16.1 Introduction

The Cooper-pair transistor (CPT) is a three-terminal superconducting device composed of a mesoscale superconducting island connected to drain and source leads via ultrasmall (~100 nm) Josephson junctions (see Figure 16.1). Due to its small size, the energy to add a single Cooper pair to the island can be large compared with the typical temperatures at which similar quantum circuits are operated, \( T \ll 100 \text{ mK} \). Because of this, the transport properties of the device can be strongly dependent on the polarization charge presented by a capacitively coupled gate electrode and operated as a transistor with high input impedance. As an electrometer, it is useful as it can be operated with very little dissipation and should minimally influence the system that is being measured. As we will see, its similarity to another quantum circuit, the Cooper-pair box (CPB), marks it as a useful device to study decoherence in superconducting quantum bits (qubits) due to unpaired electrons.

While the CPT is topologically identical to the normal-state single-electron transistor (SET), it is operated in the superconducting state, and the superconducting nature of the device modifies the transport properties in several important ways. For our purposes, we also distinguish the CPT from the superconducting single-electron transistor (SSET) as a mode of operating this particular double-junction device. As a CPT, it is operated on the supercurrent branch near zero voltage in the coherent Cooper-pair transport regime, while as an SSET, it is operated at finite voltage bias and can include several incoherent hybrid quasiparticle/Cooper-pair tunneling processes.

16.2 Theory of Operation

As shown in Figure 16.1a, the device is composed of two Josephson junctions and can, at its core, be regarded as a single Josephson-like element with an added internal charge degree of freedom. This modification is important as it allows the CPT to act as a tunable Josephson junction that is sensitive to charge. In this spirit, it is important that we begin with a conceptual understanding of a single Josephson junction.

16.2.1 The Single Josephson Junction

A superconducting Josephson junction is composed of a thin insulating barrier connecting two superconducting electrodes. In 1962, Brian Josephson made the remarkable statement that these junctions should support a supercurrent, that is, a dissipationless current at zero voltage (Josephson, 1962). This is a consequence of the finite quantum mechanical tunneling probability that arises from the spatial overlap in the superconducting order parameter on either side of a thin junction barrier. Josephson encapsulated this effect in the following equations that relate the current and the voltage to the dynamics of the phase difference across the junction:

\[
I = I_0 \sin \delta \quad (16.1)
\]

\[
V = \Phi_0 \Delta \quad (16.2)
\]

* We use the notational shorthand \( \Phi_0 = \hbar/2e \). This is simply the superconducting flux quantum divided by \( 2\pi \).
Taken at face value, these equations are the constitutive equations defining the Josephson junction as an inductor-like non-linear circuit element. $I_0$ is a "critical current" beyond which the junction cannot support a supercurrent and the junction switches to a finite voltage branch in the current–voltage ($I$–$V$) characteristic. It can be computed from the low-temperature form of the Ambegaokar–Baratoff relation* (Ambegaokar and Baratoff, 1963).

$$I_0 = \frac{2}{e R_N} \frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2} K\left(\frac{\Delta_1 - \Delta_2}{\Delta_1 + \Delta_2}\right), \quad (16.3)$$

where

- $\Delta_1$ and $\Delta_2$ are the superconducting gap energies on either side of the insulating barrier.
- $K$ is the complete elliptic integral of the first kind.

If $\Delta_1 \sim \Delta_2$, $K \to \pi/2$ and

$$I_0 \simeq \frac{\pi}{e R_N} \frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}. \quad (16.4)$$

The most commonly used superconductor for CPTs (and SETs) is aluminum, $\Delta_{Al} = 180$–$250$ μeV. Typical junction sizes are $\sim 100$ nm × $100$ nm that yield junction resistances in the range of $10 \, \Omega < R_j < 100 \, \Omega$. With these parameters, the critical current ranges from $1$ to $100$ nA.

The presence of the phase $\delta$ across the junction in the equations marks it as an inductor-like element that can store magnetic flux. Following this reasoning, one can derive a Josephson inductance by finding the scaling coefficient between the voltage and the time derivative of the current,

$$L_I(I) = \frac{L_{10}}{\cos^2 \delta} = \frac{L_{10}}{\sqrt{1 - (1/I_0)^2}}, \quad \text{where } L_{10} \equiv \frac{\Phi_0}{I_0}. \quad (16.5)$$

One can also define a Josephson energy that is a function of the phase difference $\delta$,

$$E_I = \Phi_0 I_0 \left| \cos \delta \right| = E_{10} \left| 1 - \left( \frac{1}{I_0} \right)^2 \right|, \quad \text{where } E_{10} \equiv \Phi_0 I_0. \quad (16.6)$$

This can be interpreted as the magnetic energy that is stored in the element. This energy has a deeper significance as the coupling energy between Cooper-pair charge states on either side of the junction and will be important in calculating the energy bands of the CPT. For CPT Josephson junctions, the equivalent inductances are typically $\sim 10$–$30$ nH and the energies $20$–$80$ μeV (0.2–1 K in temperature units).

### 16.2.1.1 The $I$–$V$ Curve and Phase Dynamics

#### 16.2.1.1.1 The $I$–$V$ Curve

The CPT is operated on the so-called “supercurrent branch” of the $I$–$V$ curve at (or near) zero voltage. The two measurement modes that we discuss in this chapter involve measuring the...
switching current $I_{sw}$ at which the CPT switches from the supercurrent branch to the finite-voltage branch as well as measuring the effective inductance $L_i$ of the CPT when biased at zero current. While the small size of the junctions and internal charge degree of freedom will affect these parameters significantly, it is worthwhile to first look at the single junction case and understand how the dynamics of the phase determine the structure of the $I$–$V$ curve (see Figure 16.2a).

### 16.2.1.1.2 The Resistively and Capacitively Shunted Junction (RCSJ) Model

A more realistic model of a single Josephson junction includes a parallel shunt resistance and capacitance (see inset in Figure 16.2a). The shunt resistance $R$ encapsulates the resistance of the junction to normal/quasiparticle currents through the junction, while the capacitance $C$ is simply the physical capacitance that arises from having two planar electrodes overlapping with an insulating barrier in between. In the junctions comprising a CPT, the intrinsic $R > 10 \Omega$ and is considered effectively “unshunted” since $R$ is much bigger than the impedance presented by the Josephson inductance. As noted previously, the Josephson tunnel element is a nonlinear inductance but, for small phase excursions, it looks like a linear inductor. The circuit model presented then looks very much like a parallel $LCR$ circuit or damped simple harmonic oscillator with a natural frequency

$$\omega_n = \frac{1}{\sqrt{L_i/C}} = \sqrt{\frac{2eI_0}{\hbar C}}.$$  \hspace{1cm} (16.7)

This is usually called the “plasma frequency” and has the significance of simply being the frequency of small oscillations for a Josephson junction.

Although this model can be used to describe the intrinsic Josephson junction dynamics, it is also useful in determining the dynamics when the junction is placed in an arbitrary measurement circuit. If, for instance, we want to current bias the junction, we might add a voltage source $V_j$ in series with a resistor $R_b$ and construct a Norton equivalent that is simply an ideal current bias $I_b = V_j/R_b$ in parallel with an output impedance $R_b$. In this case, we can roll $R_b$ into the shunt $R$ in our $RCSJ$ model. With this simple circuit equivalent, we can derive a set of first-order differential equations defining the equations of motion for the phase by writing down Kirchoff’s equations for the currents in each branch. We can write this system of equations as a single second-order differential equation

$$\ddot{\delta} + \frac{1}{RC} \dot{\delta} + \sqrt{\frac{2eI_0}{\hbar C}} \left( \sin \delta - \left( \frac{I_b}{I_0} \right) \right) = 0.$$ \hspace{1cm} (16.8)

For small $\delta$ at zero current bias, this describes the motion of a particle with coordinate $\delta$ oscillating in a quadratic potential with a friction term, viz., it’s a damped simple harmonic oscillator (and is consistent with our earlier statement that $L_i$ is linear for small oscillations). At finite current bias, however, this equation describes the motion of the particle in a tilted sinusoidal (or washboard) potential.\footnote{This vast oversimplification has its caveats and must be amended to account for stray capacitance in the bias lines and any other reactances relevant in the frequency range of our junction dynamics (typically $\omega < 50$ GHz for small aluminum junctions).} We know from Equation 16.2 that in order for the junction to support a finite dc voltage across

---

**FIGURE 16.2** (a) Schematic $I$–$V$ curve/characteristic for an ultrasmall Josephson junction. The ideal $T = 0 I$–$V$ is shown in light gray, switching to the voltage state at $I_s = I_c$. For $T > 0$, thermal fluctuations can drive the system to the voltage state at much lower bias currents in ultrasmall ($A_m < 0.01 \mu m^2$) junctions, as shown in black. (b) $I_b < I_c$. The phase particle is trapped in a local minima of a washboard potential and $\langle \delta \rangle = 0$ and no voltage is developed across the junction. (c) $I_b = I_c$. The washboard potential tilted at exactly the critical slope necessary to “release” the phase particle from its local minimum and enter a free-running state at finite voltage, $V = \phi_0 \langle \delta \rangle > 0$. (d) $I_b < I_c$. Phase diffusion. In ultrasmall junctions, thermal fluctuations can drive the phase to evolve in a 1D random walk of phase steps $\Delta \phi = 2\pi$. Since the washboard is tilted, this results in a slow diffusion in one direction, $V = \phi_0 \langle \delta \rangle > 0$, although it is still considered to be on the “supercurrent branch.”
it, the particle must have some net motion in the phase, that is, \( \dot{\varphi} \neq 0 \). Since the particle is trapped at a fixed phase when \( I_b < I_w \) we have a well-defined phase and the junction remains in the supercurrent state (see Figure 16.2b). By applying a bias current, we tilt the washboard and, when \( I_b = I_w \), the particle is tipped out of its well and begins to run away (Figure 16.2c). In this free-running state, the time derivative of the phase has both an ac and a dc component corresponding to ac and dc voltages across the junction. In a more realistic treatment, \( I_b \) includes a stochastic contribution from thermal fluctuations due to the dissipation in the environment and so the current at which the junction switches to the free-running state is lower than \( I_b \) since thermal excitations can drive the phase particle over the potential barrier prematurely. Because of this, the measured current is distributed according to the statistics of a thermally excited (Kramers) escape process (Fulton and Dunkleberger, 1974). We will differentiate this switching current \( I_{sw} \) from the ideal Ambegaokar–Baratoff value \( I_0 \) and remember that it is actually a statistical variable.

If we reduce the current bias in the free-running, finite voltage state, the particle velocity begins to slow down due to the dissipation in the circuit. Once the dissipation rate can compensate for the ac power generated by the free-running voltage oscillations, the particle "retraps" into a minima in the washboard potential and the junction returns to the zero-voltage supercurrent branch. The current at which this happens, \( I_{ret} \), the "return current" is an indication of the amount of damping or dissipation in the environment at the free-running oscillation frequency (Tinkham, 2004). Roughly, the closer to zero \( I_{ret} \) is, the less damping there is. Although the CPT is designed and operated in the unshunted, hysteretic regime, in many practical dc SQUID magnetometers, the junctions are intentionally shunted with damping resistors to eliminate hysteresis in the \( I-V \) measurement permitting stable finite voltage operation.

### 16.2.1.1.3 Concerns Specific to Small Junctions

While the picture above is the standard way to look at relatively large junctions (area \( \geq 1 \mu m^2 \)), the junctions in CPTs are usually about a factor of 100 smaller and have much smaller capacitances.

The main consequence of this is that the observed switching currents can easily be smaller than \( I_0 \) by an order of magnitude. One way to understand this is to return to the harmonic oscillator analogy. We can see that \( \omega_0^2 \approx 1/C \), so \( C \) is an effective mass for our "phase particle." In CPT junctions, this means that the equivalent phase particle can be relatively light compared with its larger junction cousins and so thermal fluctuations, even at 100 mK, can have a more significant effect, kicking the particle into the free-running state much more easily. This has the end effect of reducing \( I_{sw} \) far below \( I_0 \) and also widening the distribution of \( I_{sw} \) (Figure 16.2a).

A more problematic effect of the light phase particle mass is the fact that the same thermal fluctuations can "kick" the phase particle from well to well in the washboard potential in a process called "phase diffusion" (Kautz and Martinis, 1990). At zero current bias, this results in a one-dimensional random walk in the phase with zero mean. At finite current, the tilt in the washboard biases this diffusive motion in one direction, such that the mean phase velocity \( \langle \dot{\varphi} \rangle \) can be positive (negative) at positive (negative) bias (see Figure 16.2d). The result of all of this is that at finite currents below the switching current, we often see a small finite voltage. This slope is fairly uniform near zero current and gives the supercurrent branch a slight resistance contribution that can correspond to 10 s or even 100 s of \( \Omega \).

The downside of this behavior is that the junction or CPT is not truly dissipationless.

For electrometer operation, we want \( I_{sw} \) to have as narrow a distribution as possible since it is the quantity that will vary with applied gate voltage/polarization charge. It has been demonstrated, however, that one can narrow this distribution and also increase \( I_{sw} \), so that it is closer to \( I_b \) by fabricating a larger capacitance (a few picofarad) in the leads shunting the junction to "weigh down" the phase particle (Joyez et al., 1994; Joyez, 1995).

### 16.2.2 Coulomb Blockade

While the above discussion of a single junction will eventually be relevant to the CPT measurements we are interested in, we have not yet seriously considered the role of the internal charge degree of freedom in determining the overall critical current and Josephson inductance in this device. This is, after all, the knob that turns the CPT into a transistor.

We can qualitatively recognize the role of the island charge by first determining the energy required to charge the CPT island with a single electron charge \( e \),

\[
E_C = \frac{e^2}{2C_\Sigma},
\]

where

\[
C_\Sigma = C_{J1} + C_{J2} + C_g + C_{\text{stray}}.
\]

Here, \( C_\Sigma \) is the total capacitance as seen by the island, and while it includes the junction and gate capacitances, it also includes a "catch-all" term, \( C_{\text{stray}} \), that is meant to include any contributions from unintentional coupling to ground. In practice, \( C_{\text{stray}} \ll C_\Sigma \) and can usually be ignored. The energy \( E_C \) is called the single-electron Coulomb blockade energy (Devoret and Grabert, 1992).

For typical junction capacitances \( C_{J1,2} \lesssim 1 \text{ pF} \) and gate capacitances of 0.01–1 \text{ pF}, \( E_C \) can be on the order of \(-100\)–400 \mu\text{eV}. In temperature units, this temperature is \( 1\text{–}5 \text{ K} \) and is therefore a significant energy scale compared with typical operating temperatures. As noted previously, the Josephson energies of these junctions are roughly on par with the Coulomb blockade energies, so we can qualitatively assume that both energy scales will play a role in the transport.
The Cooper-Pair Transistor

16.2 Band Structure of the CPT

Since we have two junctions in series (Figure 16.3), we must define separate phases for each junction \( \delta_i \) and \( \delta_e \). We can now be more precise about the role of the charging energy by looking at the Hamiltonian for the CPT and how the contributions couple to the phases and the island charge \( n \),

\[
H = H_1 + H_2 + H_{CB} + H_{QP},
\]

(16.11)

\( H_{1,2} \) are the contributions due to the Josephson energy coupling the charge states on either side of the junctions, while \( H_{CB} \) is the Coulomb blockade contribution. Explicitly, we have

\[
H_{1,2} = -E_J \cos \delta_{i,2},
\]

(16.12)

\[
H_{CB} = \frac{E_C}{4} (n - n_e)^2.
\]

(16.13)

Here we have used the reduced gate voltage \( n_e = C_s V_g / e \) that is the polarization charge applied by the gate in electron units. \( H_{QP} \) is the contribution from quasiparticle excitations in the leads and on the island. Physically, they correspond to unpaired electrons/holes that can be introduced to the system via thermal excitations or some nonequilibrium process that can break Cooper pairs. While this is an important contribution, we ignore it now to facilitate the discussion.

We can write down a matrix equivalent of Equation 16.11 in the basis of island charge states \( |n \rangle \), noting that the Josephson coupling terms that link Cooper-pair charge states on either state of the junction also link “adjacent” Cooper-pair charge states \( |n \rangle \to |n \pm 2 \rangle \).

Defining “external” and “internal” phases \( \delta = \delta_i + \delta_e \) and \( \theta = \delta_i - \delta_e \) and setting \( \theta = 0 \) (equivalent to assuming that the island has negligible inductance), we can write \( H \) in the charge basis. For example, the Hamiltonian spanning the Cooper-pair island charge states \( |\pm 2 \rangle, |0 \rangle, \) and \( |2 \rangle \) is

\[
H = \begin{bmatrix}
E_c(-2n_e)^2 & -\frac{1}{2}(E_x e^{-i \delta_{i,2}} + E_x e^{i \delta_{i,2}}) & 0 \\
-\frac{1}{2}(E_x e^{-i \delta_{i,2}} + E_x e^{i \delta_{i,2}}) & E_c(-n_e)^2 - \frac{1}{4} (E_x e^{-2i \delta_{i,2}} + E_x e^{2i \delta_{i,2}}) & 0 \\
0 & 0 & E_c(2n_e)^2
\end{bmatrix}.
\]

(16.14)

The eigenvalues for this matrix represent the energy bands and are functions of \( n_e \) and external phase \( \delta \). The choice of basis size (how big a matrix one really needs to write down) is dictated by the ratio of \( E_x / E_c \). If this ratio is small, charge is easily localized and the corresponding external phase fluctuations are large, so the device has a weak effective Josephson energy. In this case, the charge basis may be described well by a small subspace of \( \{|0 \rangle \} \) spanning only a few charge states near \( |0 \rangle \). If \( E_x / E_c \geq 1 \), this approximation fails and one must write down a larger matrix spanning a bigger subspace of \( \{|0 \rangle \} \). In this limit, the charge on the island is no longer as well defined as its quantum mechanical state is now described by a linear superposition of several charge states. Likewise, fluctuations in the CPT external phase are then small and the CPT begins to look like a single Josephson junction.

16.2.3.1 The Uncertainty Principle at Work

We can numerically diagonalize Equation 16.14 to calculate the eigenvalues at various \( n_e \) and \( \delta \). Figure 16.4 shows the resulting eigenenergy surfaces corresponding to the ground and first excited state energies, \( \epsilon_0 \) and \( \epsilon_1 \). The quantities that we want to measure, \( I_{\|}(n_e) \) and \( I_{\perp}(n_e) \), are defined by the first and second derivatives of these surfaces in the phase. The size of this modulation is a direct consequence of how well the CPT island localizes Cooper pairs, \( 2 \epsilon_0 \), through the uncertainty relation between charge number and phase, \( \Delta n \Delta \theta \geq 1/2 \). In this sense, the CPT is a strange charge-based device because it relies on the competition between number and phase uncertainty. This is in contrast to the SET or SET where \( E_c \) is entirely dominant. In the CPT, we would like \( E_c \) to be big but would also like \( E_J \) to be comparable in magnitude, so that the switching current is not so small that it is difficult to measure well. Similarly, rf measurements of the Josephson inductance (described later in this review) become easier at bigger \( E_J \) as larger signal powers can be used while biased on the supercurrent branch.

16.2.3.2 Calculation of the Critical Current Modulation

The critical current of the CPT can now be determined from the eigenenergies calculated above. By restricting to the ground state band \( \epsilon(n_e, \delta) \), we can compute the critical current (Joyez, 1995)

\[
I_0(n_e) = \frac{1}{\varphi_0} \left. \frac{\partial \epsilon(n_e, \delta)}{\partial \delta} \right|_{\delta_{\text{max}}},
\]

(16.15)

* The phases \( \delta_i \) and \( \delta_e \) bear a strict conjugate correspondence to the number of Cooper pairs having “flown” through junctions 1 and 2. This discussion closely follows that given by Joyez (1995) and a more extensive discussion of choosing appropriate quantum variables appears there.

† For consistency with Joyez (1995), \( n \) in \( |n \rangle \) refers to the single electron number and not Cooper pair number.
where $\delta_{\text{max}}$ is the phase maximizing the derivative. In Figure 16.5, we show the calculated critical currents for several ratios of $E_J/n_e/E_C$. Note that the calculated currents are $2e$ periodic in the applied gate charge which is the result of the Josephson coupling of Cooper-pair charge states. In practice, measured switching currents in CPTs are usually significantly smaller than the calculated value due to small-junction effects noted above. However, the total magnitude of the modulation can be on the order of 10–100 nA for practical aluminum device parameters and is easily measured.

16.2.3.3 Calculation of the Effective Inductance Modulation

The ground state energy surface $\epsilon_0(n_g, \delta)$ corresponds to an effective Josephson energy that is a function of charge as well as phase. When the ratio $E_J/E_C \lesssim 1$, the phase dependence, while still $2\pi$ periodic, gets “peaky” near odd-integer $n_g$. Despite this, one may still define a Josephson inductance in a similar manner (Sillanpää et al., 2004; Naaman and Aumentado, 2006b), that is,

$$L_I(n_g) = \frac{1}{\phi_0^2} \frac{\partial^2 \epsilon_0(n_g, \delta)}{\partial \delta^2} \bigg|_{\delta=0}. \tag{16.16}$$

This is similar to Equation 16.5, but since the $\epsilon_0(n_g, \delta)$ is not strictly sinusoidal, we do not get the exactly the same result. For typical aluminum device parameters, the inductance modulation can be 10–100 nH and, like the switching current, is easily measured.

16.2.4 Quasiparticle Poisoning

Although the “quasiparticle” excitations that are talked about in superconductivity do not explicitly bear a well-defined charge, they do represent the screened excitations presented by an unpaired electron that might exist due to the breaking of a Cooper pair. A quasiparticle can tunnel onto a CPT island by “undressing” itself of its screening cloud and tunneling through a junction barrier alone. At this point, this additional charge presents a full electron charge offset to the CPT island. Since the CPT switching current and inductance are nominally $2e$ periodic in the gate charge, the addition of an extra electron can present a significant change in the way the CPT is operated, since it fluctuates the charge by $e$. In the literature, this problem was first discussed in terms of island “parity,” but more recent work has favored the more colorful term “quasiparticle poisoning.” The most important thing about quasiparticle poisoning for electrometry is that it is a stochastic process whose dynamics
can happen on timescales comparable to our measurement time, so its effect on the resulting measurement must be well understood. That being said, quasiparticle poisoning has proven to be an interesting problem in itself, and the CPT has been very useful in its study.

To see how quasiparticles enter the picture, we now include the Hamiltonian \( H_{\text{qp}} \) in our description of the CPT,

\[
H_{\text{qp}} = \sum_j \varepsilon_j \gamma_j \gamma_j^\dagger.
\]  

(16.17)

The \( \gamma_j, \gamma_j^\dagger \) are annihilation/creation operators for quasiparticle excitations in the superconductor (Tinkham, 2004), while \( \varepsilon \) is the energy of the excitation. In our case, this can be \( \Delta_i \) or \( \Delta_j \), remembering that the island and leads can have different gap energies. These quasiparticles are usually taken to be thermally generated and therefore have an exponentially small probability of existing at low temperatures. The expression for the quasiparticle density is (cf., Shaw et al., 2008),

\[
n_{\text{qp}} = D(\varepsilon_i)\sqrt{\frac{2\pi k_B T}{\Delta_i}} \exp\left(-\frac{\Delta_i}{k_B T}\right).
\]  

(16.18)

\( D(\varepsilon) = (2.3 \times 10^4 \mu \text{m}^3 \cdot \mu \text{eV}^3) \) is the density of states at the Fermi energy and \( \Delta \) is the superconducting gap energy in either film 1 or 2. Plugging in \( \Delta \approx 200 \mu \text{eV} \) (aluminum) and \( T = 100 \text{mK} \) gives us \( 2 \times 10^{-4} \mu \text{m}^{-3} \). This is a very low number considering the volume of a generic CPT island and leads; yet we know, experimentally, that the actual quasiparticle density is typically \( 10^{-1000} \mu \text{m}^{-3} \) (Mazin, 2004; Shaw et al., 2008). The disparity in the thermal prediction versus the experimentally measured numbers is the first indication that the source for the quasiparticles we see at low temperatures is distinctly nonthermal in nature. As yet, no one has determined the source of these nonequilibrium quasiparticles and we are stuck with the problem of understanding them.

### 16.2.4.1 Energetics of the Nonequilibrium Quasiparticle Poisoning Process

Since we are forced to work in an environment filled with nonequilibrium quasiparticles, we must figure out how to include them in the band picture that we constructed above. We have three states of interest (see Figure 16.6) (Aumentado et al., 2004):

- **State 0** No quasiparticles in or near the CPT island. *Even parity.*
- **State \( \ell \)** A quasiparticle is in the leads, in the vicinity of the CPT island. No quasiparticles on the island. *Even parity.*
- **State \( i \)** A quasiparticle is on the CPT island. No quasiparticles in the leads near the island. *Odd parity.*

These states can all be described with the band structure derived in the previous section by offsetting the modulated energy surfaces by the superconducting gap energy, corresponding to where the quasiparticle lives, viz.,

\[
E_{0}(n_{\ell}) = \varepsilon_{0}(n_{\ell}, \delta = 0),
E_{\ell}(n_{\ell}) = \varepsilon_{0}(n_{\ell}, \delta = 0) + \Delta_{i},
E_{i}(n_{\ell}) = \varepsilon_{0}(n_{\ell} + 1, \delta = 0) + \Delta_{i}.
\]  

(16.19)

Here, we include the possibility of different gap energies in the leads and island, \( \Delta_i \) and \( \Delta_j \), respectively, and confine ourselves to \( \delta = 0 \). In principle, phase diffusion will smear these levels somewhat, but the average phase on the supercurrent branch will always be localized in the bottom of a potential well in the phase when biased near \( I_b = 0 \).

In Figure 16.7, we show the energy bands for two different pairs of island + gap energies using typical Coulomb, Josephson, and gap energies for an aluminum device. In the "type H" device (Figure 16.7a), the island gap is greater than the lead gap, and in the "type L" device, the reverse is true. Assuming that nonequilibrium quasiparticles are present, we can limit our discussion to \( E_{\ell} \) and \( E_i \). In the \( T = 0 \) limit, we expect the system to relax to the lowest energy state and the problem is reduced to whether the \( \ell \) or \( i \) state energy is smallest (denoted in Figure 16.7a and d by the black dotted trace). For this purpose, we introduce the energy difference:

\[
\delta E_{n_{\ell}} = E_{n_{\ell}} - E_{i}(n_{\ell}).
\]  

(16.20)

At \( T = 0 \), the sign of this quantity determines what state we are in, that is,

\[
\text{sgn}[\delta E_{n_{\ell}}] = \begin{cases} 
+1 & \text{even parity} \\
-1 & \text{odd parity}
\end{cases}
\]  

(16.21)

* A brief history detour. In early treatments of quasiparticle poisoning, all quasiparticles were assumed to be thermal in nature, so the important energy in the problem is the free energy required for transition from 0 to \( i \) state (Tsomini et al., 1992, 1993; Amar et al., 1994; Joyez et al., 1994; Tinkham et al., 1995). This is basically the process of breaking a Cooper pair with thermal fluctuations. Experimentally, this model was more or less verified early on, but the situation was complicated by the anecdotal evidence that this picture failed to explain the numerous unpublished experiments that showed poisoning at low temperatures. In the absence of nonequilibrium quasiparticles, the early free energy theories are still valid and, in any case, when the system is heated sufficiently \( T > 250 \text{mK} \), the thermal quasiparticle generation rate dominates over the nonequilibrium rate. For a good review of the early theory, see Tinkham (2004).
\[ \Delta_i = 115 \text{ eV}, \Delta_j = 250 \text{ eV}, \Delta = 220 \text{ eV} \]

Type H \( \Delta_i < \Delta_j \)

\[ E_C = 115 \text{ eV}, E_{\text{ion}} = 80 \text{ eV}, \Delta = 220 \text{ eV} \]

Type L \( \Delta_i > \Delta_j \)

\[ E_C = 115 \text{ eV}, E_{\text{ion}} = 80 \text{ eV}, \Delta = 250 \text{ eV} \]

**FIGURE 16.7** (a) Type H \( \Delta_i < \Delta_j \) energy bands. \( f \) state/odd parity (light gray), \( i \) state/even parity (medium gray), \( 0 \) state/even parity (black solid). Minimum energy state is denoted as the black dotted trace. Corresponding even (light gray) and odd (medium gray) state (b) critical currents and (c) effective inductance for a type H device. (d) Type L \( \Delta_i > \Delta_j \) energy bands. Corresponding even (light gray) and odd (medium gray) state (e) critical currents and (f) effective inductance for a type L device. Gray areas mark range in \( n_g \) where the CPT island is "trap-like," that is, a potential well for quasiparticles and the parity state is bimodal and can rapidly switch when coupled to thermal excitations.

\[ \delta E_{q1} \] is an effective potential barrier height for quasiparticles. When \( \delta E_{q1} \) is positive, the CPT island looks like a barrier, and when it is negative, it looks like a trap (Figure 16.8). In reality, the system is at finite temperature, so this qualitative picture must be amended to include the possibility of thermal excitations (phonons) coupling to the quasiparticles and exciting them out of the trap. Since this is a thermal escape process, we must characterize the system in terms of the average lifetimes of the poisoned and unpoisoned states, \( \tau_o \) and \( \tau_e \). Likewise, it is also useful to talk about these lifetimes as rates, \( \Gamma_e = \frac{1}{\tau_e} \) and \( \Gamma_o = \frac{1}{\tau_o} \). \( \Gamma_e \) is known as the "poisoning rate" and \( \Gamma_o \) is the "ejection rate."

Experimentally, at \( T \leq 250 \text{ mK} \), the poisoning rate \( \Gamma_o \) has little temperature dependence and can be anywhere between \( 10^3 \) and \( 10^5 \text{ s}^{-1} \) (Aumentado et al., 2004; Ferguson et al., 2006; Naaman and Aumentado, 2006b; Court et al., 2008b). The fact that this rate is constant in this temperature range points to the notion that the source is nonthermal in nature and the rate for Cooper-pair breaking in the leads determines the poisoning rate, \( \Gamma_o \sim \Gamma_{\text{eff}} \gg \Gamma_e \). The ejection rate \( \Gamma_e \) does, however, have a temperature dependence at these low temperatures since the process of kicking a quasiparticle out of a potential well is a thermal escape process. One can derive the probability of the
even parity state (no quasiparticle on the island) using detailed balance arguments (Aumentado et al., 2004),

\[ P_e = P_{\ell} + P_i = \frac{1}{1 + \beta_{0\ell} e^{-\Delta E_{\ell}/kT}}. \] (16.22)

The factor, \( \beta_{0\ell} = \Gamma_{0\ell}/(\Gamma_{0\ell} + \Gamma_i) \), accounts for the generation and recombination rates for nonequilibrium quasiparticles in the leads. What this all means is that there is not really any strictly quasiparticle-free zone for typical energies that we would use in practice.* In Figure 16.7, we denote the places where the CPT looks trap-like and it is apparent that both the L and H devices are affected. Even with the higher island gap, the H device can trap quasiparticles, albeit in a shallower potential, \( \Delta E_{\ell} \) than presented by the L device and also over a smaller range in \( n_e \).

It is interesting to note that if there is no source for nonequilibrium quasiparticles, \( \Gamma_{0\ell} = 0 (\beta_{0\ell} = 0) \) and \( P_e = 1 \) whether or not we have a barrier-like or trap-like profile. In other words, there would be no poisoning regardless of the relative gap energies. The fact that one sees poisoning easily in trap-like devices at low temperatures is the strongest indication that the quasiparticles in these systems are generated by some nonequilibrium process. Surprisingly, the temperature dependence above also predicts that the even state probability can actually be enhanced since, even though the poisoning rate might be fixed, the ejection rate can be increased by heating the CPT and giving quasiparticles energy to escape (Aumentado et al., 2004; Palmer et al., 2007).

This model of nonequilibrium quasiparticle poisoning was first roughly outlined and tested in Aumentado et al. (2004) using CPTs with engineered gap energy profiles and measured with the ramped current technique (see below). It was subsequently confirmed in several later experiments in both CPTs and CPBs using rf and dc techniques (Gunnarsson et al., 2004; Yamamoto et al., 2006; Palmer et al., 2007; Savin et al., 2007; Court et al., 2008b; Shaw et al., 2008).

16.3 Fabrication

16.3.1 Electron-Beam Lithography and Two-Angle Deposition

The majority of CPTs (and single-charge tunneling devices in general) are fabricated from evaporated aluminum using conventional electron beam lithography and electron-gun (or thermal) deposition techniques.

The most conventional method of fabrication is to first define the island and lead pattern of the CPT in a special double-layer e-beam resist stack that is spun onto a silicon substrate. The double-layer is constructed such that the upper “image” layer defines the pattern outline, while the lower “ballast” layer is meant to provide a vast “undercut” region underneath the image. This is usually achieved using resist for the lower layer that is much more sensitive to the e-beam exposure than the upper layer. When the pattern is written, the dosage required to generate the image overexposes the lower layer resist such that the pattern is much wider in the lower layer resist, forming an “undercut” region when the sample is developed (cf., Cord et al., 2006).

The undercut region allows us to tilt the substrate so that the impinging aluminum atoms can deposit an image through the image resist mask at an angle (Figure 16.9). After the first aluminum deposition, oxygen is released into the chamber that grows a thin (<1 nm) insulating Al₂O₃ layer on the surface of the aluminum. The substrate is then rotated to another angle and a second image is deposited onto the substrate such that it overlaps with the first oxidized image. In this manner, insulating junctions can be formed in situ without exposing the device to air. Subsequent processing removes the resist, so that it can be bonded up in a suitable measurement circuit. This process is called the “Dolan bridge” or “shadow deposition” technique (Dolan, 1977) and is used in almost all single-charge tunneling devices as well as most superconducting qubit designs. There are many variations on this technique but, surprisingly, the basic method has remained unchanged for more than 30 years.

16.3.2 Aluminum and Gap Engineering

Aluminum is regarded as the material of choice for these devices because of the general ease with which one can grow a reliable oxide layer at room temperature within the deposition chamber. Aluminum is easily deposited using both thermal and e-beam deposition techniques that are common to most clean rooms and
16.4 Practical Operation and Performance

The Coulomb, Josephson, and superconducting gap energies are all 10–300 μeV, so practical measurements are performed well below 1 K in a dilution refrigerator, usually at T ≤ 100 mK. There are currently two generic modes of operation of the CPT: current switching electrometry and rf electrometry. The main advantage of the current switching electrometry is its cost. rf electrometry can require relatively expensive microwave amplifiers, generators, and fast digitizers, while current switching electrometry can be achieved with simple function generators and voltage preamps operating with less than 1 MHz of bandwidth.

16.4.1 Current Switching Electrometry

One of the earliest methods for measuring the CPT was to measure the modulation of the switching current \( I_{sw} \) as a function of an applied gate voltage. This measurement is performed by cycling the current bias between the supercurrent and voltage branches in the \( I-V \) characteristic. We can track the current at which the CPT switches to the voltage state by tracking the voltage across the device and using a simple threshold trigger (see Figure 16.10). We can subdivide this measurement approach into ramped and

![Figure 16.10](image-url)

**FIGURE 16.10** The switching current measurement. (a) Typical \( I-V \) curve for CPT device. The feature at \( -0.4 \text{ mV} \) (\( \Delta_i + \Delta_e \)) is the so-called Josephson quasiparticle peak or QJP and is due to more a complex Cooper-pair quasiparticle tunneling cycle. The device switches to the true voltage branch at \( \geq 2(\Delta_i + \Delta_e) \). Inset: Simplified measurement circuit schematic. (b) Expanded view of switching current cycle. The current is ramped along the supercurrent branch until the voltage across the CPT switches to the finite valued voltage branch. The current at which this happens is recorded, and the results of many ramp cycles are recorded in a switching current histogram as in (c).
pulsed current bias techniques. Each has its own merits, but the experimental literature is largely dominated by the ramped technique.

16.4.1.1 Ramped Switching Current Measurement

As the name suggests, the bias current through the CPT is ramped linearly in time such that the current exceeds the maximum switching current and is then returned to zero bias for the next cycle. The probability of switching at a particular current can be backed out from the histogram of switching currents accumulated over many cycles (see Figure 16.10a). This ramped bias current measurement was originally done in the late 1970s in relatively large junctions (Fulton and Dunkleberger, 1974), but the methodology is also suitable to small Josephson junction devices such as the CPT (Joyez, 1995).

This method can be used for electrometry since it yields a switching current histogram whose mean value changes with gate, but its power lies in the fact that one can obtain the switching or “escape” rate $\gamma_{sw}(I_g,n_g)$ through a straightforward transformation of the whole histogram as in the original work by Fulton and Dunkleberger (1974). This is useful since the escape rate contains information about the electron temperature and how well the system is isolated from external noise (Devoret et al., 1987).

16.4.1.2 Quasiparticle Poisoning in the Ramped Current Measurement

If quasiparticle poisoning is present, the effective Josephson energy of the CPT flickers between two different values corresponding to even and odd parities. Each of these energies has its own escape rate $\gamma_{e/o}$ and $\gamma_{o/e}$, and the observed switching current in any given ramp cycle is determined by the instantaneous state of the system as the current is ramped. If the ramp rate is fast enough such that it can span the distance separating the odd and even switching currents faster than the odd/even lifetimes, then the ramped measurement becomes a snapshot of the parity state and we see that the switching currents group around two distinct values as shown in type L device histograms in Figure 16.12.

In our example, the modulation of the switching current histograms is similar to that of the type H device, except that it shows a distinct 1e shifted image corresponding to the presence of the odd state, particularly where we predict the island potential to be trap-like. This bimodal behavior is really only evident if the measurement is faster than the state lifetimes. If the current bias ramp rate were slowed down significantly, then the system could flicker back and forth between parity states many times and we would only see switching distributions grouped around whichever parity state had the lowest switching current. That is, we would see a purely 1e modulation in the switching current. In the early literature, the available dc measurement techniques were unable to capture the dynamical nature of the quasiparticle parity states, and the periodicity (1e versus 2e) of the modulation was the only handle with which to gauge whether poisoning was present. Because of the dynamics of the poisoning process, this correlation can be misleading.

In the type L example given here, the device is definitely poisoned, particularly where we expect the island potential to look trap-like. However, if we examine the type H device’s switching current modulation, it is distinctly 2e periodic (Figure 16.11). The early picture of poisoning would naively assume that the poisoning is not present when, in fact, it is happening much faster than a simple analysis would indicate. We know, for instance, that the island potential is trap-like over a narrower range and that the trap potential is shallow compared with the operating temperature (this is due to the gap engineering for the H device). Therefore, quasiparticles that get trapped on the island

![Type H Histograms](image-url)

**FIGURE 16.11** (See color insert following page 21-4.) (a) Type H $I_{sw}$ histograms versus $n_e$. Histogram height is displayed in grayscale on the right-hand side, whereas all counts are displayed equally on the left-hand side. As in Figure 16.7, the gray box in (a) denotes regions where the island potential is trap like for quasiparticles. (b) Selected histograms corresponding to several gate voltages. Device parameters: $\Delta_i = 246 \mu$eV, $\Delta_o = 205 \mu$eV, $E_c = 115 \mu$eV, and $E_n = E_{2e} = 82 \mu$eV.
are rapidly ejected by thermal fluctuations. In this case, quasi-particles can jump in and out of the island before the CPT has time to latch into the free-running voltage state. In other words, the switching current measurement is bandwidth limited and is not able to see short-lived odd-parity events. While the average probability of being in the even state might be close to unity, quasiparticles may constantly be getting trapped and ejected at very rapid rates. In terms of the poisoning and ejection rates,

$$P_s = \frac{\Gamma_n}{\Gamma_n + \Gamma_o}. \quad (16.23)$$

If the ejection rate $\Gamma_o$ is big compared with the poisoning rate $\Gamma_n$, then $P_s$ approaches one even though $\Gamma_n$ may be arbitrarily large itself. This is an insidious effect since it was previously common for many groups working in CPB qubits to assess their quasiparticle situation from what appeared to be clean $2e$ modulation characteristics. In fact, $2e$ modulation is observed in the presence of these fast quasiparticle dynamics when the measurement technique is slow in comparison. Since quasiparticles might have lifetimes far shorter than 1 $\mu$s, most available methods (dc and rf alike) can have problems with this.

In CPTs with deeper trap potentials, such as the type L devices we show here, the ejection rate can be slowed down significantly. Since the measurement shown in Figures 16.12 and 16.13c,d was performed with a ramp rate that was comparable to the quasiparticle ejection rate, it was possible to obtain a bimodal histogram of $I_{sw}$. In this case, it is easy to correlate the derived escape function with meaningful rates. The two slopes in Figure 16.13d are the even and odd state escape rates, $\gamma_n$ and $\gamma_o$, respectively. Thus, the bias current ramp is not infinitely fast, and the parity can flip from even to odd in the time it takes to ramp between the two current values corresponding to the odd and even state switching currents resulting in several switching events in the region between the peaks. These correspond to quasiparticles jumping into the CPT island during the bias ramp. Since the plateau that connects the even and odd switching rates is derived from the histogram counts between the even and odd peaks, it is a direct indication of the poisoning rate, $\Gamma_n$. We can apply similar reasoning to the type H data in Figure 16.13a,b and notice that the escape rate for $n_g = 0.99$ is a little curvy, although the other escape rates shown look quite linear (on a semilogarithmic scale). While we do not develop a distinct plateau in this case, the deformation in the form of the escape rate is still an indication of very fast poisoning. This should not be a surprise since we expect the island to look trap-like at this gate voltage as we noted above. Thus, the lesson to be learned here is that despite appearances, the data shown in Figure 16.11 masks the fact that the type H device is poisoned.

16.4.1.3 Single Shot Measurement

In a single shot operation (Cottet et al., 2001), we pulse the current bias up from zero to some value $I_p$ for a time $\tau_s$. The probability of switching to the voltage state is given by

$$P_{sw}(n_g) = 1 - e^{-\gamma_n(I_p,n_g)\tau_s}, \quad (16.24)$$

where $\gamma_n(I_p,n_g)$ is the rate at which the CPT phase escapes to the free-running voltage state when instantaneously biased at $I_p$ and $n_g$. If we are biased at $n_g$ and wish to determine whether the polarization charge has shifted by $\delta n_g$, we need to figure out the likelihood of seeing a switching event that is actually due to the shift in switching probability versus just the probability of switching with no charge shift at all,

$$\Delta P_{sw}(n_g, \delta n_g) = e^{-\gamma_n(I_p,n_g)\tau_s} - e^{-\gamma_n(I_p,n_g + \delta n_g)\tau_s}. \quad (16.25)$$

If we want to measure this change in polarization charge in a single measurement, then we require that $\Delta P_{sw} = 1$. That is, if we pulse the bias current and see the CPT switch to the voltage state,
FIGURE 16.13  (See color insert following page 21-4.) (a,c) Switching current (I_sw) histograms and (b,d) derived switching/escape rates for a type H (barrier-like) and type L (trap-like) CPTs. For the type L device, the quasiparticle trapping behavior is evident in the bimodal I_sw distribution. In this case, the poisoning rate Γ_{eo} can be read directly from the derived escape rate in (d) as shown. Although the type H device is barrier-like for most n_g, it still looks like a trap near n_g = 1 (see Figure 16.7). This is apparent in the “curvy” structure of the escape curve for n_g = 0.99 as compared with the escape rates at other n_g in (b).

then the polarization charge has shifted by δn_e with certainty. This defines a minimum charge shift δn_{ss} or voltage change δV_{ss} (δn_{ss}e/C_p) on the gate that can be detected with one measurement. If we want to detect a smaller change in gate voltage, then we would have to take multiple measurements until the uncertainty in the shift of 〈I_d〉 is satisfactory. In many experiments, this is a completely reasonable approach. It is possible, however, to attempt to reduce the width of the switching current distribution by increasing the shunt capacitance across the CPT (increasing the effective mass of our imaginary phase particle) (Joyez, 1995).

In the end, pulsed single shot measurements have never been very popular. This may be because their chief use would have been in charge-based qubits, and other significant measurement schemes were shown to outperform it (Vion et al., 2002). However, it might still be a viable measurement method outside of superconducting quantum computing.

16.4.2 Zero-Biased rf Electrometry

The initial aim of the rf inductance measurements was to demonstrate a dissipationless (or near dissipationless) electrometer that could be used in charge-based quantum circuits such as the CPB, but the most useful application in recent years seems to have been to study the dynamics of quasiparticle tunneling.

Initial measurements of the Josephson (or “quantum”) inductance of a CPT were performed by Sillanpää et al. (2004) and soon thereafter by Naaman and Aumentado (2006). rf measurements of CPTs were also performed by Court and coworkers (Ferguson et al., 2006) as well, but these measurements focused on a modulation of the dissipation and were not strictly confined to the supercurrent branch.*

16.4.2.1 The rf Measurement Setup

Figure 16.14a shows a typical microwave circuit used in rf-CPT electrometry. In this scheme, the CPT is embedded in a tank circuit composed of the parasitic capacitance provided by the leads, the Josephson inductance of the CPT, and an extra surface mount inductor soldered to a printed circuit board near the chip to lower the tank resonance to the range of the microwave amplifier.†

Following the circuit path in Figure 16.14a, we reflect an incoming microwave signal (the carrier) off of the CPT resonator circuit and measure the amplitude and phase of the outgoing signal, that is, we measure the scattering parameter S_{11}. As shown

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* It’s important to note that measurement of the Josephson inductance (the second derivative of the ground state energy in phase) is conceptually linked to the “quantum capacitance” (the second derivative in charge). rf measurements of the capacitance have recently been shown to be an extremely useful measurement of charge-based qubits such as the CPB (Wallraff et al., 2004; Duty et al., 2005) and the transmon (Schuster, 2007).

† The word “parasitic” should be a tip-off to the reader that this scheme might be a little hit-or-miss and, indeed, it can be frustrating to target the operating frequency within 10% (this is the typical bandwidth for sub-1 GHz low-noise cryogenic amplifiers). While this setup is very similar to that used in typical rf-SET operation (Schoelkopf et al., 1998), it is not necessarily trivial to implement.
earlier, the gate-dependent Josephson inductance can be many 10's of nano-Henries and, since this inductance provides much of the total inductance available to the resonator, the frequency shift can be on the order of the bandwidth (typical $Q_s \sim 20–30$). While a purely dissipationless circuit will reflect all of the signal ($|S_{11}|^2 = 1$) (Pozar, 2004), there are, in fact, a number of sources of dissipation, including the lossy traces on the pc board and leads on chip, the losses in the surface mount inductor and wire bonds, and, finally, the intrinsic CPT losses from any phase-diffusion resistance present. The end result is that there is a visible resonance dip in the reflected power amplitude $|S_{11}|^2$ (Figure 16.14b,c). For our purposes then, the frequency shift due to the modulation of the CPT Josephson inductance can be inferred from the reflected amplitude modulation.* That is, since the CPT inductance is a function of island charge, the final output microwave power amplitude and phase modulate with the gate-induced polarization charge.

Like the rf-SET, one can characterize the ultimate noise performance of this device as an electrometer in terms of an effective charge resolution in a 1 Hz bandwidth. In the rf measurement of the CPT inductance, the best number that has been reported is $\delta Q \sim 50\mu e/\sqrt{\text{Hz}}$ (Naaman and Aumentado, 2006b). That is, if we integrate the reflected power at the end of our measurement chain for 1 s, we can resolve a change in polarization charge at the gate electrode $C_g V_g$ of $5.2 \times 10^{-5}$ electrons. If we compare this with the best rf-SSET numbers $\delta Q < 5\mu e/\sqrt{\text{Hz}}$ (Brenning et al., 2006), the rf-CPT is more than 100 times slower at resolving the polarization charge. At first blush, this seems to put the rf-CPT at a disadvantage, but the rf-SSET charge resolution comes at the expense of using a relatively complex charge transport cycle that involves both quasiparticles and Cooper pairs. The back-acting voltage fluctuations of this cycle presented at the device they are measuring are equally nontrivial and have even been used to cool an electromechanical system that it was intended to measure (Naik et al., 2006). While some would consider this a feature of the rf-SSET, it seems to get away from the notion of simply wanting to use the device as a noninvasive electrometer. In fact, quasiparticle poisoning the rf-CPT has been used as a method to measure the effect of nearby SSETs, demonstrating that the latter emits nontrivial microwave power into its environment when voltage-biased (as they would be when operated as electrometers) (Naaman and Aumentado, 2007).

16.4.2 Operation Beyond 1 GHz

Although all of the published CPT experiments were operated with carrier frequencies below 1 GHz, this is not a fundamental requirement. In fact, this limitation seems to be determined by parasitic self-resonances in the surface-mount inductors, often used in these experiments. In principle, the operating frequency can be raised to many gigahertz using coplanar resonators. The advantage of moving to higher frequencies is that wideband, low-noise HEMT amplifiers are now readily available in the 4–8 GHz range and all of the associated passive

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* In principle, this information is also accessible through the phase of the outgoing signal and can be measured using an IQ mixer. Since loss was present in many of these experiments and the frequency shift was usually very obvious, measurement of the amplitude modulation has ended up being the simplest method.
components (directional couplers and isolators) are smaller and much more common (less expensive).

16.4.2.3 Quasiparticle Poisoning in the rf-CPT

As in the switching current measurement, quasiparticle poisoning also has a dramatic effect on rf measurement. Unlike the switching current measurement, a qualitative understanding of the measurement response is very straightforward. Since the inductance of the CPT can switch instantaneously between two different parity states when the gate is biased into a trap-like regime, the reflected power also switches between two different amplitudes. This kind of response is more commonly known as “telegraph noise.” A typical example is shown in Figure 16.15a for a type L device. We note that this is the same bimodal behavior that we observe in the switching current measurements except that we can sit at zero bias and watch quasiparticles jumping in and out of the CPT island. In Figure 16.15c, we histogram the telegraph time traces as a function of gate voltage, we get data that, unsurprisingly, are very similar to our bimodal switching current histograms in Figure 16.12.

The rates \( \Gamma_{oe} \) and \( \Gamma_{eo} \) can be derived from an analysis of these time-domain traces but requires a careful characterization of the system measurement bandwidth (Naaman and Aumentado, 2006a). Although it is possible to extract the poisoning rate \( \Gamma_{eo} \) from a switching current measurement, it is far more difficult to pull out an ejection rate. This is a direct consequence of the time-ordered nature of the current bias ramp. In contrast, the rf measurement is not burdened by this kind of time ordering in any obvious way, and both the poisoning and ejection rates are available.

The availability of the poisoning and ejection rates has provided further verification of the model presented in Aumentado et al. (2004), while confirming that nonequilibrium quasiparticles are generated in the leads at a constant rate below \( \sim 250 \) mK. In addition, the ejection rate \( \Gamma_{oe} \) has been used to validate the notion that biased SSETs emit nontrivial levels of microwave power into their environment (Naaman and Aumentado, 2007). This is important as for a long time rf-SSET electrometry had been considered a viable method of measuring CPB qubit charge states.

![FIGURE 16.15](image)

(a) Reflected power (linear units) of type L CPT at \( n_g = 1 \) in the time domain. The even (upper) and odd (lower) state levels are evident in telegraph-noise time traces. (b) Histogram of the full time trace. (c) Histograms versus gate voltage. (Adapted from Naaman, O. and Aumentado, J., Phys. Rev. B, 73, 172504, 2006b.)
16.4.2.4 Relation to the Cooper-Pair Box

Earlier, we alluded to the fact that the CPT was related to the CPB. In fact, when the CPT is biased at zero, it is identical to the CPB if the biasing circuit series impedance is small at the relevant parallel-junction plasma frequency. Since the problem of quasiparticle poisoning is also important to charge-based qubits such as the CPB (Nakamura et al., 1999; Wallraff et al., 2004), quantronium (Vion et al., 2002), and transmon (Koch et al., 2007), the fact that we can study it with relatively high instantaneous bandwidth in the CPT motivated several experiments aimed at studying the dynamics of the poisoning process in detail (Aumentado et al., 2004; Ferguson et al., 2006; Naaman and Aumentado, 2006b; Court, et al., 2008b). Ultimately, it was realized that quantum capacitance measurements could achieve the same objective measuring CPBs directly and have yielded the most detailed quasiparticle poisoning studies to date (Lutchyn and Glazman, 2007; Shaw et al., 2008).

16.5 Present Status and Future Directions

The CPT is the simplest device in which dc transport characteristics can be correlated to the duality between charge and phase. Although the most prominent recent CPT experiments have focused on the problem of nonequilibrium quasiparticle poisoning, the CPT is also a promising general-purpose low-temperature, low-backaction electrometer. Since the most reliable techniques for fabricating these devices involve aluminum fabrication with gap, Josephson, and Coulomb blockade energies in the ~100–300 μeV range, operation is restricted to dilution refrigeration, so that the systems that one attaches it to must also be cold. While this seems restrictive at present, there are already several mesoscopic condensed matter systems that might benefit from minimally invasive fast electrometry.

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References


