductor using the \( Q \)-meter, the generator is set to the desired frequency, while the variable capacitor tunes the circuit to resonance as indicated by the peak reading of the voltmeter. At resonance, the impedance of the circuit is simply the ESR of the inductor. The current meter sets the current of the circuit as

\[
I = \frac{E}{R_x}
\]

where

- \( E \) is the generator voltage
- \( R_x \) is the equivalent resistance of the inductor

**FIGURE 35.4** The basic circuit of a \( Q \)-meter showing the signal source, the inductor under test, and the voltmeter.
Because the circuit is at resonance, the voltages of the two reactances are equal and of opposite phase. These voltages are

\[ V = iX_C \quad \text{or} \quad V = iX_L \]  

(35.8)

where

- \( X_C \) is the capacitive reactance
- \( X_L \) is the inductive reactance, which are numerically equal at resonance

Substituting the relationship of the circuit current, the result is

\[ V = \frac{EX_L}{R_X} = \frac{EX_C}{R_X} = EQ \]  

(35.9)

Therefore, the voltage across the reactances is equal to \( Q \) times the applied voltage. If, as an example, the voltage source were 1 V, the voltmeter would read \( Q \) directly. \( Q \) values of several hundred are common, and therefore, voltages of several hundred volts could be measured. Modern circuits do not typically encounter voltages of this magnitude, and many components cannot withstand this potential. Therefore, most \( Q \)-meters use a much smaller source voltage, typically 20 mV.

If the frequency of the source and the circuit capacitance are known, it is possible to calculate the inductance of the unknown.

### 35.4 Other \( Q \) Measuring Techniques

There are very few \( Q \)-meters being manufactured today, although there are thousands of old \( Q \)-meters still in use. Because the \( Q \)-meter was the only accepted method of measuring \( Q \) for so many years, it will take decades before alternative methodologies overtake the \( Q \)-meter.

Measuring \( Q \) without a \( Q \)-meter involves a variety of \( RLC \) measuring instruments that measure the vector impedance and calculate \( Q \). The calculated \( Q \) value is not as valid as that determined with a \( Q \)-meter unless the \( RLC \) meter is capable of measuring \( Q \) at the desired frequency. Only the more sophisticated, and expensive, \( RLC \) meters allow the use of any test frequency. Despite the new sophisticated \( RLC \) measuring instruments, there is an adapter for one model \( RLC \) instrument that allows the classic \( Q \)-meter-style measurement to be made.

Because vector impedance measurement is covered elsewhere in this handbook, the remainder of this section will be devoted to measurements using the \( Q \)-meter.

### 35.5 Measuring Parameters Other than \( Q \)

In addition to \( Q \), the \( Q \)-meter can be used to measure inductance, the \( Q \) or dissipation factor of a capacitor, and the distributed capacitance, \( C_d \), of an inductor.

If an inductor with capacitance \( C_d \) is placed in the \( Q \)-meter circuit, the total circuit capacitance includes both the \( Q \)-meter's capacitance and the additional \( C_d \). Therefore, when resonance is achieved, the actual resonating capacitance is more than what is indicated on the \( Q \)-meter capacitor's dial. If \( C_d \) is not included in the calculation of inductance, the resulting value would be too large.

In many applications, the actual inductance is not the important parameter to be measured by the \( Q \)-meter. The actual parameter being determined is “what capacitance is required to resonate the particular inductor at a specific frequency,” regardless of \( C_d \). In other applications, such as inductors that are to be used in wide-range oscillators, where \( C_d \) will limit the tuning range, \( C_d \) is an important parameter.
The Q-meter can be used to determine \( C_d \), which will also allow for an accurate calculation of inductance. Determining \( C_d \) is a matter of resonating the inductor under test at more than one frequency.

To understand how the two-frequency measurement will allow the determination of \( C_d \), assume that an inductor is resonated at a frequency \( f_1 \). The relationship between the applied frequency, \( f_2 \), and the capacitor of the Q-meter to obtain resonance is

\[
f_1 = \frac{1}{2\pi \sqrt{L(C_1 + C_d)}}
\]

(35.10)

where \( C_1 \) is the capacitance set on the Q-meter.

Resonating the same inductor at a second, higher, frequency, \( f_2 \), requires a Q-meter capacitance of \( C_2 \), such that

\[
f_2 = \frac{1}{2\pi \sqrt{L(C_2 + C_d)}}
\]

(35.11)

This implies that \( C_2 \) is a smaller capacitance than \( C_1 \). Using these two equations and solving for \( C_d \), the following result is obtained:

\[
C_d = \frac{C_2 f_1^2 - C_1 f_2^2}{f_1^2 - f_2^2}
\]

(35.12)

A convenient relationship between \( f_1 \) and \( f_2 \) is to set \( f_2 = 1.414 f_1 \). With frequencies thus related, the distributed capacitance is

\[
C_d = C_1 - 2C_2
\]

(35.13)

\( C_d \) causes errors in the measurement of Q because of current through the \( C_d \). The Q measured by the Q-meter is called the “effective Q.” Since large inductors with significant \( C_d \) are no longer in common use because of the use of active filters, the distinction between effective Q and real Q is seldom considered. For additional information about effective Q and distributed capacitance, see [3].

To measure capacitors on the Q-meter, a relatively high-Q inductor is connected to the inductance terminals on the Q-meter and resonated with Q-meter's internal calibrated capacitor. The capacitor to be measured is connected to the instrument's capacitor terminals, which are in parallel with the internal variable capacitor. This increases the circuit capacitance. The Q-meter variable capacitor is adjusted to regain resonance, which requires that the capacitance be reduced by an amount equal to the unknown capacitor.

The Q of the capacitor can be measured. The addition of the capacitor reduces the circuit Q because of the additional loss introduced by the capacitor. In the description of the Q-meter, the Q-meter's variable capacitor is assumed to have no loss.

Measuring the Q of a capacitor using the Q-meter is seldom done. This is because most capacitors have very high-Q values, usually significantly higher than the test inductor and the Q-meter's internal capacitor. Therefore, the addition of a high-Q unknown capacitor makes an imperceptible difference in the overall circuit Q making it very difficult to obtain an accurate Q measurement. There are special cases, such as measuring the Q of a capacitive transducer or a varactor diode, where low Q values are encountered and the Q-meter can be used successfully.

For best results, the Q of the inductor must be considerably greater than that of the unknown capacitor, and the unknown capacitance must be considerably greater than the internal capacitance. If these criteria are met, the Q of the unknown capacitor can be read from the Q-meter. If these criteria are compromised, corrections can be made but the equations become complex and the accuracy degrades.
Most Q-meters provide measurement ranges to about 500 or to 1000. It is sufficient for measuring inductors, which was the main purpose of the Q-meter. For measuring high-Q devices such as ceramic resonators with Q values greater than 1000 or for quartz resonators with Q values extending into the hundreds of thousands, the Q-meter technique is insufficient. A high-Q circuit implies very little energy loss, and the energy that must be removed to make a measurement must be very small if Q is to be measured accurately.

### 35.6 Measuring High-Q Capacitors

The original intent of the Q-meter was primarily for measuring the Q of inductors since it was the Q of the inductor that set the bandwidth of most tuned circuits at radio frequencies (RFs). The Q of a capacitor was usually much higher than the inductor in a resonant circuit and only inductors were typically measured. The Q of a capacitor has several other names such as dissipation factor, loss tangent, or ESR. The resistive part of a capacitor’s impedance has become increasingly more important for power supplies and circuits where capacitors could experience significant currents and a capacitor with a high dissipation factor or low Q wastes energy and could be destroyed by internal heating.

The technique described using a Q-meter with a calibrated inductor for making capacitor measurements is very limited by the Q of the resonating inductor and particularly difficult where the capacitor to be measured is a high capacitance value and high Q.

Modern circuits do not rely as much on high-Q inductors as in the past. Signal filtering once done at intermediate and audio frequencies, IF and AF, is done routinely with digital signal processing. RF filtering is still required, but new applications extend into UHF and microwave frequencies where conventional wire-wound coils are not used. The need for wire-wound inductors and their measurements is diminishing.

As the need for high-Q inductors diminishes, high-Q capacitors are becoming in more demand as the use of switching power supplies increases. When describing capacitors for power supplies, ESR is the preferred term. But \( ESR = X/Q \).

Modern switching power supplies operate at RF frequencies and therefore use components that would be suitable for RF circuits and therefore require RF style measurements.

Another factor in the increasing need for high-Q capacitors is the use of solid-state amplifiers for RF. High-powered vacuum tube amplifiers used resonant circuits that were high voltage and lower current when compared to the same power level for transistor amplifiers. These higher currents increase the heating of a capacitor due to the ESR. Capacitors used for filters usually required a high Q because of the large circulating currents, but in a transistor power amplifier, high-Q capacitors are also required for coupling and bypassing, applications not usually associated with high Q.

With the demise of the Q-meter, vector network analyzers were used to measure capacitor and inductor parameters. It is difficult to obtain accurate Q measurement for ultrahigh-Q capacitors using a vector network analyzer because of the large disparity between the real and imaginary part of the capacitor’s impedance. It requires the network analyzer to accurately separate the real and imaginary parts and to have the ability to resolve small fractions of a dB.

Modern ultralow ESR chip capacitors can have Q factors well into the thousands. The classic Q-meter, when measuring inductors, could handle Q factors from less than 100 to perhaps as high as 1000 with accuracy. A very important factor when using resonance as a method of measuring Q of either inductors or capacitors is the Q of the resonating element. When measuring the Q of inductors in the classic Q-meter, the Q of the resonating capacitor, using an air dielectric, had sufficiently high Q to measure moderately high Q's for inductors. Using resonance techniques for measuring the Q of ultralow ESR capacitors demands an inductor of extremely high Q. This need can be met with a section of coaxial transmission line.

A typical transmission line section used for measuring Q is on the order of a meter or two in length. One end is short-circuited, while the other is open circuited. To ensure low loss, air is used as the...
dielectric, and the inner, conducting, surfaces are gold or silver plated to decrease skin effect resistance. At the open-circuited end, a sleeve extends the outer conductor of the transmission line, called a choke section, to minimize radiation from the end of the transmission line. At the shorted end of the transmission line, there is the ability to insert a leadless component usually involving a jackscrew-like clamping device.

The transmission line section is more than just an inductor but a resonant circuit. The measurement techniques call for the characterization of the transmission line in terms of resonant frequency and bandwidth. The transmission line will resonate at odd multiples of one-quarter of its wavelength. Since the dielectric is air, the resonant frequencies will be \((n/4)(3\pi f/d)\) where \(d\) is the length of the line and \(n\) is an odd integer.

To determine resonance and the bandwidth of the transmission line section, a signal generator is loosely coupled into the transmission line section with a loop, and the RF energy is sampled, also loosely coupled with either a capacitive probe or another loop. It is important that the coupling be very loose to prevent energy from being lost to the signal source or the sensor. Another important reason to limit the power coupled to the measurement is that very high \(Q\) are involved, which implies high circulating currents. The heating of the capacitor under test can skew the measurements for some capacitors. The arrangement is similar to a cavity filter but with a very high amount of insertion loss. Insertion losses of 40–60 dB are not uncommon. Typical source/detector combinations are a signal generator and a sensitive RF millivoltmeter or a spectrum analyzer. Another excellent source/detector pair is a spectrum analyzer with a tracking generator or a scalar, or vector, network analyzer. The limitations of the vector network analyzer are removed from the measurement technique as there is no need to separate the real and imaginary parts in the described technique.

The transmission line is resonated and the resonant frequency and the bandwidth of the line are determined. It is not necessary to use only the 3 dB bandwidth and the 6 dB or other values may be used. The bandwidth is used to calculate the \(Q\) of the resonator, and the calculation must reflect the measurement points used.

The capacitor under test is inserted in the shorted end of the transmission line and the resonant frequency and bandwidth are measured again. The addition of the capacitor will change the resonant frequency of the transmission line because of the capacitance being placed in series with the center conductor of the transmission line. The bandwidth of the transmission line will be wider because of the capacitor’s ESR.

Using the before and after values, the value and \(Q\) of the capacitor under test can be determined. This technique is the basis behind ANSI/EIA RS 483, Effective Series Resistance (ESR) and Capacitance of Multilayer Ceramic Capacitors at High Frequencies, standard.

The \(Q\)-meter relied on causing resonance in a circuit to determine the capacitance or inductance and the attendant \(Q\). The transmission line technique also relies on resonance. The classic \(Q\)-meter may be a thing of the past, but the techniques that defined the \(Q\)-meter live on.

**Defining Terms**

**Bandwidth:** A measurement of the amount of frequency spectrum occupied by a signal or the equivalent spectrum covered by a circuit that passes a finite band of frequencies. There are a number of methods of defining bandwidth depending on the nature of the spectrum. Relative to resonant circuits, the bandwidth is measured between the -3 dB points of the passband.

**Distributed capacitance:** The amount of capacitance added to an inductor typically from the capacitance due to adjacent wires in a solenoid-type inductor. The distributed capacitance is given as a single capacitance figure for a specific inductor and can be defined as the equivalent capacitance across the entire coil. It is also important to note that the distributed capacitance with no external capacitance.
Effective inductance: Due to distributed capacitance, less capacitance than that calculated from an inductance value is required to resonate a circuit. If the actual capacitance required to resonate a circuit is used to calculate an inductance value, the resulting inductance value will be higher than the theoretical inductance value. This higher value is called the “effective inductance.” The actual inductor cannot be considered as a pure inductance of a value equal to the effective inductance because the real inductor has a resonant frequency that a pure inductance does not.

Q: A measurement of the rate of energy loss in a resonant circuit.

Q-meter: An instrument for measuring Q factor by resonating the circuit and measuring the voltage across the reactances.

RF: radio frequencies: Any frequency above about 10 kHz extending to the upper limits of the radio spectrum. Generally, radio frequencies are distinguished from power generation frequencies such as 50, 60, or 400 Hz and above the range of human hearing or 20 kHz.

References