Bridge Engineering Handbook
Fundamentals
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Bridge Aesthetics: Achieving Structural Art in Bridge Design

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It so happens that the work which is likely to be our most durable monument, and to convey some knowledge of us to the most remote posterity, is a work of bare utility; not a shrine, not a fortress, not a palace but a bridge.

Montgomery Schuyler

Writing about John Roebling’s Brooklyn Bridge, 1883

3.1 Introduction

3.1.1 Why Consider Aesthetics

People know intuitively that civilization forms around civil works for water, transportation, and shelter. The quality of the public life depends, therefore, on the quality of such civil works as aqueducts,
bridges, towers, terminals, meeting halls: their efficiency of design, their economy of construction, and the appearance of their completed forms. At their best, these civil works function reliably, cost the public as little as possible, and when sensitively designed, become works of art. The public is becoming ever more aware of this potential, and demanding an ever higher standard for the appearance of the bridges in their communities.

If civil works are to become works of art, engineers cannot just worry about the structure and leave the appearance to someone else. If a decision affects the size, shape, color, or surface texture of a visible part of the bridge, it affects how people will feel about the bridge. The shapes and sizes of the structural members themselves dominate people’s impressions of a bridge. They are the largest elements of the bridge, therefore the first elements people see as they approach and the most strongly remembered. It is impossible to correct the appearance of a poorly proportioned or detailed structure by the application of “aesthetic treatments” involving color, texture, or ornamentation, though many have tried.

Since engineers control the shapes and sizes of the structural components, they must acknowledge that they are ultimately responsible for the appearance of their structures. Thus, to meet their obligations as professionals, engineers must respond to the public’s concern. For the same reason engineers would not build a bridge that is unsafe, they should not build one that is ugly. All engineers are accustomed to dealing with issues of performance, efficiency, and cost. Now, they must also deal with issues of appearance, something the most accomplished have always done.

3.1.2 Frequent Objections to Considering Aesthetics

- It always adds cost.

  Not true. Simply paying attention to proportions and details can result in an attractive bridge with no increase in cost (Figure 3.1). Indeed, there are times when the search for economy also results in an improvement in appearance (Figure 3.2). Whether there is additional cost varies widely depending on region of the country, owner preferences and practices, contractor capabilities, span length, size of project, community aspirations, and other project specifics.

  If there is an increased cost, then the relevant question becomes: does the aesthetic improvement justify the additional cost? Few people automatically buy the cheapest car or living room

![FIGURE 3.1](image-url)
sofa no matter what it looks like. We make decisions every day to spend additional money to get a better quality product. We can make the same kind of judgments about bridges, keeping in mind that the bridges we build today will be prominent features in our communities for the next 80 years or more. If the affected community is involved, as it should be, we can take advantage of their concerns and insights as well.

- People can’t agree on what looks better.
  Also not true. People have agreed for centuries on which paintings look better, which symphonies sound better, and which buildings are more attractive. A consensus on which bridges look better and why has existed since at least 1812 as articulated in the writings of Thomas Telford. That consensus has been recognized by artists and others. For example, Robert Maillert’s Salginatobel Bridge (Figure 3.3) was formally recognized by New York’s Museum of Modern Art in 1949 and by many others since. That consensus is embodied in this chapter.

- My client/boss won’t let me.
  Show your client/boss this chapter.

- I don’t know how.
  Read this chapter and you will.

### 3.1.3 What Is the Goal?

The purpose of this chapter is to help bridge designers improve the elegance of their bridges by examining actual examples and discussing the principles on which they are based. The ultimate goal is to make every bridge efficient, economical, and elegant by giving meaningful visual expression to loads, equilibrium, and forces. With this as a goal every bridge will become an asset to its community and environment.

The American Association of Highway and Transportation Officials (AASHTO) formally recognized this ultimate goal in 2010 when its Standing Committee on Bridges and Structures adopted a *Bridge Aesthetics Sourcebook* based on these principles. The *Sourcebook* was prepared by the Bridge Aesthetic Subcommittee of the Transportation Research Board.

Both this chapter and the *Sourcebook* are inspired by David Billington’s work illuminating the history of structural art in bridge design. The next two sections of the chapter are a summary of that work.
3.2 Engineer’s Aesthetic and Structural Art

3.2.1 Aesthetic Tradition in Engineering

“Aesthetics” is a mysterious subject to most engineers, not lending itself to the engineer’s usual tools of analysis, and rarely taught in engineering schools. Many contemporary engineers are not aware that many famous engineers have made aesthetics an explicit component of their work, beginning with the British engineer Thomas Telford. In 1812, Telford defined structural art as the personal expression of structure within the disciplines of efficiency and economy. Efficiency here meant reliable performance with minimum materials. Economy implied construction with competitive costs and minimal maintenance expenses. Within these bounds, engineer/structural artists find the means to choose forms and details that express their own vision, as Telford did in his Craigellachie Bridge (Figure 3.4). The arch is shaped to be an efficient structural form in cast iron, whereas the diamond pattern of spandrel bars, at a place in the bridge where structural considerations permit many options, is clearly chosen with an eye to its appearance.

Those engineers who were most conscious of the centrality of aesthetics for structure have also been regarded as the best in a technical sense. Starting with Thomas Telford (1757–1834), we can identify Gustave Eiffel (1832–1923), and John Roebling (1806–1869) as the undisputed leaders in their fields during the nineteenth century. They designed the largest and most technically challenging structures and they were leaders of their professions. Telford was the first president of the first formal engineering society, the Institution of Civil Engineers, and remained president for 14 years until his death. Eiffel directed his own design-construction-fabrication company and created the longest spanning arches and the highest tower of the time. Roebling founded his large scale wire rope manufacturing organization while building the world’s longest spanning bridges (Figure 3.5).

In reinforced concrete, Robert Maillart (1872–1940) was the major structural artist of the early twentieth century. Maillart, first in his 1905 Tavanasa Bridge and later with the 1930 Salginatobel (Figure 3.3) and others, imagined a new form for three-hinged arches that included his own invention of the hollow box in reinforced concrete. The Swiss engineer Christian Menn (1927) has demonstrated how a deep understanding of arches, prestressing, and cable-stayed forms can lead to structures worthy of
exhibition in art museums; for example, his 1964 Reichenau Arch (Figure 3.6) and his 1992 bridge at Sunniberg (Figure 3.7).

The engineers’ aesthetic results from the conscious choice of form by engineers who seek the aesthetic expression of structure within the disciplines of efficiency and economy. Their forms are not the unconscious result of the search for economy nor the product of supposedly optimizing calculations. Instead, the engineer chooses a form from among several options with similar structural and economic
characteristics because of its superior aesthetic potential. In seeking such aesthetic expression the engineers above and many other of the best structural engineers have recognized the possibility for structural engineering to be an art form parallel to but independent from architecture. These people have, over the last two centuries, defined a new tradition, structural art, which we take here to be the ideal for an engineer’s aesthetic.
These works of structural art provide evidence that the common life flourishes best when the goals of freedom and discipline are held in balance. The disciplines of structural art are efficiency and economy, and its freedom lies in the potential it offers to the individual designer for the expression of a personal style motivated by the conscious aesthetic search for engineering elegance. These are the three leading ideals of structural art—efficiency, economy, and elegance.

### 3.2.2 Three Dimensions of Structure

Structure’s first dimension is a scientific one. Each working structure must perform in accordance with the laws of nature. In this sense, then, technology becomes part of the natural world. Methods of analysis useful to scientists for explaining natural phenomena are often useful to engineers for describing the behavior of their artificial creations. It is this similarity of method that helps to feed the fallacy that engineering is applied science. But scientists seek to discover preexisting form and explain its behavior by inventing formulas, whereas engineers seek to invent forms, using preexisting formulas to check their designs. This scientific dimension is measured by efficiency.

Technological forms live also in the social world. Their forms are shaped by the patterns of politics and economics as well as by the laws of nature. Thus, the second dimension of structure is a social one. In the past completed structures might, in their most elementary forms, be the products of a single person: in the civilized modern world, however, these technological forms, although at their best designed by one person, are the products of a society. The public must support them, either through public taxation or through private commerce. The social dimension of structure is measured by Economy.

Technological objects visually dominate our industrial, urban landscape. They are among the most powerful symbols of the modern age. Structures and machines define our environment. The locomotive of the nineteenth century has given way to the automobile and airplane of the twentieth century. Large-scale complexes that include structures and machines become major public issues. Power plants, weapons systems, refineries, river works, transportation systems, and bridges—all have come to symbolize the promises and problems of industrial civilization.

Bridges such as the Golden Gate, the George Washington, and the Sunshine Skyway (Figure 3.8) serve to function for our time and carry on the traditions set by the Brooklyn Bridge. Nearly every American knows something about these immense structures, and modern cities repeatedly publicize themselves by visual reference to these works. So it is that the third dimension of technology is symbolic, and it is, of course, this
dimension that opens up the possibility for the new engineering to be structural art. Although there can be no measure for a symbolic dimension, we recognize a symbol by its expressive power and its elegance.

The designer must think aesthetically for structural form to become structural art. All of the leading structural artists thought about the appearance of their designs. These engineers consciously made aesthetic choices to arrive at their final designs. Their writings about aesthetics show that they did not base design only on the scientific and social criteria of efficiency and economy. Within those two constraints, they found the freedom to invent form. It was precisely the austere discipline of minimizing materials and costs that gave them the license to create new images that could be built and would endure.

3.2.3 Structural Art and Architecture

The modern world tends to classify towers, stadiums, and even bridges as “architecture,” creating an important, but subtle, fallacy. The visible forms of the Eiffel Tower and the Brooklyn Bridge result directly from technological ideas and from the experience and imagination of individual structural engineers. Sometimes the engineers have worked with architects just as with mechanical or electrical engineers, but the forms have come from structural engineering ideas.

Structural designers give the form to objects that are of relatively large scale and of single use, and these designers see forms as the means of controlling the forces of nature to be resisted. Architectural designers, on the other hand, give form to objects that are of relatively small scale and of complex human use, and these designers see forms as the means of controlling the spaces to be used by people. The prototypical engineering form—the public bridge—requires no architect. The prototypical architectural form—the private house—requires no engineer. Structural engineers and architects learn from each other and sometimes collaborate fruitfully, especially when, as with tall buildings, large scale goes together with complex use. But the two types of designers act predominately in different spheres, and the results of their efforts deserve to have different names. Christian Menn’s Sunniberg Bridge is an example of structural art arising solely from structural considerations. No architect was involved. “Architecture” is what architects do. “Structural art” is what engineers do.

3.3 Structural Art and the Design Process

3.3.1 Design Versus Analysis

Today many engineers see themselves as a type of applied scientist, analyzing structural forms established by others. Seeing oneself as an applied scientist is an unfortunate state of mind for a design engineer. It eliminates the imaginative half of the design process and forfeits the opportunity for the integration of form and structural requirements that can result in structural art. Design must start with the selection of a structural form. It is a decision that can be made well only by the engineer because it must be based on a knowledge of structural forms and how they control forces and movements.

Many engineers focus on analysis in the mistaken belief that the form (shape and dimensions) will be determined by the forces as calculated in the analysis. But, in fact, there are a large number of forms that can be shown by the analysis to work equally well. It is the engineer’s option to choose among them, and in so doing, to determine the forces by means of the form, not the other way around.

Take the simple example of a two-span continuous girder bridge, using an existing structure, MD 18 over U.S. 50 (Figure 3.9). Here the engineer has a wide range of possibilities such as a girder with parallel flanges, or with various haunches having a wide range of proportions (Figure 3.10). The moments will depend on the stiffness at each point, which in turn will depend on the presence or absence of a haunch and its shape (Figure 3.11). The engineer’s choice of shape and dimensions will determine the moments at each point along the girder. The forces will follow the choice of form. Within limits, the engineer can direct the forces as he chooses.

Now, let’s examine which form the engineer should choose. All of them can support the required load. Depending on the specifics of the local contracting industry, many of them will be essentially
equal in cost. That leaves the engineer a decision that can only be made on aesthetic grounds. Why not pick the one that the engineer believes looks best?

That, in a nutshell, is the process that all of the great engineers have followed. Maillart’s development of the three-hinged concrete box arch (Figure 3.3), for example, shows that the engineer cannot choose form as freely as a sculptor, but he is not restricted to the discovery of preexisting forms as the scientist is. The engineer invents form, and Maillart’s career shows that such invention has both a visual and a rational basis. For Maillart, the dimensions were not to be determined by the calculations alone, and even the calculations’ results could be changed (by adjusting the form) because a designer rather than an analyst is
at work. Analysis is the servant of design, not the source of it. Design, the development of the form, is by far the more important of the two activities. Before there is any analysis, there must be a form to analyze.

### 3.3.2 Role of Case Studies in Bridge Design

Looking at built bridges is a good way for engineers to improve their skills. Bridge design, especially of highway overpasses, often involves standard problems but always in different situations. Case studies can help designers with standard problems by showing models and points of comparison for a large number of bridges without implying that each such bridge be mere imitation.

The primary goal is to look carefully at all major aspects of the completed bridge, to understand the reasons for each design decision, and to discuss alternatives, all to the end of improving future designs. Such cases help to define more general ideas or principles. Case studies are well recognized by engineers when designing for acceptable performance and low cost; they can be useful when considering appearance as well. The parts of a case study for a typical highway overpass are as follows:

1. **An overall evaluation** of the bridge as a justification for studying it. Is it a good example that can be better? Is it a model of near perfection? Is it a bad example to be avoided?, and so on.
2. **A description** of the complete bridge, divided into parts roughly coinciding with easily identifiable costs and including modifications to each part as suggested improvements. In this major description there is an order to the parts that implies a priority for the structural engineer.
   a. **The Concept and Form** of the completed bridge goes together with a summary of the bridge performance history (including maintenance) and of its construction cost, usually given per square foot of bridge. Required clearances, foundation conditions, hydraulic requirements, traffic issues, and other general requirements would be covered here.
   b. **The Superstructure** includes primarily the main horizontal spanning members such as continuous girders, arches, trusses, and so on. In continuous steel girder bridges, the cost is primarily identified with the fabricated steel cost. Modification in design by haunching, changing span lengths, or making girders continuous with columns would be discussed including their influence on cost.
   c. **The Piers** are most frequently columns or frames either in the median or outside the shoulders or at both places in highway overpasses. These are normally highly visible elements and can have many possible forms. Different designs for the relationship among steel girder, bearings, and columns can make major improvements in appearance without detriment to cost or performance.
   d. **The Abutments** are also highly visible parts of the bridge that include bearings, cantilever walls, cheek walls, wing walls, and so on.
   e. **The Deck** includes the concrete slab, overhangs, railings, parapets, provisions for drainage, and so on, all of which have an influence on performance as well as on the appearance either when seen in profile or from beneath the bridge.
   f. **The Color** is especially significant for steel structures that are painted and texture can be important for concrete surfaces of piers, abutments, and deck.
   g. **Other Features** include lighting, signing, plantings, guardrail, and other elements and their transitions onto the bridge, all of which can have important visual consequences to the design.

The order of these parts is significant because it focuses attention on the engineering design. The performance of a weak structural concept cannot be saved by good deck details. An ugly form cannot be salvaged by color or landscaping. The first four parts are structural, the fifth is in part structural, whereas the last two, while essential for the bridge engineer to consider, involve primarily nonstructural ideas.

3. **A comparison** to other similar bridges or bridge designs for similar conditions as a critique of the concept and form. Including bridges with very different forms creates a useful stimulus to design imagination.
4. A conclusion with a discussion of the relationship of this study to a theory of bridge design. Clearly any such discussion must be based upon a set of ideas about design that implicitly bias the writer, who should make these potential biases explicit. This conclusion should show how the case study illustrates a theory and even at times forces a modification of it. General ideas form only out of specific examples.

The images of the MD 18 bridge (Figures 3.9 through 3.11) shows how a case study can be used to improve future designs. Understanding costs is always important, but breaking them down by components often identifies opportunities to make changes that significantly improve appearance without materially increasing cost. In this case recognizing that the cost of the steel girders was a relatively small part of the total bridge cost suggested improving the bridge’s appearance by haunching the girder. Indeed, depending on the practices in a given state and the capabilities of local fabricators, the added fabrication cost of the haunch might be offset by the savings in steel, resulting in no cost increase at all. Any increase in girder cost might seem like a significant amount when measured relative to the cost of the girders by themselves. But seen relative to the cost of the bridge as a whole it could be a very small increase. Thus, a change in this one component can make a significant improvement in the appearance of the bridge but have little impact on the cost of the bridge as a whole.

3.4 Conceptual Engineering, the Neglected Phase of Design

3.4.1 Creating the Concept and Form

Creative engineering design consists neither in applying free visual imagination alone nor in applying rigorous scientific analysis alone nor in applying careful cost analysis alone, but of applying all three together at the same time. Creative engineering design starts with a vision of what might be. Development of the structural vision requires what many engineers call conceptual engineering. Conceptual engineering is the stage when the basic concept and form of the bridge is determined. Conceptual engineering is the most important part of design. All that follows, including the aesthetic impression the bridge makes, will depend on the quality of the concept and form selected. Unfortunately, it is a stage that is often ignored or foreclosed by the application of unwarranted assumptions, preconceived ideas, or prior experience that may not in fact apply.

Reasons given for short changing this stage include “Everybody knows that (steel plate girders, precast concrete girders, cast in place concrete) are the most economical structure for this location,” or “Let’s use the same design as we did for the _____ bridge last year.” Or the selection of form is based largely on precedents and standards established by the bridge-building agency. For example, the form of a highway overpass may be predetermined by the client agency to be (steel plate girders, precast concrete girders cast in place concrete), because that is what the agency is used to or what local contractors are used to or even because the (steel plate girders, precast concrete girders, cast in place concrete) industry is a dominant political force in the state. Or the decision to forgo conceptual engineering may be simply habit—either the engineer’s or the client’s—often expressed in the phrase “We’ve always done it that way.” The assumption underlying all of the above is that the current bridge cannot benefit from changes in ideas, practices, or materials that might have occurred since previous designs were done.

Some will protest conceptual engineering is unnecessary because costs will indeed differentiate and determine the form. Such beliefs often rely too much on unit costs from past projects, ignoring changing conditions, or unique aspects of the current bridge that might result in different unit costs. Or, the engineer might be assigning unwarranted precision to the results of his cost calculations. The cost of the bridge will not be the cost the engineer calculates; it will be whatever cost a contractor is willing to build it for. Rarely do engineer’s estimates get within 5% of the contractor’s bid. Given the engineer’s lack of knowledge about the precise cost, differences of 5% or less might as well be treated as cost—neutral.
Perhaps the most insidious reason conceptual engineering is shortchanged is the practice of bridge building agencies to budget design separately from construction. This leads to pressure to minimize the cost of design, and the design phase most likely to be cut is conceptual engineering. Such decisions are usually based on the types of unexamined assumptions listed above. Thus, potential savings of hundreds of thousands of dollars in construction are foregone in order to save tens of thousands of dollars on design, the ultimate example of penny/wise—pound/foolish thinking.

Accepting these assumptions and beliefs places an unfortunate and unnecessary limitation on the quality of the resulting bridge. It often results in hammering a square peg into a round hole, creating a suboptimal bridge and unnecessary construction cost that far outweighs the cost of conceptual engineering. It also sacrifices the chance for innovation for, by definition, improvements must come from the realm of ideas not tried before. As Captain James B. Eads put it in the preliminary report on his great arch bridge over the Mississippi River at St. Louis:

Must we admit that, because a thing has never been done, it never can be, when our knowledge and judgment assure us that it is entirely practicable?*

Unless these assumptions are challenged, no design will occur. Instead, there will be a premature assumption of the bridge form, and the engineer will move immediately into the analysis of the assumed type. That is why so many engineers mistake analysis for design. Design is more correctly the selection of the concept and form in the first place, which many engineers have not been permitted to do.

### 3.4.2 Doing Conceptual Engineering

Conceptual engineering should be the stage when all of the plausible options, and some not so plausible, are considered. Thus the engineer’s first job is to question all limiting assumptions and beliefs. From that questioning will come the open mind that is necessary to develop a vision of what each structure can be at its best. Unless such questioning is the starting point it is unlikely that the most promising ideas will ever appear.

The options are then examined at a rough level of precision, with consideration of the design intent (see Section 3.5.2.2), various materials, size and form of major members, constructability, project cost, life cycle economics, and appearance. The most promising ideas are then taken to greater levels of refinement. Solutions will emerge that fit the requirements of the site and that are roughly equivalent in terms of structural efficiency and economics. The form of the bridge can then be selected based on which of these solution best appeals to the aesthetic sensibilities of the designer, the owner, and the public.

### 3.5 Application to Design

#### 3.5.1 Ten Determinants of Appearance

“How people react to an object depends on what they see and the order in which they see it. This means the largest parts of the bridge—the superstructure, piers and abutments—have the greatest impact. Surface characteristics (color/texture) come next, then details (Sourcebook, p. 15).” This underlies the idea of a hierarchy of design as introduced in Section 3.3 and laid out in more detail here. In summary, design decisions should be approached in the following order of importance:

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1. Horizontal and vertical geometry
2. Superstructure type
3. Pier/support placement and span arrangements
4. Abutment placement and height
5. Superstructure shape (including parapets, overhangs, and railings)
6. Pier shape
7. Abutment shape
8. Color
9. Texture, ornamentation, and details
10. Lighting, signing, and landscaping

It is the last five elements that are usually considered the “aesthetic” elements, but they are the least important in determining the final result. The aesthetic impact of the first five elements must be considered from the very beginning, or the resulting bridge will be a disappointment.

### 3.5.2 Thinking about Aesthetics in Design

Before a designer can start on the bridge itself, he or she must understand what the bridge is expected to accomplish, functionally as part of a transportation system and socially, visually and symbolically as part of a living community and environment. The designer must have an idea of all of the criteria that the structure must meet and all of the concerns that will act on the structure. In recent years, the Federal Highway Administration (FHWA) and many other transportation agencies have recognized that this is a broad task, requiring the coordination of many, often competing, interests. This process has been given the name Context Sensitive Design. (*Sourcebook*, p. 5; see the *Sourcebook* for more detail on how to use Context Sensitive Design techniques to address all of the concerns involved in a project.)

#### 3.5.2.1 Understand the Goals and the Site

##### 3.5.2.1.1 Owner Requirements

These requirements begin, of course, with the transportation goals that must be met. These include the widths and design speeds of the roadways being carried or traversed, what types of traffic will the bridge be expected to carry or traverse, and whether that includes pedestrians and/or transit. Among other things, these requirements will determine clearance envelopes the bridge must provide.

The owner may have conducted or be bound by a previous feasibility study, Environmental Assessment, Environmental Impact Study or other document, or there may be a formal project purpose and need statement that defines what the intended result of the project is. Other owner requirements will include design standards and policies, including in many cases existing aesthetic design guidelines. Finally, the owner may have established cost limitations that have to be considered.

##### 3.5.2.1.2 The Community and Other Stakeholder Requirements

Potential stakeholders include communities, elected officials, businesses, public review agencies, and the people that will live with the bridge after it is constructed. All concerned parties should be involved from the very beginning, before putting pencil to paper. If people know that they have been included from the beginning, and that the designers have no preconceived notions, the process will run more smoothly, the final result will address the most strongly held desires of the community (Figure 3.12) and it will meet with their approval.
3.5.2.1.3 The Site

The obvious concerns are the physical features. Bridges over canyons or deep cuts will require a structural type that may be inappropriate for a highway crossing. Rivers have a certain width that must be crossed. Geology may favor a certain type of foundation or substructure layout.

Aesthetically, the site establishes the visual field or background against which the bridge will be seen and the context within which it will be judged. Some examples are as follows:

- A bridge on a high profile crossing a canyon or deep valley or a side hill alignment will be visible from a distance. The relationship of the bridge form to the sides of the canyon/valley will likely define the aesthetic impact.
- A bridge located at the top of a crest on a ridgeline will frame views of the distant landscape (Figure 3.13).
- A bridge on a flat coastal plain or over open water is often seen from a distance and in silhouette. The overall composition of its forms and its parts may be the most defining visual image.
- A bridge over a depressed highway will usually be the most prominent feature in the driver's visual field. The form and proportions of the superstructure, piers, and abutments will be critical to its aesthetic impact.

A rural site will have a background of natural features; an urban site will have a background of adjacent buildings and structures with their own architectural features.

How all of this looks will be affected by the daily movement of the sun and the change of seasons. The viewpoints and areas from which the bridge will be seen and by whom need to be understood and, in many case prioritized. It is not always possible to make a bridge look good from all angles.

The best way to understand the visual field is to go to the site at different times of day, at night and in as many different seasons as possible, and take lots of photos. For both aesthetic and technical reasons, there is no substitute for first-hand familiarity with the bridge site.
3.5.2.1.4 Corridors and Interchanges

If the bridge is part of a larger project, an interchange or corridor, where multiple structures will be seen at the same time or in quick succession, the relationships between them needs to be considered. This situation often leads to the establishment of a theme that mandates similar forms for similar parts of the bridges in order to reduce the visual cacophony that results when bridges with different forms and details are seen together (Figure 3.14).

3.5.2.2 Develop a Design Intention

The Sourcebook describes this step as a written list of all of the factors that will influence the design of the bridge in their order of importance. The designer should solicit comments from all involved parties, make appropriate revisions, and then get it approved by the owner. This will be the basis of all future design work.

This step may have already been taken as part of an Environmental Impact Statement or planning study, which often result in a “purpose and need statement” or similar document. However, such statements need to be carefully reviewed to insure that they fully incorporate all of the bridge design issues and that no important component is missing.

3.5.2.3 Do a Conceptual Engineering Study

The importance of conceptual engineering is discussed in Section 3.4. Here are some techniques for making a conceptual engineering study successful.

1. Involve All Stakeholders in Identifying Options

“Communities and review agencies will have opinions about what types of bridges are appropriate. Testing their ideas in the conceptual engineering phase will avoid the need to go back and look at their options later when they object that their ideas are not being considered. It will also encourage their support of the final decision. It may even result in the adoption of a superior but previously unconsidered bridge type (Sourcebook, p. 12).”
2. Make sure all involved stakeholders know all of the implications of the alternatives, including comparative costs. Knowing all of the facts, they will be more likely to support the final decision.

3. Test promising options with 3D views taken from the important viewpoints. Even seasoned design professionals have a hard time anticipating all of the visual implications of a design from 2D engineering drawings. For nonprofessionals it is almost impossible. Showing 3D views of a planned bridge gives all participants a common image of each option to work from (Figure 3.15).

4. Evaluate the options and make the selection based on efficiency, economy, and elegance. Only by applying all three criteria of efficiency, economy, and elegance to multiple alternatives can the process narrow down to the concept that best satisfies all of the requirements.

5. Produce the Conceptual Engineering report. Many agencies call the product of conceptual engineering the type, size, and location report.

3.5.2.4 Proceed to Detailed Analysis and Design
Section 3.6 provides practical ideas for the detailed design of aesthetically pleasing bridges.

3.5.3 Working with Architects, Landscape Architects, and Artists
Gifted engineers working without the assistance of architects, artists, or other visual professionals have produced masterpieces. Thus, it is not necessary for all bridge design teams to include visual professionals. The engineer should seek to develop his or her skills in this area. However, for reasons of time or personal inclination, this is not always possible. Accordingly, engineers have often sought the advice of
other visual professionals—experts in aesthetics who are consulted in the same way as experts in soils, traffic, and wind. Many memorable bridges illustrate the potential success of this approach. The Golden Gate Bridge is a famous example.

Such collaboration does not relieve the engineer of the responsibility to be knowledgeable about aesthetics. As the leader of the design team, he or she remains responsible for the final result. Many over-decorated and expensive failures have been created when the collaboration was done poorly or when someone other than the engineer took over the lead role. The visual professional’s role should be as aesthetic advisor and critic, making comments and suggestions for the engineer’s consideration. In this role a landscape architect, urban designer, architect, or artist can have a positive impact, but the engineer must have the last word (Sourcebook, p. 48).

Figure 3.16, for example, shows a successful collaboration between an engineer and architect.

If the involvement of aesthetic advisors is to be successful, the engineer must be sure that they understand the basic issues involved in bridge design. Most visual professionals are used to dealing with buildings and their immediate surroundings, but bridges are significantly different than buildings. They are much larger, they are often seen at high speeds, and they typically have few surfaces that are flat and level. The architect/landscape architect needs to take the time to understand these differences, and the engineer needs to insist that he or she does. Effectively working with other visual professionals also requires that the engineer develop sufficient knowledge about aesthetics and sufficient self-confidence to recognize valuable ideas and reject inappropriate ideas.

Some have observed that the public seems to more readily accept bridges designed by teams that include architects, urban designers, or landscape architects than those that do not. People sometimes feel that more of their goals will be met when such professionals are involved, in part because most people in these professions are skilled at discussing and responding to community concerns. Unfortunately, engineers have a reputation for being insensitive to community wishes, due in part to many engineers’ inability to speak clearly and knowledgeably in this area.

The engineer needs to develop the vocabulary and knowledge to remain the project’s spokesman to the client and community groups, even concerning aesthetic ideas. Gaining the vocabulary and

**FIGURE 3.15** A photo simulation showing how a proposed bridge will affect an existing recreational lake and nearby commercials trip. Proposed Ken Burns Bridge over Lake Quinsigamond, Worchester, Massachusetts.
knowledge to respond to a community's aesthetic concerns allows an engineer to fulfill the leadership role and retain the community's confidence.

3.5.4 Replacing Historic Bridges/Designing Bridges in Historic Places

Some communities see themselves as historic enclaves and view a bridge as a chance to restate local architectural traditions (Figure 3.17). In those situations a formal historic review process may be in place. The result is often pressure to build a new bridge that looks just like a previous bridge or matches a nearby architectural style. These projects are seldom an aesthetic success. Indeed, in situations governed by the National Commission for Historic Preservation, such an approach violates the Secretary of Interior Standards for Historic Preservation. A better approach is to develop new designs which respect and emulate elements of the previous bridge and any surrounding historic district. More detail on handling these concerns can be found in the Sourcebook.

FIGURE 3.16 An example of a successful collaboration between an engineer and an architect/urban designer which considers nearby land uses and views as well as technical requirements. Clearwater Memorial Causeway, Clearwater, Florida. (From AASHTO, Bridge Aesthetics Sourcebook, American Association of State Highway and Transportation Officials, Washington, DC, 2010. With permission.)

FIGURE 3.17 Two existing traditional bridges, both eligible for the National Register of Historic Places, were replaced by bridges of innovative contemporary design in tune with the community’s aspirations and self-image. Rich Street and Main Street Bridges, Civic Center, Columbus, Ohio.
3.6 Design Guidelines

The following section presents a quick outline of the practical ideas for developing an aesthetically successful bridge. The full details can be found in the *Sourcebook*. Ideas are presented for the Ten Determinants of Appearance in the order of their importance as discussed above.

3.6.1 Horizontal and Vertical Geometry

Before there is a concept for a bridge, the roadway geometry creates a ribbon in space that in itself can be either attractive or unattractive. The geometry establishes the basic lines of the structure, to which all else must react. A graceful geometry will go a long way toward fostering a successful bridge, while an awkward or kinked geometry will be very difficult to overcome. The structural engineer must work interactively with the project’s highway engineers during development of the project geometry. A proactive approach is highly recommended since it is extremely difficult to change the project geometry during later stages (*Sourcebook*, p. 15).

This is a topic that particularly benefits from 3D studies, especially if the geometry can be overlaid on the topography through photos or digital terrain models.

3.6.2 Superstructure Type

The superstructure type refers to the structural system used to support the bridge. It can be an arch, girder, rigid frame, truss, or cable-supported type structure. Because of their size and prominence, the most memorable aspect of the structure will be provided by its structural members. The following are few highlights from the *Sourcebook* (pp. 17 and 18):

- Generally, thinner structures with longer spans are more visually transparent and pleasing than deeper structures or structures with shorter spans (see Figure 3.15).
- The superstructure can be shaped to respond to the forces on it so that the bridge visually demonstrates how it works. For example, haunched girders demonstrate the concentration of forces and moments over the piers. They also reduce the midspan structure depth and provide a more visually interesting opening beneath the structure (see Figure 3.18).
- Use of different structure types over the length of a bridge should be avoided as it usually interrupts the visual line created by the superstructure and is contrary to developing a sense of unity and integrity. If different structural types are unavoidable then a common parapet profile or other feature needs to be found to tie them together.

![FIGURE 3.18](image) The haunch gives this girder a more interesting and attractive shape that also tells a story about the flow of moments and forces in the structure. The stiffener is utilitarian but its placement and curvature make it ornamental as well, so that it reinforces the story told by the haunch. I-81, Virginia. (From AASHTO, *Bridge Aesthetics Sourcebook*, American Association of State Highway and Transportation Officials, Washington, DC, 2010. With permission.)
• For multispan girder bridges, it is preferable to use the same depth of girder for the entire bridge length and not change girder depths based on the length of each individual span except for haunches at the piers.
• The underside of the bridge or soffit as a ceiling will be important for bridges over pedestrian traffic or recreational trails, where the underside will be readily visible because of the slow speed and close proximity of the observers.

3.6.3 Pier/Support Placement and Span Arrangements

Most bridges are linear frameworks of relatively slender columns and girders. A bridge viewed from its side will appear as a transparent silhouette (Figure 3.19). A bridge viewed looking along its length will appear to be a collection of massive structural forms. Pier placement will largely determine how attractive these views are.

The success of the visual relationship between the structure and its surrounding topography will depend heavily on the apparent logic of the pier placement. For example, a pier placed at the deepest point in a valley will seem unnecessarily tall. A pier placed in the water near the shore will seem less logical than one placed on the shore.

Pier placement establishes not only the points at which the structure contacts the topography but also the size and shape of the openings framed by the piers and superstructure. It is desirable to keep constant the height/span ratio of these openings.

3.6.4 Abutment Placement and Height

The abutment is the location where a bridge reaches the ground and the transparency of the structure transitions to the mass of the adjoining roadway or topography. Abutments may become visually massive structures (Figure 3.20) or practically disappear (Figure 3.21), depending on their height and the nature of the grading at the bridge ends. Abutment placement is visually more important on shorter bridges than on longer bridges, since an observer is more likely to view a short bridge in its entirety. Shorter abutments placed farther up on the slope widen the opening below the bridge and allow a more inclusive view of the landscape beyond. Taller abutments placed closer to an undercrossing roadway more strongly frame the opening and create a gateway effect. Passage through the bridge seems more of an event.

The abutment placement also influences the attractiveness of the space below the bridge for pedestrians. The abutment needs to be set back far enough to allow for a decent sidewalk width and shaped to avoid niches and offsets that might become hiding places or maintenance headaches.

**FIGURE 3.19** The substructure for this high-level crossing with slender piers is virtually transparent. Meadows Parkway over Plum Creek, Castle Rock, Colorado (Sourcebook 3.6). (From AASHTO, Bridge Aesthetics Sourcebook, American Association of State Highway and Transportation Officials, Washington, DC, 2010. With permission.)
3.6.5 Superstructure Shape (Including Parapets, Overhangs, and Railings)

The superstructure elements such as deck overhangs, parapets, and railings establish and enhance the form of the structural members. The shapes of these elements and the shadows they cast will strongly influence the aesthetic interest of the structure. For example, the overhang dimension between the edge of the bridge deck and the girder fascia can range between two extremes.

- A wide overhang can create a deep shadow. When used in conjunction with a thin deck slab line and a relatively transparent barrier, the bridge is perceived as being slender and lighter.
- A narrower overhang will put the face of the parapet closer to the face of the fascia girder, making them look like one surface and making the superstructure seem thicker.

Railings and parapets are among the most visually prominent elements of a bridge (Figure 3.22). They are located at the highest point, are usually visible from a distance, and are the bridge components that are closest in proximity to drivers and pedestrians. From a cost perspective, modifications to railings and parapets
are often less expensive than modifications to girders or other bridge components. Thus improvements to the railings and parapets can be a cost-effective way to improve the appearance of a bridge (Sourcebook, p. 21).

3.6.6 Pier Shape

The pier shape refers to the form and details of the piers. From many viewpoints, particularly those at oblique angles to the structure, the shapes of the piers will be the most prominent element of the bridge. The majority of piers for many bridges are structural frames consisting of circular or rectangular columns with a cap beam that supports the superstructure girders. They look like an assembly of different parts rather than a unified form. Improving their appearance requires integrating the parts by, for example, aligning exterior columns with the outside end of the cap. This eliminates the cap cantilevers, integrates the columns with the cap and thus simplifies the overall appearance of the pier (as shown in Figure 3.22).

A major improvement can be obtained by integrating the pier cap within the plane of the superstructure. This type of cap is commonly used for concrete box girder bridges, and is an important reason for their visual appeal. It can also be used on other bridge types, such as steel plate girders. With this design the pier cap is invisible. The pier appears much simpler because the transverse lines of the cap are eliminated (see Figure 3.14). This change is particularly helpful on skewed bridges, where the length of a dropped pier cap makes it a sizable and distracting element.

3.6.7 Abutment Shape

As we saw in the section on abutment placement, abutments may become visually massive structures or practically disappear. Much depends on the height of the abutment together with the grading around it. Mid height and full height abutments create large surfaces that strongly influence how the bridge is perceived. Abutment shapes are typically more important visually on shorter bridges than on longer bridges, since an observer is more likely to view a short bridge in its entirety. For structures involving pedestrians, either on the bridge or below it, the provisions made for them at the ends of the bridge can be among the most memorable aspects of the structure for them.

Abutments may also have an important symbolic function, as these are the points where travelers begin and end their passage over a bridge. The abutment shape and/or elements placed on it can also be used to emphasize the bridge as a gateway to communities, parks, or other significant places (as in Figure 3.20). These elements need to be visually consistent with the bridge itself and large enough to have an effect when seen at the distances from which the bridge is usually observed. This is particularly necessary for elements

FIGURE 3.22  This pedestrian fence uses standard chain link, but the arched top edge, carefully designed details and distinctive color improve the appearance of the bridge without a significant increase in cost, I-235 Reconstruction, Des Moines, Iowa (Sourcebook 4-24). (From AASHTO, Bridge Aesthetics Sourcebook, American Association of State Highway and Transportation Officials, Washington, DC, 2010. With permission.)
that will be seen from a multi-lane high speed roadway. Such elements will need to be on the order of 50 feet high and at least 10 feet wide in order to be noticed at all (note height of abutment feature in Figure 3.15).

3.6.8 Color

Color has a long history of application on bridges because of its strong visual impact at a low cost. Color, or lack thereof, will influence the effect of all the decisions that have gone before. It provides an economical vehicle to add an additional level of interest. The colors of uncoated structural materials as well as coated elements and details need to be considered.

Since bridges are almost always relatively small elements within the visual field of which they are a part, it is necessary to select color in relation to the surroundings. This can be done by means of colored photographs taken at various times of the day and at various seasons, or, better yet, by actually going to the site. There are several plausible strategies outlined in the Sourcebook (p. 27), such as

• Integrate the bridge into the surrounding landscape by selecting colors similar to nearby vegetation, rock formations, and so on. Many designers select shades of green, red, or brown with this in mind.
• Create a strong identity for the bridge by visually contrasting it with its surroundings. This may be particularly appropriate in the case of sites with little vegetation where the bridge can be viewed from a distance. The Golden Gate is a famous example of this strategy.
• Identify the bridge with a geographic region or culture through the use of colors that will form this association. For example, New Mexico has a tradition of coloring bridge surfaces to relate to its distinctive Native American culture.

Selection of a strategy should be an outgrowth of the vision statement. It is foolhardy to select a color for something as large as a bridge or wall by looking at Federal Standard color chips in the office. At the very least, take the chips out to the site. Better yet, require the contractor to provide large (at least 4’ x 8’) sample panels on site on which candidate colors can be tested and a final selection made (Sourcebook, p. 27).

Color choices are complex decisions requiring specialized technical knowledge and refined visual sensibility. Architects and especially landscape architects frequently make color selections for outdoor environments. They can be helpful consultants.

3.6.9 Texture, Ornamentation, and Details

Ornamentation, texture, and details are elements that can add visual interest and emphasis. Structural elements themselves, such as stiffeners and bearings, can serve this function. Indeed, traditional systems of architectural ornament started from a desire to visually emphasize points where force is transferred, such as from beam to column through an ornamental capital.

Patterns of grooves or insets and similar details are other examples. Surface texturing, often produced by formliners, can be used to create patterns, add visual interest and introduce subtle surface variations and shading, which in turn soften or reduce the scale or visual mass of abutments, piers, and walls (Sourcebook, p. 27).

Ornament is best used sparingly. Less is generally better than more (Figure 3.23). As bridge engineer J.B. Johnson put it in 1912:

In bridge building...to overload a structure or any part thereof with ornaments... would be to suppress or disguise the main members and to exhibit an unbecoming wastefulness. The plain or elaborate character of an entire structure must not be contradicted by any of its parts.

Above all, do not use false arches or other fake structural elements as cosmetic “make up” to disguise an inappropriate or uninteresting design. Aside from requiring additional costs to construct and maintain, adding false structure will rarely improve a design and is often viewed as extraneous clutter.
Using formliners that mimic other materials is particularly tricky. Using one material to simulate another creates a type of visual dishonesty that is always a problem in aesthetics. Some highlights from the Sourcebook (pp. 27 and 28):

- When using formliners to simulate another material, avoid suggesting a material that would not be utilized in that application. For example, stone texturing on the surfaces of a cantilevered pier cap creates visual disharmony because a cantilever could not in fact be constructed with stone.
- When simulating traditional material, such as stone or brick, formliner-created surfaces should be made as realistic as possible. For example, “mortar lines” should line up when the pattern turns a corner, just as they would on real masonry. Use color in addition to texture to assist in the simulation by, for example, differentially staining the “stones” of a simulated stone wall to reflect the variations in color found within and between actual stones (see Figure 3.20).
- The use of horizontal lines in patterns requires special attention to avoid conflict with the profiles of roadways and bridges, which are rarely straight or level. For example, the lines produced on a bridge parapet by a formliner with a strong horizontal pattern are likely to conflict with the top and bottom edges of the girder and/or parapet.
- Consider the speed and position of the observer. When texture is viewed up close and at slow speeds, the depth of relief and the details of the pattern can be fully appreciated. However, on a bridge over a rural freeway, finely textured surfaces and complicated patterns will not be perceptible to travelers moving at high speeds. In freeway conditions features with minimum dimensions of 3 or 4 inches and textures with a minimum relief of 2–3 inches are necessary to create a visible effect.
- The use of textures needs to be closely monitored in construction, since poor detailing or construction can severely affect the appearance. For example, the contractor should be required to align features of formliners between one formliner panel and the next so that no construction joints interrupt the overall pattern.

Utilitarian details, such as electrical conduits or bridge drainage, often create major and unforeseen visual impacts on bridge appearance (Figure 3.24). Every visible element of the bridge, no matter how utilitarian or seemingly inconsequential, must be anticipated and integrated into the concept. If it is visible, it must be designed to be seen.
3.6.10 Lighting, Signing, and Landscaping

Though not actually part of the structural system, these elements can have great influence on the aesthetic impression a bridge makes.

3.6.10.1 Lighting

Roadway lighting is governed by the illumination requirements of the owner, but still requires multiple choices of pole type, height, and spacing as well as fixture and lamp type. Although an individual pole may not seem to be much of a visual element, a row or array of them on a bridge will exert surprising influence on the appearance of the bridge. Close coordination with the lighting engineer is necessary to make sure that this influence is positive.

The most important step is to simplify the array. For example, place all of the poles on either the median or the sidewalk, but not both. Another goal is to coordinate the pole location with bridge features by, for example, lining the poles up with pier locations or, at the very least, centering the longitudinal pattern of pole placement at the midpoint of the bridge.

Lighting of the bridge itself is a good way to draw attention to the bridge and make it an asset to the nighttime environment (Figure 3.25). Such lighting must be sensitive to motorists, pedestrians, boaters,
and other users. It should be selected and located to enhance and highlight the structure, yet minimize glare and unnecessary distraction. The lighting must respond appropriately to the context, both in terms of surrounding structures and environmental conditions. Considerations of impact on wildlife and light pollution in the night sky should be weighed together with those of aesthetics. Aesthetic lighting design for bridges requires specialized technical knowledge and refined visual sensibility beyond the capabilities of many lighting engineers. It is a consulting specialty of its own. Engineers should consider including such specialists when developing an aesthetic lighting design.

3.6.10.2 Signing

There are two types of signs mounted on bridges. The first and most common is where the bridge itself is used as a support for a sign serving the under passing roadway. The second is when a sign structure is erected on a bridge to serve the bridge’s own roadway. This is often necessary on long viaducts and ramps. In both situations the sign usually blocks and/or complicates the lines of the bridge itself. The result is rarely attractive. Thus, the most desirable option is to keep signing off bridges. Saddling a bridge with an unattractive sign or sign structure defeats the purpose of creating an attractive aesthetic bridge design. The first goal should be to seek alternate locations for signs away from bridges. This will inevitably mean more specialized structures for the signs themselves (Sourcebook, p. 32).

The Sourcebook makes suggestions for what to do when signs must be mounted on bridges.

3.6.10.3 Landscaping

Landscaping is defined here to include planted areas and hardscape: stone, brick, or concrete paving, often colored and/or patterned, used primarily for erosion control or pedestrian circulation. Landscaping should enhance an already attractive structure. It should not be relied upon to cover up an embarrassment or hide some unfortunate detail. Conversely, it should not be allowed to grow up to hide some important feature that is crucial to the visual form of the bridge. Landscaping can be a more economical and effective way to add richness and interest to a design rather than special surface finishes or materials (see Figure 3.26). For example, a large, plain concrete abutment can be effectively enhanced by well-chosen landscaping (Sourcebook, p. 32).

FIGURE 3.26 Well-integrated landscaping at a bridge over I-5 in Olympia, Washington.
3.7 The Engineer’s Challenge

The design guidelines just outlined will improve most everyday structures, but they will not guarantee structural art. There are no hard and fast rules or generic formulas that will guarantee outstanding visual quality. Each bridge is unique and should be studied individually, always taking into consideration all the issues, constraints, and opportunities of its particular setting or environment. Nevertheless, observing the successes and failures of other bridges and using design guidelines can improve an engineer’s aesthetic abilities and help avoid visual disasters.

Society holds engineers responsible for the quality of their work. No one has the right to build an ugly bridge. Bridge designers must consider visual quality as fundamental a criterion in their work as performance, cost, and safety. Engineers can learn what makes bridges visually outstanding and develop their abilities to make their own bridges attractive. They can achieve outstanding visual quality in bridge design without compromising structural integrity or significantly increasing costs.

The ideal bridge is structurally straightforward and elegant, providing safe passage and visual delight for drivers, pedestrians, and people living or working nearby. It is an asset to its community and its environment. The engineer’s challenge is not just to find the least costly solution. The engineer’s challenge is to bring forth elegance from utility: we should not be content with bridges that just move cars and trucks and trains; they should move our spirits as well.

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Bibliography

As described in the Introduction, much more discussion of the guidelines in this chapter and much additional discussion of related topics are contained in following:


Much more detail and background on the discussion and ideas in this chapter can be found in the following book by the author:

The following should be part of the reference library of any engineer interested in bridge aesthetics, in alphabetical order:

A complete discussion of the topics in Sections 3.2, 3.3, and 3.34 can be found in the following three books by David Billington: