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Physical Image Quality Evaluation of X-ray Detectors for Digital Radiography and Mammography

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This chapter describes methods and tools for the assessment of image quality in digital radiography and digital mammography, including detector characterization.

24.1 Introduction

In many medical imaging departments, the transition from analog imaging systems using screen/film (S/F) technology to digital radiography (DR) systems based around some form of digital X-ray detector is complete. The images generated by these systems provide vital information in the management and treatment of patients within the hospital. In the case of mammography, this often extends outside the hospital environment, and can involve the imaging of asymptomatic women within the context of a mammography screening program. Image quality is a crucial aspect of system performance, and the body of literature dealing with the evaluation of radiography and mammography system image quality is rich and diverse. Medical images are generated for a specific purpose, and the quality of said images should be evaluated within this context; the notion of image quality thus relates to the ability of the device to perform the clinical task for which it was designed/used (Loo et al. 1984; Wagner and Brown 1985). An almost endless number of different imaging tasks are routinely undertaken and considering all is not a practical proposition. Furthermore, the Gold Standard metric of task-based evaluation in radiology is currently the receiver operating characteristic (ROC) or alternative free-response operating characteristic (AFROC) study, and the observer reading sessions required for these studies are time consuming.

We are, therefore, forced to look for alternatives that can be implemented practically, yet give useful, relevant results. The technique employed depends to some extent on the reason for the image quality evaluation, which could be anything from a simple, routine quality control (QC) check to optimization of the imaging chain using detailed knowledge of the sub-components, commissioning testing, or even “ad-hoc” trouble shooting. Broadly, there are currently two main practical approaches in routine use: test object based evaluation and physical image quality assessment using transfer function analysis. These methods can be applied in a system scenario, with the system assessed as a whole or at a sub-component level, such as explicit detector evaluation. There is value in both of these techniques and evaluation scenarios and, thus, both are discussed.

This chapter begins with a discussion of test object measures of image quality in mammography and general radiography, with the focus on threshold contrast-detail (CD) curves. This is followed by five sections dealing with the various steps involved in the physical evaluation of factors that influence image quality using transfer function methods. The first step is measurement of the detector response function, followed by measurement of resolution/sharpness using the modulation transfer function (MTF) and then noise is quantified using the noise power spectrum (NPS). Finally, global metrics of detector performance are introduced.
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via the concepts of noise equivalent quanta (NEQ) and detective quantum efficiency (DQE) (see also Section I, Chapters 14 and 15 of this book). In each of these sections, some theory relevant to the metric is given, followed by discussion of the measurement methods and then some practical examples. The last section discusses the link between the NEQ and signal detection theory methods, with the simple detection task used in the CD test object methods that were introduced at the beginning of the chapter.

24.2 Test Object Evaluation of Image Quality

For many years, test objects have formed the basis of image quality evaluation techniques employed by medical physicists. Although there has been a move towards metrics such as the MTF and NPS, based on transfer function analysis, test object methods still form a crucial aspect of image quality evaluation for the purposes of QC. Threshold contrast-detail (CD) curves or low contrast detectability (resolution) are widely used to obtain a measure of image quality that can be compared against typical performance data for similar detector or imaging system types (IPEM 2010; Marshall et al. 2011; NHSBSP 2013). Alternatively, the results can be compared to reference data to establish whether the system meets the minimum image quality criteria required (if they have been established) before the system can be used clinically (Hendrick et al. 1999; EC 2006).

CD curves are typically measured using a homogeneous polymethyl methacrylate (PMMA) phantom containing a pattern of thin discs or holes and are often referred to Burger–Rose phantoms, after the pioneers who developed these methods (Rose 1948; Burger 1950). The targets are usually rows of circular discs or holes that vary in diameter along one direction and vary in thickness or depth along the other direction. The background signal in the image must be homogeneous. When imaged, this regular pattern of discs can be used in detection tests by human observers or input to software that implements detection algorithms for automated scoring. The variation in thickness or depth generates a variation in contrast and, thus, object detectability; these are, therefore, threshold test objects that require the observer (or software) to establish the last disc considered visible (i.e., the threshold).

There are two basic paradigms for finding thresholds. In the first method (that could be termed “free response”), the observer counts to the last disc considered visible (Rose 1948; Burger 1950) and repeats this for the range of diameters present in the test object. This is a basic method that is quick to perform, and, hence, useful for QC purposes (Hay et al. 1985; Cowen et al. 1987), but has limitations—primarily that the individual observer detectability rate is not controlled. The observer is free to decide on the last disc and has to maintain this decision threshold constant over the duration of the reading session, and in fact between reading sessions if CD methods are used to track imaging system performance over time (years). However, experienced observers are able to some extent to hold their threshold constant between reading sessions, at least over a period of several weeks (Cohen et al. 1984).

The second method of generating a threshold is through the use of a (multiple) alternative–forced choice (M-AFC) technique, whose implementation requires a different design of test object. Typically, there is a grid of cells, each cell with a disc (or circular hole) at the center and an identical target positioned randomly in one of the four corners. The observer decides on the corner that contains the stimulus, this is repeated with several images and observers for a given imaging condition. The known truth of the phantom layout means that the mean probability of correct (PC) response can be estimated. The relationship between PC and physical stimulus generally has a sigmoid shape (often modeled with the psychometric function) that grows from 1/M at the random guessing level to 1.0 (certain detection).

A curve fit is applied to the PC as a function of stimulus magnitude (Ohara et al. 1989; Veldkamp et al. 2003; Young et al. 2006). The great advantage compared to the Rose free response type CD test object is that threshold contrast or thickness can be defined at some PC value (typically 62.5% or 75%), yet the evaluation remains subjective. Repeating as a function of disc diameter yields the CD curve that links the disc diameter (area) and threshold contrast, usually visualized via a log–log plot of threshold stimulus (contrast, thickness, or hole depth) (y-axis) against detail diameter (x-axis). The curve can be compared against earlier measurements for constancy testing over time or against performance standards, if available. Note that the phantom described by Burger (1950) has two sections, one with rows of discs (i.e., Rose–like), and a smaller section with a 4-AFC design.

For radiographic images, the uncertainty on the result depends on the within-reader variance \(S_b\), the between reader variance \(S_l\), and the case sample variance \(S_c\) (i.e., variation between images due to different realizations of the background noise pattern) (Cohen et al. 1984). These sources of uncertainty can be combined to give the standard error (SE) as:

\[
SE = \sqrt{\frac{S_b^2}{n} + \frac{S_l^2}{l} + \frac{S_c^2}{nlm}}
\]  

(24.1)

where \(n\) is the number of images, \(l\) is the number of observers, and \(m\) is the number of independent readings of a given image. Average values for SE expressed as fractional standard deviation taken from Cohen et al. (1984) are \(S_l = 0.137\), \(S_b = 0.143\), and \(S_c = 0.108\); hence, two observers reading three images once incur a fractional standard deviation of approximately 14%.

CD test objects tend to be used in one of two setups, that could be termed system or detector geometries. In the system approach, the test object is positioned at a patient equivalent position (such that geometric (source) unsharpness is included) and imaged with added tissue simulating materials, a clinical beam quality, and typically the anti-scatter grid in place. The resultant CD curve is influenced by all the elements of the X-ray imaging chain, including the spectrum and scatter rejection efficiency. In the detector approach, the test object is positioned at the detector input plane, and a calibrated X-ray spectrum set, typically with additional metal filtration at the X-ray tube (Hay et al. 1985). This method attempts to minimize the influence of all elements in the imaging chain with the exception of the X-ray detector and the detector air kerma or image. As dose/image influences image quality score, some measure of dose has to be specified. For test objects used in the system context, this tends to be a measure of patient dose (such as entrance air kerma or breast dose in mammography), while, for test objects used in a detector geometry, the air kerma/image at the X-ray detector is the parameter specified or recorded.
24.2.1 Mammography

A number of test objects are used in routine assessment of image quality in mammography (including mammography screening), including the American ACR phantom, the Leeds TORMAM and TORMAX test objects and CDMAM (Reis et al. 2013) (see also Section IV, Chapters 55 through 57 of this book). With the exception of TORMAX, these are system test objects that contain a range of targets including calcification-like and mass-like objects. The CDMAM phantom is a CD test object with a 4-alternative forced choice (AFC) design, containing circular details composed of gold, with diameters ranging from 0.06 to 2.00 mm. Imaged with 40 mm PMMA, this test object generates a system CD curve and is used throughout Europe to assess image quality in mammography. Between 8 and 16 images are acquired for a given imaging condition, with a slight shift between acquisitions to have different realizations of partial volume effect. Scoring can be manual (human observer) or via dedicated scoring software (CDCOM) (Karssemeijer and Thijssen 1996; Young et al. 2006), with uncertainties estimated using a bootstrap approach that are generally below approximately 10% for the 0.1 mm diameter detail. The resultant CD curve (expressed in gold thickness) can be compared against levels in EC guidance that give Acceptable and Achievable levels of performance of systems for use in breast screening. Recent work by Warren et al. (2012) has shown that CDMAM CD data correlate with radiologist detection performance of simulated microcalcification clusters.

Figure 24.1 shows CD curves obtained with the CDMAM phantom on a mammography system for different detector air kerma/image values. Also shown are the corresponding mean glandular dose (MGD) values calculated for the acquisition setting (30 kV and W/Rh). The system meets the Acceptable image quality level for gold thickness at ∼0.6 mGy and the Achievable level at ∼1.8 mGy, within the Achievable level of 2.4 mGy (for 50 mm PMMA) set in the European Protocol for screening mammography (EC 2006). Note that many European countries have implemented the image quality and dose performance levels given in EC (2006), and systems must meet these levels before use in a breast screening program.

24.2.2 General Radiography

CD test objects for use with general imaging systems (including fluoroscopy, digital subtraction angiography, and digital radiography) include phantoms developed at Leeds medical physics department (Hay et al. 1985; Cowen et al. 1987) and the CDRAD test object developed by Thijssen (1993). Both consist of a PMMA baseplate with circular details, but there are notable differences in both design and application of these test objects. Stimuli in the Leeds test object are generated by circular discs of unspecified composition arranged in both straight and circular rows. There is a √2 step in contrast (∼thickness) between successive discs along a given row, and there is also a √2 step between diameters (ranging from 0.25–11.1 mm). Hence, for a quantum noise limited detector, a factor 2 increase or decrease in detector air kerma (DAK) will (on average) increase or decrease the number of visible discs at each diameter by one (on average). Scoring is of the Burger–Rose “free response” type, where the observer counts to the last disc considered visible. CDRAD is composed of a matrix of cylindrical holes of different diameters and depths drilled into the PMMA. Both hole depth and hole diameter range from 0.3 to 8.0 mm, with an average step of 20% between discs/diameters. As with CDMAM, layout and scoring is via a 4-AFC paradigm.

In use, the Leeds test objects are normally positioned at the detector input plane and imaged with a tube voltage between 65 and 80 kV with added filtration of 1.0, 1.5, or 2.0 mm Cu. The number of discs seen at a given diameter is converted to a contrast for the beam quality used to acquire the image using tables supplied with the test object. Hence, the influence of beam quality is (to some extent) removed from the evaluation, and the threshold contrast gives a measure of the low contrast detectability of the X-ray detector. CDRAD, on the other hand, is typically used as a system test object, imaged with 20 cm PMMA, using a patient geometry and a freely selected tube voltage (typical clinical). There is generally no attempt to estimate the radiation contrasts generated by the holes and, hence, threshold hole depth is plotted against detail diameter. Reference CD data are available for use with the Leeds CD test objects for various modalities,
Threshold CD test objects provide a summary measure of X-ray detector or system low contrast detectability and, hence, provide the QC physicist with useful information regarding the absolute value of the quality score and whether there have been changes in system performance since the previous QC visit. However, these test objects combine a number of aspects related to the quality of images (the large area contrast, the sharpness/resolution and noise) in a single score and, under fault conditions, for example, this can make it difficult to identify the particular aspect of the imaging chain resulting in the fault. An alternative approach is to make an explicit assessment of various aspects of detector performance that directly influence image quality, including detector response function and dynamic range, the sharpness (MTF), noise (NPS), and overall detector efficiency (the detective quantum efficiency [DQE]). These physical parameters derive from transfer function analysis of linear imaging systems (Rossman 1969; Cunningham 2000) and form the basis current protocols for medical X-ray detector imaging performance characterization in general radiography (IEC 2003) and mammography (IEC 2007). This section describes the first step in the application of such a protocol, namely measurement of the detector response function.

### 24.3.1 Images for Quantitative Measurements

Step one is to ensure that the system produces the required image types. X-ray detectors used for medical applications save digital images so that they can be transferred using the Digital Imaging and Communications in Medicine (DICOM) protocol. Image types with DICOM tag [0008,0068 Presentation Intent Type] set to “For Processing” are generally suitable for physical image quality assessment. Images with this tag set to “For Presentation” are intended for display with the final clinical image processing applied, which is almost certain to be strongly non-linear and, therefore, unsuitable for analysis using Fourier based metrics. The first requirement in physical image quality assessment is, therefore, access to “For Processing” images; availability of these images currently ranges from ~100% for most digital mammography systems, to ~0% for digital fluoroscopy or angiography systems, with access for general radiography systems lying somewhere in between these figures. In order to acquire this image type, many systems provide specific program settings (e.g., “Pattern” image types for Carestream devices or “System Diagnosis; Flat Field 400” for Agfa systems). It may be necessary to contact the application specialist or service engineer to determine the program or settings to be used: note that this is a vital step if transfer function analysis is to be applied. While Fourier based analysis can be applied to images with clinical image processing (McDonald et al. 2012; Urbanczyk et al. 2012), great care must be taken in the calculation and interpretation of results.

Although the images are declared as “For Processing,” corrections will have been applied to the raw detector images. For flat panel detectors (FPDs), these will include a pixel correction map to correct non-responsive or defective pixels, and a flat-field correction that corrects for radiation field non-uniformity and for the offset and gain variations of individual pixels (Schmidgunst et al. 2007). Further corrections of noise level for small exposure time variations are also allowed. While these are standard corrections, it is unlikely that their exact specification will be made available to the operator. Manufacturers testing against one of the IEC standards (e.g., IEC 2003) have the advantage of access to the raw images and full control over the applied corrections; however, the standards make it clear that, when measuring DQE, all of these corrections must be applied as for normal clinical use.

### 24.3.2 Geometry and Collimation

Assessment of DQE explicitly involves a characterization of detector sharpness and noise properties via the MTF and normalized noise power spectrum (NNPS). To minimize the influence of X-ray source on measured sharpness, a long focus to detector surface distance (FDD) should be employed, along with a small focus. The International Electrotechnical Commission (IEC) standard describes collimation in addition to the primary X-ray
tube collimation, with the intention of controlling the influence of scattered radiation on the results, to be placed at positions between the source and the detector. However, this collimation may be omitted if it can be shown that this does not influence the results (IEC 2003, 2007). For general detectors, Dobbins et al. (2006) found only a small influence (2.3%) from collimation on the measured NNPS, while Samei et al. (2006) found a larger influence (up to 8%) on the MTF, and encouraged the use of the X-ray tube collimator, rather than additional collimation between tube and detector.

### 24.3.3 Radiation Quality and Exposure Range

The radiation quality should be relevant to the application for which the detector is used. For this purpose, IEC have defined standard beam radiation qualities (RQA spectra) for radiography and mammography detector characterization (IEC 2003, 2007), implemented using aluminum and designed to simulate patient exit spectra. An example of implementing the RQA3 quality: the 10.0 mm Al filter is positioned at the X-ray tube output, the nominal tube voltage of 50 kV is selected (Table 24.1), along with some tube current-time product (mAs), and a number of baseline exposure readings are acquired. An additional Al filter equal to the half-value layer (HVL) for the RQA beam is then also positioned at tube exit (4.0 mm Al for RQA3). The exposure is measured again, and the ratio of exposure taken with the HVL filter to that without the filter is calculated. This ratio should be 0.50 ± 0.01; if not then the tube voltage is adjusted and the procedure repeated until the addition of the HVL filter reduces the exposure by a factor of 2.0.

Alternative beam qualities have been proposed and shown to give results close to those of the RQA spectra (e.g., Samei et al. 2013). For the purposes of quantitative QC measurements, IPEM propose a spectrum of 70 kV, filtered with an additional 1.0 mm of copper. Radiation quality has some influence on the measured MTF and, therefore, the detector response function (or conversion function (IEC 2005, 2007) should be measured at the same radiation quality used for the response function and NNPS measurements.

The air kerma should be measured using an appropriate monitoring device, calibrated at the energy used for the characterization. Ideally, the measurement should be made at the entrance plane of the detector, with the detector removed from the beam. If this is not possible, then the radiation meter should be positioned at some point away from the detector surface and towards the source, such that the influence of backscattered radiation is minimized. The air kerma range at the detector surface covered in the response function measurement should be appropriate to the clinical range and levels used. A normal (or reference) air kerma level should be identified that is equal to the level at which the detector is operated in clinical practice. This can be established by acquiring an image under automatic exposure control, with the relevant filter in position and measuring the air kerma at the detector. The NPS is also evaluated at two additional levels, above and below the normal operating level. IEC (2003) states that the exposures to obtain the response function should include an image acquired at zero kerma (a dark image), the minimum air kerma level should not be greater than one fifth of the normal level, while the highest level should be 20% greater than the maximum air kerma, for which NPS (and DQE) is evaluated. The incremental air kerma steps should have a logarithmic (base 10) form, and should not be greater than 0.1.

### 24.3.3.1 Practicalities for Mammography Detectors

For mammography detectors, FDD is fixed at between 600 and 700 mm for current systems (see also Section II, Chapter 23 of this book). Collimation should be used to give an irradiated field area of 100 mm × 100 mm, although the collimation may be omitted if it is shown not to influence the results. The IEC standard (IEC 2007) specifies qualities covering the range of anode/filter settings currently available on commercial systems, operated at 28 kV nominal X-ray tube voltage and a 2 mm Al filter supported at the tube exit port. Beam quality parameters for the IEC mammography spectra are listed in Table 24.2. An approximate mean energy for each spectrum is given, estimated using the model of Boone et al. (1997). The air kerma at the detector surface (DAK) is measured at the reference point centrally situated at 6 cm from the thorax side. Corrections for the dosemeter-detector distance are made via the inverse square distance law point source/FP.

### Table 24.1

Radiation Qualities Used for Mammography Detector Characterization and Associated HVL, Number of Photons mm⁻² μGy⁻¹, and Approximate Mean Energy

<table>
<thead>
<tr>
<th>Radiation Quality</th>
<th>Filter Thickness (mm)</th>
<th>Added Filter (mm Al)</th>
<th>Nominal X-ray Tube Voltage (kV)</th>
<th>Approx Mean Energy (kV)</th>
<th>Nominal Half-Value Layer (mm Al)</th>
<th>Photons (mm⁻² μGy⁻¹), i.e., SNRᵢᵣ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mo/Mo (RQA-M 1)</td>
<td>0.032</td>
<td>2</td>
<td>25</td>
<td>18.5</td>
<td>0.56</td>
<td>4639</td>
</tr>
<tr>
<td>Mo/Mo (RQA-M 2)</td>
<td>0.032</td>
<td>2</td>
<td>28</td>
<td>19.5</td>
<td>0.60</td>
<td>4981</td>
</tr>
<tr>
<td>Mo/Mo (RQA-M 3)</td>
<td>0.032</td>
<td>2</td>
<td>30</td>
<td>20.4</td>
<td>0.62</td>
<td>5303</td>
</tr>
<tr>
<td>Mo/Mo (RQA-M 4)</td>
<td>0.032</td>
<td>2</td>
<td>35</td>
<td>23.0</td>
<td>0.68</td>
<td>6325</td>
</tr>
<tr>
<td>Mo/Rh</td>
<td>0.025</td>
<td>2</td>
<td>28</td>
<td>20.2</td>
<td>0.65</td>
<td>5439</td>
</tr>
<tr>
<td>Rh/Rh</td>
<td>0.025</td>
<td>2</td>
<td>28</td>
<td>21.1</td>
<td>0.74</td>
<td>5944</td>
</tr>
<tr>
<td>W/Rh</td>
<td>0.050</td>
<td>2</td>
<td>28</td>
<td>20.9</td>
<td>0.75</td>
<td>5975</td>
</tr>
<tr>
<td>W/Al</td>
<td>0.500</td>
<td>2</td>
<td>28</td>
<td>22.0</td>
<td>0.83</td>
<td>6575</td>
</tr>
</tbody>
</table>


Note: RQA radiation qualities are those in the IEC standard 62220-1-2.
TABLE 24.2

Radiation Qualities Used for Characterization of Diagnostic Detectors and Associated HVL, Number of Photons mm$^{-2}$ μGy$^{-1}$, and Approximate Mean Energy

<table>
<thead>
<tr>
<th>Radiation Quality</th>
<th>Approximate X-ray Tube Voltage (kV)</th>
<th>Additional Filtration</th>
<th>Approx Mean Energy (kV)</th>
<th>Half-Value Layer (mm Al)</th>
<th>Photons (mm$^{-2}$ μGy$^{-1}$), i.e., SNR$_{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQA 3</td>
<td>50</td>
<td>10.0 mm Al</td>
<td>40.3</td>
<td>4.0</td>
<td>21,759</td>
</tr>
<tr>
<td>RQA 5</td>
<td>70</td>
<td>21.0 mm Al</td>
<td>53.3</td>
<td>7.1</td>
<td>30,174</td>
</tr>
<tr>
<td>RQA 7</td>
<td>90</td>
<td>30.0 mm Al</td>
<td>64.0</td>
<td>9.1</td>
<td>32,362</td>
</tr>
<tr>
<td>RQA 9</td>
<td>120</td>
<td>40.0 mm Al</td>
<td>77.0</td>
<td>11.5</td>
<td>31,077</td>
</tr>
<tr>
<td>RQA 5 alternative*</td>
<td>70</td>
<td>0.5 mm Cu + 1.0 mm Al</td>
<td>52.1</td>
<td>7.1</td>
<td>29,846</td>
</tr>
<tr>
<td>RQA 9 alternative*</td>
<td>120</td>
<td>1.04 mm Cu + 1.0 mm Al</td>
<td>75.8</td>
<td>11.5</td>
<td>31,505</td>
</tr>
<tr>
<td>IPEM 70</td>
<td>70 dial</td>
<td>1.0 mm Cu</td>
<td>56.6</td>
<td>7.9</td>
<td>32,300</td>
</tr>
</tbody>
</table>


Note: RQA radiation qualities are those in the IEC standard 62220-1-2.
*RQA 5 and RQA 9 alternative spectra given in Samei et al. (2013).

24.3.3.2 Practicalities for Diagnostic Detectors

For general detectors, FDD is generally adjustable, but for the characterization this should be fixed to be at least 150 cm, and the area of detector irradiated should be at least 160 mm $\times$ 160 mm (IEC 2003). Beam quality parameters, including the RQA spectra, are listed in Table 24.1. For a given RQA spectrum, the tube voltage should be adjusted, starting from the approximate tube voltage, such that the measured HVL matches that in the standard.

Again, the IEC protocol (2003) gives no explicit advice on the normal air kerma level. One option is to compare the DR detector with a general purpose S/F system (sensitivity 400), which requires a DAK of $\sim$2.5 μGy to reach an optical density of 1.0 above base plus fog. Taking 2.5 μGy as the normal level for general radiography detectors, then the IEC standard states that the two additional levels to be evaluated are 2.5/3.2 = 0.78 μGy and 2.5 $\times$ 3.2 = 8.00 μGy. This is consistent with Figure 4 in IEC 62220-1 (IEC 2003). The IEC air kerma range for the response is, therefore, 0.5 μGy (2.5/5) to 9.5 μGy (1.2 $\times$ 8) in 14 steps. We would recommend measuring the response until at least 20 μGy.

24.3.4 Measuring and Fitting the Response Function

The response function is established by measuring the mean pixel value (PV) at the center of the irradiated field, using a 100 $\times$ 100 pixel region of interest (ROI). This is performed on the unprocessed (DICOM “For Processing”) images. While recording the mean PV, the variance should also be recorded, as this can then be used for a basic three component decomposition of the noise (Section 24.5.9.2). The response function (or conversion function [IEC 2003]) is formed by plotting PV against the estimated total number of photons mm$^{-2}$ at the detector surface (Q). The value of Q is established by multiplying DAK (μGy) by the input photon fluence mm$^{-2}$ μGy$^{-1}$ (Φ). In the IEC standards (IEC 2003, 2007), Φ is referred to as SNR$_{in}$, and is calculated by integrating the input X-ray spectrum, generated using the Boone spectral models (Boone and Seibert 1997; Boone et al. 1997), over the photon energy range. For energies RQA5 and above, the number of photons mm$^{-2}$ μGy$^{-1}$ is roughly constant, at approximately 31,200 mm$^{-2}$ μGy$^{-1}$, with a maximum deviation of approximately 4%.

Alternatively, this plot is commonly made against detector air kerma (μGy) or exposure (mR). The measured PV is then fitted by a model function; for current clinical systems this will either be a linear, logarithmic or power model. There should be a good correspondence between the model and the measured data, with IEC (2003, 2007) requiring an $R^2$ value ≥0.99 and that no individual experimental data point should deviate from the fitted value by more than 2% relatively. Before calculating any of the detector parameters described in the next sections, the images used for the analysis must be linearized by applying the inverse of the response function. An example of linearization for three common response functions plotted against detector air kerma (DAK) is:

- $PV = a + b DAK \Rightarrow PV_{in} = \frac{PV - a}{b}$ (24.2)
- $PV = a + b \ln(DAK) \Rightarrow PV_{in} = \exp\left(\frac{PV - a}{b}\right)$ (24.3)
- $PV = a + b DAK^c \Rightarrow PV_{in} = \left(\frac{PV - a}{b}\right)^{1/c}$ (24.4)

Following this step, the PV data takes on units of the response function, for example Q photons mm$^{-2}$, air kerma (μGy), or even...
mAs, if PV has been plotted against mAs. Common features are seen: removal of the offset, division by the gain, and removal of any (non-linear) function that was applied.

Figure 24.3a shows response functions plotted against DAK for four general detectors. These curves cover the four common relationships seen for medical imaging devices: linear (generally with some offset), logarithmic (negative, in this case), and power. There is a good fit between these chosen model curves and the measured data, with \( R^2 \) values \( \geq 0.996 \). Figure 24.3b shows the influence of energy on the response curve for an a-Se based mammography detector. The response was measured for four X-ray beams covering the typical range utilized by the automatic exposure control (AEC) of this system; the added polymethyl methacrylate (PMMA) blocks used to simulate breast tissue were positioned at the tube exit port. The generated X-ray beams have average energies ranging from \( \sim 19.6 \) to \( \sim 24.6 \) keV, and the gradient of the response curve (plotted against DAK) is seen to increase as a function of energy. Figure 24.3c takes all of these data points together, and plotting as a function of DAK gives a weak relationship between PV and

\[
\begin{align*}
7 \text{ cm: PMMA; } & 34 \text{ kV W/Ag} \\
6 \text{ cm: PMMA; } & 30 \text{ kV W/Ag} \\
4 \text{ cm: PMMA; } & 28 \text{ kV W/Rh} \\
2 \text{ cm: PMMA; } & 25 \text{ kV W/Rh}
\end{align*}
\]

\[
\begin{align*}
y = 5.03x + 64.24 \\
R^2 = 0.86
\end{align*}
\]

\[
\begin{align*}
y = 34.31x + 45.69 \\
R^2 = 1.00
\end{align*}
\]
24.4 Transfer Function Analysis and Sharpness

24.4.1 Mathematical Considerations

24.4.1.1 Input/Output Relationships: Signal and Contrast

Contrast is a basic means of quantifying a signal, and is an important component of object detectability. Two definitions of contrast are commonly used. The Weber contrast ($C_w$) is relevant to situations describing small intensity changes on a large uniform background, as is the case for contrast-detail test objects, and is defined as:

$$C_w = \frac{\overline{d}_o - \overline{d}_n}{\overline{d}_o + \overline{d}_n},$$

(24.5)

where $\overline{d}_o$ and $\overline{d}_n$ are the mean pixel value (pixel intensity) within the object and in the neighborhood background, respectively. The Michelson contrast ($C_{\text{m}}$) applies to scenarios in which low and high intensities occupy similar fractions of the image area:

$$C_{\text{m}} = 2 \cdot \frac{\overline{d}_{\text{max}} - \overline{d}_{\text{min}}}{\overline{d}_{\text{max}} + \overline{d}_{\text{min}}},$$

(24.6)

This definition applies to periodic signals, for example in gratings, and is useful in derivations of the modulation transfer function (MTF).

24.4.1.2 Linearity and Stationarity of Digital Detectors

The first requirement for quantitative evaluation of detector signal and noise properties using transfer function analysis is that of linearity (see also Section I, Chapter 14 of this book). For a system with transfer characteristic $T\{\}$, acting on two input signals, $d_1(x)$ and $d_2(x)$ (i.e., in 1D), a system is linear if:

$$T[d_1(x) + d_2(x)] = T[d_1(x)] + T[d_2(x)].$$

(24.7)

Furthermore, multiplication of the input by a constant ($\Lambda$) multiplies the output by the same constant:

$$T[\Lambda d(x)] = \Lambda T[d(x)],$$

(24.8)

thus, the output is the same, regardless of whether the system acts on the inputs together or on each input individually followed by summation.

The second requirement is that the system must have a shift invariant response, such that the image of a point is independent of its position within the image. This property of shift invariance allows the definition of a system characteristic called the impulse response function (IRF) that describes how a point source incident on the system is imaged. If we consider an impulse, $\delta(x-x_0)$, incident on a linear system, at a position $x = x_0$, the result of the system acting on this impulse is $T[\delta(x-x_0)]$, thus the impulse response function:

$$\text{IRF}(x, x_0) = T[\delta(x-x_0)].$$

(24.9)

Using this concept of the IRF, for an input expressed as a superposition of many impulse functions incident on a linear system, then each impulse is imaged as one IRF in the image plane, independently of the others. The total image will be a sum of all the IRFs across the image plane: this is the superposition principle, whereby images can be built by summing many IRFs, each scaled by a constant. This superposition of IRFs leads to the superposition integral:

$$T[d(x)] = \int_{-\infty}^{\infty} d(x') \text{IRF}(x, x') dx'.$$

(24.10)

For a shift invariant IRF, this integral can be simplified to the convolution integral:

$$T[d(x)] = \int_{-\infty}^{\infty} d(x') \text{IRF}(x - x') dx',$$

(24.11)

or using the $*$ operator for convolution:

$$T[d(x)] = d(x) \ast \text{IRF}(x).$$

(24.12)

This equation describes the output signal when the input, $d(x)$, is transferred through a linear and shift invariant system. Cunningham (2000) uses the term “IRF” to describe the impulse response for the 1D case, and the point spread function (PSF) for a 2D imaging system.

24.4.1.3 Sampling and Aliasing

We are dealing with digital X-ray imaging systems, in which the incident X-ray photon image is first converted to charge or light photons and then sampled spatially. This is achieved by a discrete array of detector elements (referred to as “dels” or pixels [used in this work]) for FPDs while, for flying spot CR detectors, pixel spacing is determined by the diameter of the laser and the reader scanning pitch (Rowlands 2002). Sampling has important consequences for image formation. If $d(x,y)$ denotes the original continuous physical parameter that constitutes the image signal, then (perfect) sampling of this representation is obtained by multiplication by a 2D spatial sampling function, $s(x,y)$, composed of an infinite array of Dirac delta functions with even spacing ($\Delta x$, $\Delta y$) across the sampling grid:
\[
s(x,y) = \sum_{k_x=-\infty}^{\infty} \sum_{k_y=-\infty}^{\infty} \delta(x - k_x \Delta x, y - k_y \Delta y).
\]
(24.13)

The sampled image, \(d_s(x,y)\), is given by the sampling (comb) function acting on the input signal:

\[
d_s(x,y) = d(x,y)s(x,y)
\]
(24.14)

We take the Fourier transform (FT) of the sampling function:

\[
S(f_x, f_y) = \text{FT}\left[ \sum_{k_x=-\infty}^{\infty} \sum_{k_y=-\infty}^{\infty} \delta(x - k_x \Delta x, y - k_y \Delta y) \right]
\]
(24.15)

Then, by the Fourier transform convolution theorem, the Fourier transform of the sampled image can be written as

\[
D_s(f_x, f_y) = D(f_x, f_y) * S(f_x, f_y)
\]
(24.16)

The spectrum of the sampled image is seen as the spectrum of the (ideal) image infinitely repeated over the frequency plane, on a grid with spacing \((1/\Delta x, 1/\Delta y)\) (Albert and Maidment 2000). Figure 24.4 shows this for two cases, (a) in which the sampling grid frequency \((1/\Delta x, 1/\Delta y)\) is greater than twice the maximum frequency in the original spectrum \((f_{m,x}, f_{m,y})\), and (b) in which \(f_s\) sampling frequency is less than half the maximum frequency in the original spectrum.

For case (a), sampling is made at a sufficiently high frequency, such that no spectral overlap occurs, while, in (b), given a higher maximum frequency in the input, the sampling frequency is too low, leading to spectral overlap. This phenomenon is known as aliasing, and occurs when the sampling frequency is less than a critical frequency called the Nyquist frequency \((f_N)\). Practically, \(f_N\) is related to the pixel spacing in a given direction \((\Delta x)\), as \(f_N = 1/(2\Delta x)\) and, hence, the sampling period must be equal to or smaller than one half the period of the finest detail present.
in the image, if aliasing is to be avoided. Images sampled at a frequency higher than twice \( f_m \) are termed over-sampled, while images sampled at a rate lower than twice \( f_m \) are under-sampled. Dobbins (1995) discusses the implications of under-sampling on the interpretation of objective image quality metrics for digital X-ray imaging systems. Indirect conversion detectors can use correlation or blurring by the X-ray scintillator layer to band-limit the input signal, reducing or removing aliased frequencies. For systems with very sharp X-ray converters (e.g., photoconductors), steps can be taken to reduce aliasing by the introduction of a controlled degree of blurring at the pixel prior to sampling (“pre-sampling”) (Ji et al. 1998). Valid use of transfer function analysis to assess physical image quality requires the assumptions of linearity and shift invariance for the system under investigation. Application to digital detectors leads to the assumption of cyclostationarity (see e.g., Albert and Maidment 2000 or Cunningham 2000). The requirements for the fulfilment of shift invariance depend on the size of the objects to be imaged, and, hence, will be different for the imaging of microcalcifications (of the order of 100 \( \mu \text{m} \) or smaller) in mammography, compared to general radiography. For practical purposes, most systems can be considered linear or linearizable, and exhibit at least local stationarity.

24.4.2 Definition of Sharpness

One of the fundamental parameters relating to the quality of the images produced by a detector is image sharpness, which describes the ability of a system to transfer detail present in the input (X-ray) image through to the output image. This, in turn, is related to detector spatial resolution and, hence, the ability of a detector to separate or resolve adjacent features in the image. The starting point in the analysis of sharpness is the point spread function (PSF\((x,y)\)), given earlier and describing the intensity distribution in the image resulting from irradiation by an infinitesimally small aperture. In a perfect detector this distribution is localized to an idealized point; however, in real detectors the energy spreads out around the ideal point to a degree characteristic of the detector unsharpness (Rossman 1969). Instead of direct measurement of the PSF, it has been common to measure the line spread function (LSF\((x,y)\)), which is the response of the system to an extended (line) delta function. Instead of using the PSF or LSF to characterize unsharpness, the assumption of linearity and stationarity discussed in Section 24.4.1.2 allows the use of transfer function analysis to transform from the spatial to the spatial frequency domain. The PSF and the optical transfer function (OTF) are Fourier transform pairs:

\[
\text{OTF}(f_x,f_y) = \int \int_{-\infty}^{\infty} \text{PSF}(x,y) e^{-j2 \pi (f_x x + f_y y)} \, dx \, dy. \tag{24.17}
\]

The modulation transfer function (MTF) and the phase transfer function (PTF) are the modulus and phase of the complex function OTF, respectively:

\[
\text{OTF}(f_x,f_y) = |\text{MTF}(f_x,f_y)| e^{j\text{PTF}(f_x,f_y)}. \tag{24.18}
\]

In many cases, it is the transfer of contrast or signal modulation through the imaging system that is of interest, characterized using MTF, which can be calculated from the Fourier transform (FT) of the LSF for a given direction:

\[
\text{MTF}(f_x) = \text{FT}[\text{LSF}(x)]. \tag{24.19}
\]

The LSF can be generated using a slit or wire test object or alternatively using an edge test object from which the LSF is formed by differentiation:

\[
\text{LSF}(f_x) = \frac{d}{dx} [\text{ESF}(x)]. \tag{24.20}
\]

Cunningham and Reid (1992) explore the signal to noise ratio (SNR) associated with the slit, wire, and edge methods of measuring MTF, and show the SNR for the edge method to be superior at low spatial frequencies, while that of the slit method is superior at higher spatial frequencies.

It is helpful to consider the imaging system as a series of \((n)\) sub-systems, where the output of one component forms the input to the next. In the spatial domain, the full system PSF is joint 2D convolution of the individual component PSFs:

\[
\text{PSF}(x,y) = \text{PSF}_1 * \text{PSF}_2 * \ldots \text{PSF}_n. \tag{24.21}
\]

Transformation to the spatial frequency domain, the OTF of the complete system is the product of the sub-system OTFs:

\[
\text{OTF}(f_x,f_y) = \prod_i \text{OTF}_i(f_x,f_y). \tag{24.22}
\]

Hence

\[
\text{MTF}(f_x,f_y) = \prod_i \text{MTF}_i(f_x,f_y) \tag{24.23}
\]

and

\[
\phi(f_x,f_y) = \sum_i \phi_i(f_x,f_y). \tag{24.24}
\]

The utility of the linear systems approach is readily appreciated here, where once established, the system MTF is seen as a cascade of MTFs or spatial frequency filters, something that is much easier to conceptualize than a series of convolutions. The MTFs of the stages prior to sampling can be cascaded and then multiplied by the sampling comb to give the digital (sampled) MTF (Giger and Doi 1984):

\[
\text{MTF}_d(f_x,f_y) = \text{MTF}_\text{geom}(f_x,f_y) \text{MTF}_\text{conv}(f_x,f_y) \text{MTF}_\text{aperture}
\]

\[
(f_x,f_y) \sum_{k_x=-\infty}^{\infty} \delta\left(f_x - \frac{k_x}{\Delta x}\right) \sum_{k_y=-\infty}^{\infty} \delta\left(f_y - \frac{k_y}{\Delta y}\right), \tag{24.25}
\]

where \(\text{MTF}_\text{geom}(f_x,f_y)\) is the source/geometric unsharpness, \(\text{MTF}_\text{conv}(f_x,f_y)\) is the detector converter unsharpness,
MTF_{aperture}(f_x, f_y) is the pixel aperture and, finally, the sum of delta functions represents the sampling action of the pixel matrix. The product of the source, detector, and pixel aperture is termed the pre-sampling MTF (MTF_{pre}(f_x, f_y)):

$$\text{MTF}_d(f_x, f_y) = \text{MTF}_{\text{pre}}(f_x, f_y) \sum_{k_y=-\infty}^{\infty} \delta \left( f_y - \frac{k_y}{\Delta y} \right) \sum_{k_x=-\infty}^{\infty} \delta \left( f_x - \frac{k_x}{\Delta x} \right).$$

(24.26)

and this is the parameter used in the IEC protocol to characterize detector unsharpness. MTF_{source}(f_x, f_y) and MTF_{aperture}(f_x, f_y) are detector properties; a long FDD is employed to minimize the influence of source unsharpness. Readers are referred to Metz and Doi (1979) and Cunningham (2000) for in-depth application of transfer function analysis to X-ray imaging systems/detectors.

### 24.4.3 Measurement Method: Sharp Edge

One could use a direct measurement of the PSF from which the pre-sampling MTF is calculated to estimate the detector unsharpness; however, this is difficult for a number of reasons. The aperture used to generate the point source incident on the detector must be small compared to the PSF; fabrication of such a pinhole in a radio-opaque sheet is challenging. The resultant input signal to the detector is of very low intensity, and this can lead to noise influencing the measurement. Furthermore, using this method with pixelated detectors is problematic, as the measured PSF will depend on the position (i.e., the phase) of the aperture with respect to the pixel matrix. Some of these problems can be overcome by using line or knife-edge input sources to respectively assess the LSF or edge spread function (ESF), as these are somewhat easier to manufacture and yield greater intensities; however, the problem of position dependence remains. The solution to the phase dependence of stimuli used to digital imaging systems sharpness, first described by Judy (1976) for Computed Tomography (CT) scanners, and later by Reichenbach et al. (1991) and Fujita et al. (1992) for projection digital imaging devices, involves the angulation of the slit or edge test device against the pixel (or voxel) matrix to measure the pre-sampling MTF. While earlier work described the use of slit methods (Fujita et al. 1992; Dobbins et al. 1995), the edge method has become the preferred method for a number of reasons, including ease of manufacture and ease of positioning (Samei et al. 1998; Buhr et al. 2003).

The angled slit or edge is used to generate a composite LSF or ESF that is sampled at a higher rate than the native detector sampling pitch (“oversampled”). Figure 24.5 (adapted from Buhr et al. 2003) shows the basic approach. Taking a horizontal profile (along a row) in this figure generates an ESF sampled at the native pitch of $\Delta x$ mm. The small angle of the edge against the pixel matrix results in a shift of $p$ in the sampling position with respect to the edge position that is related to the angle of the edge:

$$p = \Delta x \tan \alpha.$$  

(24.27)

The average number of rows ($N_{ave}$) that result in a shift of 1 pixel is:

$$N_{ave} = \frac{\Delta x}{p} = \frac{1}{\tan \alpha},$$

(24.28)

thus, the edge must travel $N_{ave}$ rows vertically before the edge crosses one pixel in the $x$-direction. This information, derived from knowledge of the edge angle, enables the construction of a composite, over-sampled ESF. An alternative method is described by Samei et al. (1998), in which the 2D PV data in a region containing the edge are re-projected along the direction of the estimated angle to form a 1D ESF array, sampled at a sub-pixel pitch.

### 24.4.3.1 Location and Angle of Edge

Two commonly used methods are employed to find the edge angle required for these methods. An ROI containing the edge angle is differentiated and the position of the maximum ($\alpha$) along each row ($\gamma$) recorded. A first-order fit is applied ($\gamma = a + bx$, and the edge angle is given by $\alpha = \tan^{-1}(1/b)$, Alternatively, the ROI is converted to a binary image, differentiated (gradient operator), and a Hough transform applied from which the edge angle is retrieved.

### 24.4.3.2 ROI Dimensions for Oversampled ESF

The IEC document specifies an ROI of 100 mm $\times$ 50 mm to determine the MTF, positioned such that 50 mm extends under the radio-opaque edge and 50 mm extends into the adjacent bright field. This enables a reasonably accurate evaluation of the low-frequency drop (LFD) due to glare/long distance scattering in the X-ray converter layer and detector housing (Samei et al. 2006); ROI dimensions smaller than this may underestimate the LFD.
24.4.3.3 Construction of Oversampled ESF

In the methods of Fujita et al. (1992) and Buhr et al. (2003), a group of consecutive rows is selected with the condition that the shift of the edge transition in going from the 1st row to the \( N_{\text{row}} \)th row should be as close as possible to one pixel in the \( x \)-direction. A typical angle would be 2.5° and, hence, 22.9 (i.e., 23) lines are required to form one oversampled ESF. For a detector with a pixel spacing of 0.15 mm, this is only \( \sim 3.5 \) mm and, hence, the process can be repeated up the edge, generating successive oversampled ESFs whose edge \( x \)-position moves one pixel across the detector. These (contiguous) ESFs can realigned by estimating the center of the edge transition and averaging to form the final ESF. Buhr et al. (2003) show that ESFs should be averaged rather than the individual MTF curves (which introduces bias that inflates the MTF at high spatial frequencies). Furthermore, averaging two MTFs estimated from ESFs taken from overlapping sets of lines (by \( N_{\text{row}}/2 \) if possible) increases accuracy due to the averaging of phase errors.

The method of Samei et al. (1998) uses a re-projection of PV data in an ROI approximately centered on the edge transition. The oversampled ESF is assembled by addressing every pixel with the ROI and generating two arrays containing, respectively, the PV for the current pixel, and its distance \( s \) from the edge:

\[
s(y,x) = p(x \cos \alpha - y \sin \alpha),
\]  

where \( x \) is the column and \( y \) is the row number. This results in a fine but uneven sampling along the \( s \)-axis and, therefore, the samples are re-ordered and re-binned, with a re-binning size (\( \Delta s \)) chosen as a compromise between noise and resolution, typically \( \Delta s = 0.1 \Delta x \).

24.4.3.4 Data Conditioning of ESF

Images used for MTF evaluation will contain some noise, which will tend to limit the accuracy of the MTF estimate, especially at high spatial frequencies. The numerical differentiation step further amplifies the noise and, hence, some form of conditioning or smoothing is often applied to the ESF before differentiation. Given that MTF is used to characterize sharpness, application of smoothing to the ESF must be made with care. Samei et al. (1998) have used a fourth-order Gaussian-weighted moving polynomial fit, while Maidment and Albert (2003) describe a monotonicity constraint applied to the ESF that can greatly reduce noise without introducing bias, provided the constraint is appropriate to the ESF in question.

24.4.3.5 LSF and MTF Calculation

Once conditioned, the ESF is differentiated using a central difference algorithm to give the LSF:

\[
\text{LSF}_s = \frac{\text{ESF}_{s+1} - \text{ESF}_s}{2\Delta s},
\]  

where \( \Delta s \) is the oversampling pitch. A fast Fourier transform (FFT) is then applied to the LSF to obtain the MTF. This analysis type assumes that the submitted function is periodic and, hence, some authors apply a window function (e.g., Hanning) to the LSF that forces the extreme tails of the LSF to zero. While reducing noise in the LSF, windowing tends to suppress any long distance trends that may be present, over-estimating the MTF at low spatial frequencies, and, hence, should be used with caution. The final steps are FFT of the LSF, to give the MTF followed by normalization to MTF\([0\]). The frequency resolution in mm\(^{-1}\) of the final MTF is related to the physical length of the LSF (\( L \) mm): \( df = 1.0/L \).

24.4.4 Pixel Aperture, Fill Factor, and Signal Aliasing: EMTF

These steps describe the measurement of the pre-sampling MTF and, from Equation 24.25, it can be seen that the MTFs from source and detector unsharpness are multiplied by the pixel sampling MTF. Hence, the influence of the sampling pixel fill factor should be evident in the measured pre-sampling MTF for systems where these unsharpness sources are low in magnitude (Zhao et al. 1997). For systems with rectilinear pixels of size \( a_x \) and \( a_y \), the transfer function associated with the pixel aperture, \( P(f_x,f_y) \), is a sinc function:

\[
P(f_x,f_y) = \frac{\sin(\pi f_x a_x)}{\pi f_x a_x} \frac{\sin(\pi f_y a_y)}{\pi f_y a_y}, \tag{24.31}
\]  

Hence, the fill factor (e.g., \( \eta \)) governs the point at which the pre-sampling MTF touches the respective spatial frequency axis (e.g., \( f_0 \)) and can be calculated as:

\[
\eta_s = \frac{1}{a_x f_{0,x} \Delta s}; \quad \eta_s = \frac{1}{a_y f_{0,y} \Delta s}. \tag{24.32}
\]  

Thus, the smaller the fill factor, the higher the touch point above the nominal frequency.

For the case of under-sampled systems, such as those in which the sample distance is not sufficiently high enough to capture the frequency content of the input image without aliasing, Dobbins (1995) suggests that under-sampling leads to phase dependence of the digital MTF. Dobbins (1995) has proposed the expectation MTF (EMTF) as a parameter that can be used to quantify average “signal” response, in the face of this phase dependence for under-sampled systems. The EMTF is calculated by averaging the modulus of the sampled digital modulation transfer function (e.g., \( MTF_d(f_x, a_i) \)) over all possible phase values for a systematic (phase) shift (\( x_0 \)) of the edge across a pixel of size \( a_i \):

\[
\text{EMTF}(f_x) = \frac{1}{a_x} \int_{-a_x}^{a_x} \left| \left| \text{MTF}_d(f_x,x_0) \right| \right| dx_0. \tag{24.33}
\]  

For practical calculation, the modulus of \( \text{MTF}_d \) for a given phase shift \( x_0 \) is determined by summing the pre-sampling MTF (\( \text{MTF}_{\text{pre}} \)):

\[
|\text{MTF}_d(f_x,x_0)| = \sum_{k=0}^{\infty} e^{-\frac{i2\pi k L}{a_x}} \text{MTF}_{\text{pre}} \left| f_x - \frac{k}{a_x} \right|. \tag{24.34}
\]  

Only the first two or three terms are needed for this sum, due to the limited bandwidth of \( \text{MTF}_{\text{pre}} \) (Dobbins 1995). Figure 24.6a
FIGURE 24.6  (a) Pre-sampling MTF and expectation MTF (EMTF) curves calculated for two mammography systems: system 1 has a large pixel pitch and is under-sampled given the pre-sampling MTF, while system 2 has a small pixel pitch. (b) 2D pre-sampling MTF measured for an a-Se digital mammography detector using a disc test object.
shows MTF_{pre} and EMTF for two mammography detectors, with quite similar pre-sampling MTF curves, but with pixel pitches of 0.085 and 0.050 mm, respectively. EMTF is close to MTF_{pre} for the finely sampled detector (system 2), while for the detector with large pixel spacing (system 1), EMTF remains high at the Nyquist frequency (5.88 mm^{-1}), indicating the presence of considerable signal aliasing.

### 24.4.5 2D MTF

While standard edge or slit based methods give MTF_{pre} in two orthogonal directions across the detector, a number of methods have been described for the measurement of the full 2D pre-sampling MTF. Fetterly et al. (2002) imaged a 0.5 mm Cu plate containing 256 holes of diameter 0.107 mm that, when imaged, produced an array of points. An over-sampled PSF is generated from which MTF_{pre} is calculated by Fourier transformation. Båth et al. (2003) described a similar method utilizing an aperture mask containing 100 holes of diameter 0.1 mm in a PMMA/Pb foil. More recently, Monnin et al. (2016) implemented a variation of the Thornton method, described initially for CT imaging (Thornton and Flynn 2006), but for 2D projection images. A 50 mm diameter W disc of thickness 0.5 mm is imaged; the center of mass of the disc image is established and the pixel data in some angular arc re-ordered according to their distance from the center of mass to give a 1D pre-sampling ESF for that arc. This is repeated to form ESFs for all the angular arcs around the disc. The corresponding MTF_{pre} curves are then used to build a 2D MTF using weighting matrices. Figure 24.6b shows 2D MTF measured for an a-Se based digital mammography detector with pixel spacing 0.085 mm. Although largely isotropic, some slight MTF_{pre} anisotropy can be seen between the two directions across the detector.

### 24.4.6 Practicalities (Edge Size, Material, Thickness, Straightness, Exposure Level)

Some practicalities are common to the edge test devices used with mammography and general radiography detectors. The edge must be radiopaque, with a straight edge polished such that no ripples larger than 5µm are seen on the edge, when imaged with screen-less film. Neitzel et al. (2004) have modeled the effect of using non-opaque (semi-transparent) materials for the edge, and showed that scattered radiation generated within the edge reaches the detector, ultimately leading to an increase in MTF at all spatial frequencies. The edge must be sufficiently large such that the long distance scatter within the detector (i.e., glare) responsible for the LFD is captured in the over-sampled ESF.

#### 24.4.6.1 Mammography Detectors

A standard edge test device is defined in IEC 62220-1-2, composed of stainless steel, of minimum dimensions: 0.8 mm thick, 120 mm long, and 60 mm wide. The edge is positioned immediately in front of the detector, such that the center of the edge is 60 mm from the center of the chest wall, in a radiation field that is at least 100 mm × 100 mm. IEC guidance (IEC 2007) states that the irradiation of the edge should be made at one of the three exposure levels; to control the influence of noise on the MTF, the DAK should be 2-times the reference value (i.e., typically 100–200 µGy) using the chosen RQA quality (Table 24.2). Two acquisitions should be made: one with the edge approximately oriented with the detector matrix columns, and the next oriented to the rows. For CR detectors, the edge should be rotated 90° between acquisitions, enabling MTFs to be estimated for low to high and high to low exposure transitions.

#### 24.4.6.2 Diagnostic Detectors

For diagnostic detectors, IEC (2003) describe an edge test device composed of a 1 mm thick tungsten plate (100 mm × 75 mm) set into a 3 mm thick lead plate (200 mm × 100 mm). Again, the edge is positioned immediately in front of the detector and imaged using the chosen spectrum (Table 24.1) at one of the three DAK levels. If 2.5 µGy is taken as “reference,” then the DAK should be 3.2-times the reference exposure (8–10 µGy) to reduce the influence of noise on the MTF. The edge device is aligned as closely as possible to the central axis of the X-ray beam; the number of exposures is the same as for mammography detectors.

#### 24.4.7 Some Example MTF Results

We briefly discuss some examples of MTF_{pre} curves for mammography and diagnostic detectors. Figure 24.7a shows pre-sampling MTF measured for two orthogonal directions across a CsI-phosphor based mammography DR detector. MTF_{pre} is almost identical for the two directions, indicating excellent sharpness isotropy, a typical MTF property of scintillation phosphor based DR detectors. Also indicated on the curve is the sudden drop at low spatial frequency (the LFD discussed earlier), consistent with long distance scattering of photons in the phosphor/detector assembly. Figure 24.7b plots MTF_{pre} measured in the left-right and front-back directions of an a-Se based mammography detector, along with the sinc function associated with the pixel (nominally a_{x} = a_{y} = 0.085 mm). First, it can be seen that there is some sharpness non-isotropy between two directions. The influence of the pixel is seen clearly in the MTF_{pre} curves, where we expect f_{0,1} = 11.8 mm^{-1} for a fill factor η = 1.0. In fact, f_{0,1} ≈ 12.55 mm^{-1} (front-back) and f_{0,1} ≈ 11.89 mm^{-1} (left-right) yielded approximate fill factors of 0.94 and 0.99.

Application of MTF measurements to a flying focus digital breast tomosynthesis (DBT) system illustrates the value of cascading MTF curves of sub-components (Figure 24.7c). Just the sharpness in the tube-travel direction is considered. The starting point is MTF_{pre} measured in conventional 2D mode (i.e., static X-ray tube). MTF curves associated with the X-ray focus size (0.42 mm, measured at 6 cm from the chest wall edge), the focus motion (21 ms pulse length, giving focus travel length of 1.04 mm during one exposure), and detector binning to 0.14 mm pixels are shown, calculated using sinc functions. The focus motion MTF is calculated for an object position 4 cm above the breast table. The measured MTF_{pre} is indicated using points in this figure; good agreement is seen with the calculated MTF_{pre}, estimated from the detector MTF (static 2D mode), and the motion and binning MTFs using Equation 24.25. Finally, MTF_{pre} curves for a diagnostic CR detector are plotted in Figure 24.7d, measured in the laser readout scan and sub-scan directions using an edge method. Sharpness is reduced in the scan direction, where the laser moves...
FIGURE 24.7  (a) Pre-sampling MTF for a CsI-phosphor digital mammography detector showing some LFD, (b) pre-sampling MTF measured in two directions across an a-Se mammography detector showing influence of pixel aperture on MTF, (c) MTF curves for DBT system illustrating cascading of component MTFs, and (d) pre-sampling MTF for a diagnostic needle structure CR detector showing anisotropy between laser scan (fast) and sub-scan (slow) directions.
24.5 Noise

24.5.1 Definition

Image noise is defined as the statistical fluctuation of the signal (pixel values) in the image from one pixel to another around the mean signal value. The importance of noise in radiography is clear: random variations in the image limit our ability to discern small/low contrast objects in medical images. The constraint of minimizing patient exposure imposes limits on the available SNR of images, and makes photon statistical noise unavoidable.

24.5.2 Noise Sources

Noise sources in digital imaging can be assigned to three sources: quantum, electronic, and fixed pattern (structure) (Nishikawa and Yaffe 1990). Quantum noise in radiography arises from several causes, for example random spatial fluctuations of photons emitted from the X-ray tube, random fluctuations of photon attenuation and scattering in the materials traversed by the beam, and random capture of photons by the detector. Production and detection of X-rays in medical imaging are stochastic processes that give rise to random spatial fluctuations in the image signal. The quantum noise in a radiography image is determined by the stage with lowest number of photons along the imaging chain, a point termed the “quantum sink” (Cunningham et al. 1994). Amplifying the number of photons after the quantum sink in the imaging chain cannot reduce the quantum noise any more—only an increase in the number of photons at the quantum sink is capable of decreasing quantum noise. In properly designed and functioning X-ray systems, quantum noise is the principal noise source in a radiological image; this is spatially stationary for a homogeneous X-ray field.

Electronic noise introduced by a detector is a stochastic, additive signal arising, for example, from pixel dark current noise, noise fluctuations on the gate, and data lines in the readout process and amplifier noise (Siewerdsen et al. 1997), and is independent of the other noise sources and of the number of incident X-ray photons. Structured or fixed pattern noise is a superimposed static signal arising from dead pixels, structural detector linearity, or spatial fluctuation in X-ray sensitivity, non-uniformities in exposure from the X-ray beam (heel and geometric effects). Fixed pattern noise has a deterministic pattern that is largely removed in flat-panel detectors using a flat-fielding correction (Schmidgundt et al. 2007). Physical characteristics of the three noise sources are developed in Section 24.5.9.

24.5.3 First-Order Statistics: Standard Deviation and Variance

The PV at spatial position \( i \) is defined as the signal \( (d_i) \). It is a random variable that can be characterized in a region of interest (ROI) of \( N \) pixels by its average value \( \langle \bar{d} \rangle \) and variance \( \sigma^2 \)

\[
\bar{d} = \frac{1}{N} \sum_{i=1}^{N} d_i \tag{24.35}
\]

\[
\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (d_i - \bar{d})^2. \tag{24.36}
\]

These parameters are quantified from homogeneous images using first-order statistics, namely the standard deviation (\( \sigma \)) of the signal. This is the quadratic average of the difference between each PV and the mean PV in the ROI; the statistical distribution of the PVs around the mean value defines the noise amplitude, the symmetry of the PV distribution, the upper and lower limits of PVs, and the likelihood of observing given values. Figure 24.8 shows the histograms of PV frequency in these two cases. A variance image can be generated by calculating the variance within nominal (but generally small) ROIs (e.g., \( 2 \times 2 \) mm) across the entire image, and assigning a grey scale value to the variance (Marshall 2006a,b; Monnin et al. 2014). This gives considerable insight into the properties of the noise at a local level, and can help to identify artefacts and regions of local blurring (Marshall 2006b).

24.5.4 Auto-covariance and NPS

Using first-order statistics neglects the spatial characteristics of the noise structure (i.e., the “texture”) and its possible correlation. Signal scatter (charge sharing) between PVs can occur during the detection process, introducing short range signal correlation. This form of correlation takes place in real detectors and reduces signal variability (noise) over short distances (high spatial frequency), but not at long distances (low spatial frequency). Correlation in the spatial domain can be quantified with second-order statistics such as the auto-covariance function defined by Equation 24.37 for data sampled in a discrete space grid with a regular sampling interval equal to \( \Delta x \).

\[
\text{cov}(k) = \frac{1}{N - k - 1} \sum_{i=1}^{N-k} (d_i - \bar{d})(d_{i+k} - \bar{d}) \tag{24.37}
\]

The auto-covariance function \( \text{cov}(k) \) provides a measure of similarity for signal variations between two pixel separated by a distance, \( k \cdot \Delta x \). In the absence of spatial correlation, the auto-covariance function will be equal to the variance at the zero-value and zero at non zero-values. One notices the zero-value of the auto-covariance function is always equal to the signal variance.

The same statistics are more commonly described in the spatial frequency domain by the noise power spectrum (NPS), also called Wiener spectrum (Dainty and Shaw 1974; Cunningham 2000). The NPS is the modulus of the Fourier transform of the auto-covariance function (Wiener Khinchin theorem), and can be calculated from a ROI of \( N_x \times N_y \) pixels in the image as:

\[
\text{NPS}(f_x, f_y) = \frac{\Delta x \Delta y}{N_x N_y} \left| \int \int (d(x,y) - \bar{d}) e^{-j2\pi(xf_x + yf_y)} dx dy \right|^2. \tag{24.38}
\]
where the brackets indicate an average over several images. The NPS is the noise power per unit frequency. The small area \( \text{NPS}(f_x, f_y) \, df_x, df_y \) in the Fourier plane is the contribution to the variance for spatial frequencies between \( f_x \) and \( f_x + df_x \), and \( f_y \) and \( f_y + df_y \). The NPS represents the variance as a function of spatial frequency. The NPS versus spatial frequency is constant in the absence of noise correlation (i.e., a “white” noise, with equal power at all spatial frequencies within the bandwidth), and decreases with frequency when PVs are spatially correlated.

**24.5.5 Measurement Method**

**24.5.5.1 Homogeneous Images**

Noise assessment requires spatially uniform images in which the signal variations are only due to detector noise. Homogeneous radiographic images could be acquired without anything in the beam (only air), but, in practice, an attenuator is placed in the beam such that the detector exposure and beam quality resemble those seen clinically and, hence, simulate noise characteristics relevant to medical images. See Section 24.3 for more details.

**24.5.5.2 Region of Interest for Noise Calculation**

Noise characterization is made using images that are uniform or in ROIs where the signal is uniform—typically using square ROIs taken from the image center. The ROI has to include the reference point used for dose measurements, fixed at the center of the beam in standard radiography and at 6 cm from the chest wall edge in mammography. The precision of the noise measure will depend on the number of pixels in the ROI. The number of pixels required for noise assessment is determined by the statistical accuracy of the calculation—for an accuracy of 5%, a minimum of 16 million pixels is required for 2D NPS, and 4 million pixels for 1D NPS (IEC 2005). This number of independent pixels requires multiple images obtained in the same conditions. The standard deviation of the irradiations used to get the different images shall, therefore, be less than 10% of the mean (IEC...
Care shall be taken there is no correlation introduced by lag effects between the same pixels of consecutive images, reducing the noise compared to independent noise samples. Lag is evaluated separately (IEC 2005); see for example Siewerdsen and Jaffray (1999) for a detailed discussion on this topic.

An accurate estimate of fixed pattern noise requires an average noise estimation over many realizations of FP noise taken on several different ROIs. Each ROI should be shifted between successive images used for NPS calculation at a given DAK level, starting from the image center. This will ensure that a fresh sample of FP noise is included in each ROI to improve ergodicity. If this were not done then noise averaged from a sequence of static ROIs on successive images (temporal average) may yield a different result from a spatial average over many ROIs performed on one image of the sequence (Dobbins 2000).

24.5.5.3 Ergodicity and Stationarity (Spatial and Temporal Averages)

A process having spatial stationarity of the mean value and autocovariance is a wide-sense stationary (WSS) process (Cunningham 2000). This means that the first moment (the mean) and autocovariance are spatially constant across the image and will not change as a function of the spatial position and size of the ROI. Only the statistical precision of the measured parameters will change with the ROI size for a WSS signal. The integral of the NPS will be equal to the PV variance in the ROI for a WSS signal. Wide-sense stationarity is only true in radiographic images if exposure is spatially uniform, and the detector does not add fixed pattern noise.

Ergodicity is required when noise assessment is made from several identical images. Ergodicity claims that the average noise calculated from ROIs taken at various locations in an image is equal to the noise averaged from a (temporal) sequence measured of images acquired using identical settings. Hence, ergodicity holds that the evolution of a random signal over time (subsequent images) provides the same information as a set of noise ROIs taken in the same image.

24.5.5.4 Data Conditioning for Noise Assessment (Linearization and De-Trending)

In the framework of imaging system performance assessment, the NPS or standard deviation shall be calculated using linearized data, such as PV re-expressed in values proportional to the number of photons incident on each pixel. The noise magnitude is dependent on PV units: the standard deviation is expressed in pixel values, and the NPS unit is (PV × mm)². Comparison of noise between systems needs a common unit for pixel values, and noise is generally calculated using linearized images where the pixel values are re-expressed in detector air kerma (DAK) or photon fluence per unit area (Q) using a conversion function determined at the same beam quality and different DAK (Section 24.3.3).

Practically, signal and noise in radiographic images are never completely homogeneous, and requirements of WSS are never fully met. Large area spatial variations in detector dose due to the heel effect or beam divergence should, therefore, be removed before noise measurements (Williams et al. 1999; Dobbins 2000). An efficient de-trending method is, therefore, necessary for an accurate evaluation of variance and NPS. Low-frequency trends can be effectively reduced by fitting and subtracting a 2D second-order polynomial to the ROI used for noise assessment before noise calculation, as prescribed by the IEC protocol (IEC 2005). Figure 24.9 shows a typical image de-trending correction.

Fixed pattern (non-stochastic) noise may be completely suppressed by dividing each ROI used for noise estimation by the average ROI of all the ROIs before noise calculation. The average

![Figure 24.9](image-url) Use of a 2D second-order polynomial for low-frequency noise subtraction. (a) Actual image region for the NPS calculation. (b) The 2D polynomial fit to this region. This 2D array is subtracted from the region in (a) to give the de-trended array from which the NPS ROIs will be extracted.
ROI contains a fraction of stochastic noise correlated to each of the ROIs, and the resulting NPS has to be finally corrected by the factor \( \frac{N}{N-1} \), where \( N \) is the number of ROIs. This correction is the same as that used to remove background variations by flat-fielding techniques (Schmidgunst et al. 2007), and it is important to note that this operation will also remove structural noise of the detector from the measured noise. Figure 24.10 shows an example of the difference between the raw NPS and NPS obtained after image correction.

### 24.5.5.5 2D NPS and Reduction to 1D

(Axial and Radial Sectioning)

The 2D NPS is calculated with Equation 24.38, and then reduced to a 1D NPS curve along the \( x \) - and \( y \) - axes (NPS\(_x \) and NPS\(_y \)) by averaging 14 rows or columns of the 2D NPS around each axis (seven rows or columns on both sides of the corresponding axis) (IEC 2005). The Fourier axes are usually omitted since the zero \( x \) - and \( y \) - frequency NPS are not representative of detector properties (Dobbins et al. 1995). The frequency bandwidth of the 1D axial NPS will be limited within \( \left\{ \frac{1}{\text{FOV}_x}, \frac{1}{2a_x} \right\} \), where \( \text{FOV}_x \) and \( a_x \) are the image field of view and pixel size in the \( x \)-direction, respectively. The exact frequencies of all NPS points shall be determined and reported for the axis considered for calculation.

The radial 1D NPS is obtained by expressing NPS as a function of radial frequency, \( \sqrt{f_x^2 + f_y^2} \). A cloud of NPS points will be obtained, excluding the points on the Fourier axes. An averaged radial NPS curve may be obtained with an interpolated curve, which passes best through the points, generally binned into frequency bins of 0.05 mm\(^{-1}\). Figure 24.11 shows an example of 2D NPS and the corresponding radial 1D NPS points and interpolated curve. The frequency bandwidth of radial NPS will extend until \( \sqrt{2/a_x} \) for NPS points at 45°. Compared to axial NPS, radial NPS has the advantage of being averaged over all the directions, and not only for two directions, and is, therefore, more representative of the global noise characteristics of the image.

Examination of the 2D NPS is important and may show up noise anisotropy, spikes, or artifacts in the Fourier plane that will not be seen on 1D NPS. It is, therefore, important to examine 2D NPS for a complete noise analysis, and a 1D radial averaged or axial 1D NPS should be plotted only for isotropic NPS. Figure 24.12 shows the presence of noise spikes on the 2D NPS of a CR system (left), and an example of strongly non-isotropic 2D NPS with correlation only along the horizontal axis.

#### 24.5.5.6 Uncertainties of NPS Estimate

Noise in a radiographic image is a random process, and samples of noise contain an inherent statistical uncertainty in comparison to the (true) noise of the whole population of pixel values. The uncertainty in the Fourier decomposition is on both the NPS amplitude and for the frequencies; the uncertainty in the amplitude at a given frequency can be reduced by averaging several NPS estimates obtained from different noise realizations in several independent ROIs. By averaging a number of \( n \) NPS, the relative uncertainty of the NPS estimate will be reduced by a factor, \( 1/\sqrt{n} \), for normally distributed PVs, as Poissonian noise can be considered as normally distributed for large PV sample numbers. The IEC norm requires, therefore, a minimum of 960 half-overlapped ROIs, thus 480 independent ROIs samples for an accuracy less than 5% in the NPS amplitude, \( 1/\sqrt{480} \approx 4.6\% \).

The spectral resolution is, however, inversely proportional to the number of data points used to generate each NPS (Williams et al. 1999). The relative spectral uncertainty (\( \Delta f \)) obtained from a sample of \( N \) pixels will, therefore, be equal to \( 1/N \). Thus, for a fixed amount of pixels, there is a tradeoff between frequency and NPS amplitude uncertainties. If \( N \) pixel values are divided into \( n \) independent ROIs, the relative uncertainty in NPS amplitude (\( e_{nPS} \)) and frequency (\( e_f \)) are

\[
e_{nPS} = \frac{\Delta NPS}{NPS} = \frac{1}{\sqrt{n}} \tag{24.39}
\]

and

\[
e_f = \frac{\Delta f}{f} = \frac{n}{N}. \tag{24.40}
\]
FIGURE 24.11 (a) Grayscale representation of 2D NPS. (b) Point cloud showing each NPS value in (a) plotted against radial frequency. The line shows the interpolated 1D radial NPS.

FIGURE 24.12 NPS of a CR system showing spikes (a) and non-isotropic NPS colored (correlated) only in one direction (b).
24.5.6 Practicalities

The noise level for a given imaging system will depend on the beam quality, determined by the tube voltage (kV) and total beam filtration, and on the photon fluence at the detector surface (Q). See Section 24.3 for information on image types and standard beam qualities used for detector characterization.

24.5.6.1 Mammography Detectors

Compared to diagnostic systems, which tend to be based exclusively on tungsten targets with aluminum filtration, mammography systems use a wider range of targets and filters, including molybdenum or rhodium targets with molybdenum, rhodium, and silver filtration. Table 24.2 lists some common beam qualities used for mammography detector characterization. These additional aluminum filters may, however, add low-frequency fixed pattern mottle, especially below 0.5 mm⁻¹, coming from the internal structure of aluminum plates. De-trending is not entirely effective in removing this low-frequency structure and may form a significant component of the sampled noise, by visual inspection of images, and by calculation of the NPS and checking for peaks at low spatial frequency.

24.5.6.2 Diagnostic Detectors

Radiation qualities and exposure levels for diagnostic detectors are given in Section 24.3. In contrast to mammography detectors, the air kerma at the detector surface is measured at the reference point centrally situated in the beam. Similar comments apply regarding possible structured noise that can be introduced from the Al RQA filtration.

24.5.7 Interpretation of 2D NPS

The amplitude of the 2D digital NPS will be limited by the pixel aperture of the detector (a in the x-direction and a in the y-direction). The digital quantum NPS is given by the product (in the Fourier domain) of the analog quantum NPS of the radiant image (NPS_rad) and the pixel aperture function, the absolute square of the 2D sinc function (the Fourier transform of a 2D boxcar function). The result is the pre-sampling quantum NPS (NPS_quad). The pre-sampling NPS is, therefore, always colored by the deterministic blurring, due to the finite pixel aperture in the x- and y-directions (a and a).

\[
\text{NPS}_{\text{quad}}(f_x, f_y) = \text{NPS}_{\text{rad}}(f_x, f_y) \cdot \frac{\sin^2(\pi \cdot a \cdot f_x)}{a^2} \cdot \frac{\sin^2(\pi \cdot a \cdot f_y)}{a^2}.
\]  

(24.41)

When sampled by the pixel grid, frequency components of the pre-sampling quantum NPS above the Nyquist frequency are aliased, and may form a significant component of the sampled quantum NPS. The sampled quantum NPS will be white if no signal blur occurs within the detector (Zhao and Rowlands 1997). The 2D digital NPS has finite length given in each dimension by the number of pixels (N_x or N_y) in the ROI used for NPS estimate and the sampling interval (pixel pitch). The highest NPS frequency is given by the Nyquist frequency (f_N), determined by the pixel pitch in the given direction (\(\Delta x\) or \(\Delta y\)). The field of view of the ROI used for NPS calculation determines the lowest NPS frequency (f_min).

\[
f_{x,\text{min}} = \frac{1}{N_x \cdot \Delta x} \quad \text{and} \quad f_{y,\text{min}} = \frac{1}{N_y \cdot \Delta y}
\]

(24.42)

\[
f_{x,\text{max}} = \frac{1}{2\Delta x} \quad \text{and} \quad f_{y,\text{max}} = \frac{1}{2\Delta y}.
\]

(24.43)

24.5.8 Normalization and Units: NPS versus NNPS

The magnitude of the NPS is dependent on the PV units. Following the IEC protocol, the images used for the calculation are multiplied by the inverse of the conversion function (i.e., the inverse of the response function), resulting in images with units of mm⁻². The IEC gives this equation for the NPS:

\[
\text{NPS}_{\text{out}}(f_x, f_y) = \frac{\Delta x \Delta y}{N_x N_y} |\text{FFT}[\text{PV}(x,y) - \overline{\text{PV}}]|^2,
\]

(24.44)

giving NPS_out with units of [mm²][mm⁻²]², thus mm⁻². Note that NPS_out increases as exposure to the detector increases. The NPS is often expressed in normalized form by dividing by the square of the mean pixel value of the images from which NPS is calculated:

\[
\text{NNPS}_{\text{out}}(f_x, f_y) = \frac{1}{\overline{\text{PV}}} \frac{\Delta x \Delta y}{N_x N_y} |\text{FFT}[\text{PV}(x,y) - \overline{\text{PV}}]|^2,
\]

(24.45)

If the PV data are given some unit, for example, of air kerma (using a response function with PV plotted against air kerma), then it can be seen that the normalization always cancels units in the PV data. Hence, this normalization is valuable for comparing data when the NPS is not expressed in the same units, or for compensating for differences in gains between different systems (provided any PV offset is first removed). The NNPS, therefore, has units [mm²], and its value does not depend on the pixel value unit any more. In contrast to NPS_out, NNPS_out decreases as exposure to the detector increases (and has the inverse units). The NPS normalization can be pushed further for compensating the noise for small exposure differences by multiplying the NNPS by the photon fluence at the detector (Q). The expression NNPS \(\times Q\) is unit-less and allows a direct comparison of system performance in terms of noise.

24.5.9 Noise Decomposition

Information on image noise composition and generation may be obtained by splitting the noise into its three main sources: quantum, electronic, and fixed pattern components. Noise decomposition can show how detector parameters such as detector...
composition and thickness affect the noise sources in relation to the detector air kerma and beam quality. In this section, the term noise and the variable $S^2$ are used for either the variance ($\sigma^2$) or the NPS for each frequency bin ($S^2 = \sigma^2$ or $S^2 = \text{NPS}(f)$ for a given frequency bin $f$).

### 24.5.9.1 Basis of Noise Decomposition: Exposure Dependencies of Noise Sources

For a homogeneous X-ray field, quantum noise can be modeled with a Poisson distribution (Rimkus and Bailey 1983). The variance of a Poisson distribution is equal to the mean value of the distribution. In medical imaging devices, the signal is generated by a large numbers of photons per pixel, and the Poisson distribution can be approximated by a Gaussian (normal) distribution with high precision. Thus, the quantum noise distribution will be symmetric around the mean pixel value, and the probability of having a pixel value in a given interval can be calculated using statistical tables, based on the normal law with a variance equal to the mean value.

In the presence of quantum noise only, $S^2$ will be proportional to the average number of “events” incident on the detector. For a mean value of $q$ photons detected per pixel, the standard deviation of the number of detected photons from a pixel to another will be proportional to $\sqrt{q}$. For a digital detector which produces a signal proportional to the number of detected photons per pixel, with a proportionality factor (global detector gain) equal to $\overline{q}$, the quantum noise ($S^2_q$) will be proportional to $q$: 

$$S^2_q = \overline{q} \cdot q. \quad (24.46)$$

Quantum noise is unavoidable in an X-ray image, and will be present in a perfect detector (i.e., this would be the lowest possible noise level for some incident fluence). Real detection systems have higher levels of quantum noise (due to imperfect absorption and conversion to signal), but due also to additional noise sources. These include conversion noise due to the variability in the number of secondary quanta produced for a given input X-ray photon (Cunningham 2000) and Lubberts noise (Lubberts 1968) due to the variation in the magnitude of light (signal) pulses reaching the pixels. The magnitude of these sources scales linearly with the incident X-ray fluence.

Simple models of X-ray devices consider two other noise components, in addition to quantum noise: electronic and fixed pattern noises. Electronic noise ($S^2_e$) arises from stochastic signal fluctuations introduced in signal amplification and quantization stages, and is independent of signal level. Detectors contain structural fluctuations in sensitivity, due, for example, to detector layer granularity, leading to a spatial variation in gain. These pixel-to-pixel differences remain constant, and result in a fixed pattern signal (noise) on the images, ($S^2_{fp}$). This non-stochastic signal can be assimilated to a spatially fixed noise proportional to the square of the number of detected photons:

$$S^2_{fp} = \overline{q}^2 \cdot \overline{q}^2. \quad (24.47)$$

These three noise components are independent, and their variances simply add to form the resulting noise ($S^2$) on the image (Nishikawa and Yaffe 1990):

$$S^2 = S^2_q + S^2_e + S^2_{fp}. \quad (24.48)$$

The noise (variance) in an image will, thus, increase quadratically with the number of incoming photons (or with DAK).

### 24.5.9.2 Noise Decomposition with the Polynomial Model

Using this simple three component model, noise may be decomposed using a second-order polynomial function of the photon fluence ($Q$) or DAK fitted to the noise data (variance or NPS at a given frequency) using least squares regression (Borasi et al. 2003; Burgess 2004; Bouwman et al. 2009), for example:

$$\sigma^2_{oa} = e + qQ + fpQ^2 \quad (24.49)$$

and

$$\text{NPS}_{oa}(f) = \text{NPS}_e(f) + \text{NPS}_q(f)Q + \text{NPS}_{fp}(f)Q^2. \quad (24.50)$$

The weighting of the fitted noise data in least square regressions can have a strong influence on the accuracy and precision of the results (Monnin et al. 2014). If used, the choice of weights should ideally be made such that each noise data provides equally precise information during the fitting (i.e., the error term minimized during the fitting is weighted as a function of the variability of noise data at the different exposure levels). This may be achieved by choosing a weight inversely proportional to the variance of the noise data at each dose level. This requires measurement of variance of the noise samples at the different dose levels (many noise images are required at each dose level). It is important to note that the weighting is based on the assumption that the weights are known exactly, but in practice this is never the case, since an infinite set of noise measurements at a given dose level would be required. However, when the weights are poorly estimated, the regression can be adversely and unpredictably affected. Rather than using a limited number of noise samples at each exposure level, estimated weights can be used instead—photon fluence can be used as an estimate of the weighting instead of measured variance of noise, when the sample variance is not known with sufficient precision (Monnin et al. 2014). Although something of a simplification, this model enables reasonably realistic modeling of the noise in detectors, and is an important element of image simulation and related detection studies in mammography, for example (Mackenzie et al. 2012).

Figure 24.13a shows decomposition of the variance for an a-Se mammography detector: electronic noise is seen to dominate at low exposures, while fixed pattern noise dominates at high DAK levels. The approximate quantum limited range for this example (in terms of image variance) can be estimated as follows: electronic noise dominates below levels $< qfp \sim 5.427 \times 10^{-1}/2.587 \times 10^{-2} \sim 21 \mu\text{Gy}$, while fixed pattern noise dominates above $qfp \sim 1450 \mu\text{Gy}$. The same image data can be used for decomposition into NPS, with results shown in Figure 24.13b–d. At very low DAK (much lower than typically seen in clinical images), electronic NPS (NPS$_e$) is the dominant noise source (largest component of NPS$_{tot}$). It can be seen that the
magnitude of $NPS_e$ is constant, and the shape is approximately flat, consistent with a white noise source. At the clinical operating point of the detector ($\sim 100 \mu GY$), $NPStot$ is composed primarily of X-ray quantum noise, as expected for a well-designed detector. At high exposures, fixed pattern noise starts to dominate the image, but only for spatial frequencies below $\sim 3 \text{ mm}^{-1}$; above $3 \text{ mm}^{-1}$, quantum noise remains the dominant noise. Furthermore, spikes (periodic structures) in $NPStot$ start to appear in the total NPS.

### 24.5.9.3 Noise Decomposition without Noise Model

An explicit method of noise decomposition with minimal assumptions regarding the magnitude and signal dependency of the three noise components may be used instead of the polynomial model (Monnin et al. 2014). The first step is to isolate the stochastic and non-stochastic components of the noise. Twenty images are acquired for a given exposure level, and a wide detector exposure range is covered. For the set of images at a given dose level, centrally positioned $512 \times 512$ ROIs are de-trended using a 2D polynomial, and the total noise is calculated ($S_2$). An average ROI is then calculated from all the 20 images acquired at the exposure level, giving an ROI that primarily contains the structured noise of the detector. The division of each ROI by the averaged ROI gives the noise from the divided image ($S_2^{\text{div}}$). The division largely removes fixed pattern (non-stochastic) noise of the detector, and isolates the stochastic...
part of the noise (Granfors and Aufrichtig 2000). The averaged image contains a fraction of stochastic noise correlated to each of the 20 images; hence \((S^2_{av})\), it has to be corrected by the factor \(N/(N-1)\) to give the stochastic part of the noise \((S^2_s)\) or \(\text{NPS}_s\), where \(N\) is the number of images used to compute the averaged image (i.e., \(N = 20\) here). The non-stochastic part of the noise \((S^2_p)\) is taken as the difference between total and stochastic noises:

\[
S^2_p = S^2 - \frac{N}{N-1}S^2_{av}.
\]

(24.52)

Electronic and quantum noise are independent (stochastic) noise sources. Electronic noise is exposure independent and, hence, is present even without an X-ray signal. A linear extrapolation to stochastic noise data, \((S^2_s)\), in the low and middle dose range is performed to find the electronic noise component. The linear fit for electronic noise has to be made using weights equal to the inverse of the variance of the noise data at each exposure. Quantum noise, \((S^2_q)\), is finally taken as the residual stochastic noise after subtracting the electronic noise component.

\[
S^2_q = S^2_p - S^2_s.
\]

(24.54)

24.5.10 Noise Transfer in Cascaded Imaging Systems: Quantum Amplification, Swank Noise, Noise Aliasing

Models for the signal and noise transfer through cascaded linear detector models have already been developed for CR systems (Vedantham and Karellas 2010), for scintillation phosphors (Rabbani et al. 1987; Rabbani and van Metter 1989; Siewerdsen et al. 1997; Cunningham et al. 2004; Kim et al. 2006), and a-Se photoconductors (Zhao and Rowlands 1997), and have recently been extended to photon counting (Tanguay et al. 2013; Xu et al. 2014) detectors. In these models, signal and noise is transferred through three types of stage: (1) a gain stage represented as a binary selection process, (2) a stochastic blurring stage represented as a convolution integral with the PSF of the detector, and (3) a deterministic blurring stage such as an integration over the pixel aperture (Rabbani et al. 1987). For a gain stage “i,” for input signal and input NPS, respectively, \(d_{i-1}\) and \(\text{NPS}_{i-1}\), the output signal and NPS will be noted as \(d \_i\) and \(\text{NPS}_i\). The signal and NPS transfer through a gain stage of mean gain, \(g_i\), and gain variance, \(\sigma^2_{g_i}\), can be stated as (Rabbani et al. 1987):

\[
d_i(f, f_i) = g_i \cdot d_{i-1}(f, f_i)
\]

(24.55)

\[
\text{NPS}_i(f, f_i) = g^2_i \text{NPS}_{i-1}(f, f_i) + d_{i-1} \sigma^2_{g_i}.
\]

(24.56)

For a stochastic blur stage characterized by a convolution process with a PSF, and where \(T(f, f_i)\) is the Fourier transform of the PSF for the stage “i,” the resulting signal and NPS are given by (Rabbani et al. 1987):

\[
d_i(f, f_i) = d_{i-1}(f, f_i) \cdot T(f, f_i)
\]

(24.57)

\[
\text{NPS}_i(f, f_i) = \text{NPS}_{i-1}(f, f_i) T^2(f, f_i) + (T^2(f, f_i) - 1) d_{i-1}(f, f_i). \]

(24.58)

For a deterministic blur stage, the NPS transfer is simpler (Cunningham et al. 1994):

\[
\text{NPS}_i(f, f_i) = \text{NPS}_{i-1}(f, f_i) T^2(f, f_i) - d_{i-1}(f, f_i). \]

(24.59)

The signal and noise propagations of an incident X-ray fluence \(Q\) through X-ray detectors are modeled by dividing the detection chain into six successive elementary stages.

24.5.10.1 Capture of Incident X-rays by the Detector Converter

This is a stochastic gain stage with a gain (probability of X-ray capture or primary detection efficiency) \(\alpha\) and gain variance \(\alpha(1 - \alpha)\) (binary selection process) (Rabbani et al. 1987).

\[
\bar{d}_i = \alpha Q
\]

(24.60)

\[
\text{NPS}_i = \alpha^2 Q + \alpha(1 - \alpha)Q = \alpha Q.
\]

(24.61)

24.5.10.2 Conversion of X-rays into Secondary Quanta

This describes the stochastic gain conversion of X-rays into secondary quanta (light quanta or electronic charges for a-Se or PC detectors): gain \(\beta\) and gain variance \(\sigma^2_{g_i}\). The fluctuations in this gain term are caused by two principal factors (Swank 1973). The use of a polenergetic spectrum gives a variation in X-ray photon energy, which then leads to a variation in generated light or charge, and stochastic variations in this conversion gain add additional noise. The variation in the absorbed energy distribution (AED) is called radiation Swank noise, while optical Swank noise represents the variation in the optical pulse distribution (OPD) (Swank 1973; Chan and Doi 1984). For photon counting systems, a noise factor, \(f_{\text{scp}}\), equal to the probability that a true photon count is recorded given an interaction event (true positive fraction), has been introduced, based on a depth dependent interaction statistical model (Tanguay et al. 2013). The noise associated with this gain stage is described by the Swank factor, \(A_x\)

\[
\bar{d}_2 = \alpha \beta Q
\]

(24.62)

\[
\text{NPS}_{q2} = \beta^2 \text{NPS}_{q1} + \sigma^2_{\phi_i} = \left(\beta^2 + \sigma^2_{\phi_i}\right) \alpha Q = \alpha \beta^2 \left[1 + \frac{\sigma^2_{\phi_i}}{\beta^2}\right] Q = \frac{\alpha \beta^2}{A_x} Q.
\]

(24.63)

For a stochastic blur stage characterized by a convolution process with a PSF, and where \(T(f, f_i)\) is the Fourier transform of the PSF for the stage “i,” the resulting signal and NPS are given by (Rabbani et al. 1987):
with
\[ A_j = \frac{1}{1 + \sigma_j^2 b^2}. \]  

\[(24.64)\]

**24.5.10.3 Spreading of Secondary Quanta Within the Converter**

This stage represents stochastic spreading of secondary quanta in the detector described in the frequency domain by multiplication with a transfer function describing the scatter spreading characteristics of the detector converter \((T)\) (scintillator or photocoherent layer) (Rabbani et al. 1987).

\[
\overline{a}_j(f_*, f_j) = \alpha \beta Q \cdot T(f_*, f_j) 
\]

\[ \text{NPS}_{\psi}(f_*, f_j) = \text{NPS}_{\psi} \cdot T^2(f_*, f_j) \]

\[ + (1 - T^2(f_*, f_j))\phi_2 = \alpha \beta \left[ \left( \frac{1}{A_x} - \frac{1}{\beta} \right) T^2(f_*, f_j) + 1 \right] Q. \]

\[ \text{(24.66)} \]

**24.5.10.4 Capture of Secondary Quanta (Optical or Electrical Coupling)**

This is a stochastic stage that describes the optical or electrical coupling of the X-ray converter to the electronic readout sub-system. The coupling efficiency is a gain stage, with a gain \(\alpha\) and gain variance \(\kappa\) (1 - \(\kappa\)) (binary process), which gives the probability that the generated secondary quanta are converted to electronic signal (Rabbani et al. 1987):

\[
\overline{a}_d(f_*, f_j) = \alpha \beta \kappa Q \cdot T(f_*, f_j) \]

\[ \text{NPS}_{\psi}(f_*, f_j) = \alpha \beta \kappa \left[ \frac{1}{A_x} - \frac{1}{\beta} \right] T^2(f_*, f_j) + 1. \]

\[ \text{(24.68)} \]

**24.5.10.5 Collection of Secondary Quanta in the Detector Elements (Pixel Aperture)**

This is a stochastic gain and deterministic spreading stage, with a gain equal to the pixel fill factor

\[ \eta = \eta_x \eta_y = \frac{a_x}{\Delta x} \frac{a_y}{\Delta y}, \]

where \(\Delta x\) and \(\Delta y\) are the detector pixel spacing, and \(a_x\) and \(a_y\) the active pixel size in the \(x\)- and \(y\)-directions, and gain variance, \(\eta\) (1 - \(\eta\)) (binary process). The spreading is determined by the rectangular pixel aperture.

\[
\overline{a}_d(f_*, f_j) = \alpha \beta \kappa \eta Q \cdot T(f_*, f_j) |\text{sinc}(\pi a, f_j)| \text{sinc}(\pi a, f_j) |
\]

\[ \text{|sinc}(\pi a, f_j) = \alpha \beta \kappa \eta Q \cdot \text{MTF}_{\text{pre}}(f_*, f_j), \]

\[ \text{(24.69)} \]

\[ \text{NPS}_{\psi}(f_*, f_j) = \alpha \beta \kappa \eta Q \cdot \text{EMTF}(f_*, f_j) \]

\[ + |\text{sinc}(\pi a, f_j)| |\text{sinc}(\pi a, f_j)| \text{Q}. \]

\[ \text{(24.70)} \]

**24.5.10.6 Sampling**

Stage 6 describes the sampling of signal and noise by the pixel lattice. As discussed in Section 24.4.1.3, sampling causes signal and noise spectra to be replicated, with a replica centered on every harmonic of the sampling frequency, \(1/a\), where \(a\) is the pixel pitch. Noise after sampling consists of the infinite sum of aliased pre-sampling NPS centered at the frequencies \(k/\Delta x\) and \(k/\Delta y\). The noise components at frequencies above the Nyquist frequency are aliased into the image noise at lower frequencies. The mean sampled signal at stage 6 can be obtained by multiplication of the expression for stage 5 with an infinite train of \(\delta\) functions, uniformly spaced by intervals equal to the pixel sampling (Giger and Doi 1984):

\[
d_s(f_*, f_j) = \alpha \beta \kappa \eta Q \cdot T(f_*, f_j) \text{lsinc}(\pi a, f_j) \text{lsinc}(\pi a, f_j) |
\]

\[ \sum_{k_x = -\infty}^{\infty} \sum_{k_y = -\infty}^{\infty} \delta \left( f_x - \frac{k_x}{\Delta x} \right) \delta \left( f_y - \frac{k_y}{\Delta y} \right). \]

\[ \text{(24.71)} \]

The product between the converter transfer function, \(T\), and the pixel aperture function is equal to the modulus of the pre-sampling optical transfer function, \(\text{OTF}_{\text{pre}}\):

\[
|\text{OTF}_{\text{pre}}(f_*, f_j)| = \text{MTF}(f_*, f_j) |\text{lsinc}(\pi a, f_j)| |\text{sinc}(\pi a, f_j)| |
\]

\[ \text{(24.72)} \]

The digital (sampled) OTF is given by Giger and Doi (1984) and Dobbins (1995), and corresponds to the digital MTF \(d\):

\[
\text{MTF}_d(f_*, f_j) = |\text{OTF}_{\text{pre}}(f_*, f_j)| |\text{lsinc}(\pi a, f_j)| |\text{lsinc}(\pi a, f_j)| |
\]

\[ \sum_{k_x = -\infty}^{\infty} \sum_{k_y = -\infty}^{\infty} \delta \left( f_x - \frac{k_x}{\Delta x} \right) \delta \left( f_y - \frac{k_y}{\Delta y} \right). \]

\[ \text{(24.73)} \]

The expectation value of MTF \(d\) averaged over all phases (EMTF) is usually used to describe the digital MTF, since it satisfies the stationarity property:

\[
d_s(f_*, f_j) = \alpha \beta \kappa \eta Q \cdot \text{EMTF}(f_*, f_j). \]

\[ \text{(24.74)} \]

The mean sampled quantum NPS at stage 6 can be obtained using the same formalism:

\[
\text{NPS}_{\psi}(f_*, f_j) = \text{NPS}_{\psi}(f_*, f_j) \sum_{k_x = -\infty}^{\infty} \delta \left( f_x - \frac{k_x}{\Delta x} \right) \delta \left( f_y - \frac{k_y}{\Delta y} \right) = |k_x \text{EMTF}^2(f_*, f_j) + k_y^2| \text{Q}. \]

\[ \text{(24.75)} \]
where
\[ k_{q1} = \frac{1}{\alpha} \left( 1 + \frac{\sigma_0^2}{\beta^2} - \frac{1}{\beta} \right) = \frac{1}{\alpha} (1 + n_{ex}) \quad (24.76) \]
\[ k_{q2} = \frac{1}{\alpha \beta \kappa \eta^2} \quad (24.77) \]

The coefficient \( k_{q1} \) represents the amplitude of the correlated noise component, and depends on the X-ray absorption efficiency (\( \alpha \)) and the Poisson excess noise (\( n_{ex} = (\sigma_0^2 - \beta \lambda \beta^2) \)), which arises if conversion noise is not Poisson-distributed (Mackenzie and Honey 2007). The coefficient \( k_{q2} \) will always be greater than 1. The coefficient \( k_{q2} \) represents the non-correlated noise component, and is expected to be close to zero, since a large average conversion gain (\( \beta \)) is required to ensure no secondary sink occurs and the DQE scales with the quantum absorption efficiency, \( \alpha \). Practical application of cascaded modeling to the problem of quantum noise in mammography detectors is described by Monnin et al. (2016).

### 24.6 Global Measures of Image Quality and Detector Efficiency

#### 24.6.1 Noise Equivalent Quanta (NEQ)

The signal transfer of a linear digital imaging system is represented in the spatial frequency domain by the pre-sampling MTF and, hence, we can use the NPS to form the spatial frequency dependent SNR: the noise equivalent quanta (NEQ) represents the \( \text{SNR}^2(f) \) in frequency space,

\[ \text{NEQ}(f) = \frac{\langle d \cdot \text{MTF}(f) \rangle^2}{\text{NPS}(f)} = \frac{\text{MTF}^2(f)}{\text{NNPS}(f)} \quad (24.78) \]

The NEQ is a fundamental index of image quality, first introduced by Shaw (1963), that describes how many X-ray quanta an image is worth. It is linked to the detection performance of low contrast structures in a uniform noise limited image by an ideal human observer. A greater NEQ at a given frequency corresponds to a lower normalized image noise at this frequency. The NEQ has the unit \( \text{mm}^{-2} \), and can be interpreted as the (lower) photon fluence that a perfect imaging system would need to produce the measured image noise (“noise equivalent quanta”). A perfect imaging system is defined as a system that does not increase the input quantum noise (via absorption losses), and does not add other sources of noise (it does not degrade the SNR). Such a system would produce a NNPS equal to \( \text{MTF}^2/Q \), where \( Q \) is the mean photon fluence at the detector input, and would give the highest possible NEQ, equal to the photon fluence \( Q \). The NEQ of an image will, therefore, increase with the photon fluence \( Q \) and with the detector DQE.

#### 24.6.2 Detector Quantum Efficiency (DQE)

X-ray detector imaging performance can be described by the detector quantum efficiency (DQE) (IEC 2003), a parameter that quantifies the ability of a detector to transfer spatial frequency domain signal-to-noise ratio squared (\( \text{SNR}^2 \)) from the input X-ray beam to the image. Any degradation in SNR within the detector results in a DQE less than 1.0. As we have seen, a Poisson-distributed photon with mean fluence \( Q \) also has a mean quantum noise variance of \( Q \) and, hence, the input \( \text{SNR}^2 \), \( \langle \text{SNR}_{\text{in}}^2 \rangle \), is simply equal to \( Q \). The DQE is the ratio between the output (image) \( \text{SNR}^2 \), \( \langle \text{SNR}_{\text{out}}^2 \rangle \), and the input \( \text{SNR}^2 \), \( \langle \text{SNR}_{\text{in}}^2 \rangle \) (Dainty and Shaw 1974), and can, therefore, be calculated as:

\[ \text{DQE}(f) = \frac{\text{NEQ}(f)}{Q} = \frac{\text{MTF}^2(f)}{\text{NNPS}(f) \cdot Q} \quad (24.79) \]

The DQE is dimensionless and is independent of \( Q \) if the detector does not add noise components such as electronic or fixed pattern noise. The value of \( Q \) is established from the measured DAK, and \( \Phi \) taken from tables (e.g., Table 24.1). The DQE may also be defined as the ratio of the real NEQ to the NEQ that an ideal (perfect) detector would give, as an ideal detector would not degrade the input NEQ and would produce an NEQ equal to \( Q \). A distinction drawn between DQE and NEQ is that the DQE is not an explicit measure of image quality, but rather the efficiency of the detector/imaging process. The DQE ranges between 0 and 1, and can be used to rank detectors, but gives no information on the absolute level of quantum noise present in a given image. Such information is obtained from an incident number of quanta per unit area (\( Q \)) and the DQE, such as the NEQ.

#### 24.6.3 Cascaded Imaging Systems

Linear systems theory allows a cascaded transfer of signal and noise through successive component stages. Linear models that describe the signal and quantum noise transfer through detector models decomposed into successive stages have been widely used to establish theoretical models of DQE for various detector types (Rabbani et al. 1987; Siewerdsen et al. 1997; Zhao and Rowlands 1997; Vedantham and Karellas 2010; Tanguay et al. 2015). If the stages can be assumed to be linear (Section 24.2.2) then cascaded systems analysis is a useful tool for designing imaging systems and assessing their SNR transfer performance. Section 24.5.10 described signal and noise transfer using gain, stochastic blur, and deterministic blur stages. A “generic” digital X-ray detector can be modeled as a cascade of six elementary stages:

1. Detection of primary quanta with a given quantum efficiency (stochastic gain).
2. Conversion of primary quanta to secondary quanta, such as a conversion of X-rays to light photons or electronic charges (stochastic gain).
3. Spatial blurring of secondary quanta within the detector layer characterized by a PSF (stochastic blur).
4. Coupling of the secondary quanta to the active detector elements (stochastic gain).
5. Integration of the secondary quanta in the pixel aperture (stochastic gain, deterministic blur).
6. Sampling of the signal and noise by the pixel lattice.

The number of quanta at each stage can be displayed graphically in quantum accounting diagrams (QAD) for different...
24.7 Detector and System Characterization

24.7.1 Measurements of MTF, NPS, and DQE

This section presents examples of the technical characterization of X-ray detectors described in the previous sections, including examples of sharpness, noise, and DQE dependencies. Figure 24.15 shows greyscale extracts of homogeneous images acquired using two different 2 mm Al filters on an a-Se mammography detector. The high purity Al filter gives images that have a strong mottled pattern, while the 99.0% purity filter has notably less structure. Examining the NNPS curves calculated from these images (Figure 24.15a) shows a large bump at low spatial frequencies for the image acquired with the high purity sheet compared to the less pure sheet. At higher spatial frequencies (>~1 mm⁻¹), the NNPS is almost identical. The increased NNPS from the structure mottle in the Al filter strongly suppresses DQE <~1 mm⁻¹, while DQE is identical above ~1 mm⁻¹ (Figure 24.15b). Hence, filters to be used in detector characterization should be evaluated for the presence of mottled patterns by comparison against images acquired with homogeneous PMMA plates (Ranger et al. 2005).

Figure 24.16a shows pre-sampling MTF curves measured using the RQA5 beam quality for three diagnostic X-ray detectors: a CsI-phosphor indirect conversion DR detector and two different CR detectors, one with a needle structure based phosphor screen and one with a powder phosphor. Despite notable differences in detector design and conversion material, the MTF_pre curves are remarkably similar for all three detectors. A simplistic evaluation of image quality using a high contrast bar pattern would probably lead to the conclusion that image quality is similar between these detectors. However, measured NNPS curves at 2.5 μGy target DAK/image (Figure 24.16b) show a different picture, namely that the magnitude of noise in the image (characterized using the NNPS) is considerably greater for the powder CR device compared to the DR and needle CR detectors, for the same nominal input DAK/image. Given that the detectors have similar MTF_pre curves, the calculated DQE data reflect the changes seen in the NNPS results: efficiency is similar for the DR and needle CR detectors, but notably lower for the powder CR device (Figure 24.16c). A similar sharpness (MTF_pre) between powder and needle CR could be achieved by reducing the linear thickness of the powder phosphor. This, however, reduces primary quantum absorption—the result is higher NPS and lower DQE. The difference in DQE is reflected in the CD performance measured using the TO20 test object: Figure 24.16d shows similar performance for the DR and needle CR detectors at 2.5 μGy, while the threshold contrast is higher for the powder CR detector. In order to approximately match threshold contrast performance using the powder CR detector, the DAK/image should be scaled by the ratio of the peak DQE values, such as by ~0.5/0.2 and, thus, by a factor of ~2.5 (Aufrichtig 1999).

Spatial frequencies (Cunningham et al. 1994). QAD values for a given spatial frequency are directly related to the DQE at the corresponding frequency. QADs are a convenient means of locating the quantum sink within the chain, thus the stage with the fewest quanta determines DQE and image noise. A quantum sink is more likely to occur at high spatial frequencies where the signal is low (low MTF). Two elemental rules can be observed from the simplest cascaded models:

1. The DQE scales with the primary quantum absorption efficiency.
2. A large secondary quantum gain is required to ensure the DQE remains close to the primary absorption efficiency—otherwise a secondary quantum sink occurs that will severely decrease the DQE.

Cascaded models can also be used to predict the effect of various physical processes such as signal and noise aliasing, Poisson excess noise, or Swank noise (Cunningham et al. 1994; Monnin et al. 2016). Figure 24.14 shows an example of cascading DQE through seven stages for a digital X-ray detector. (Adapted from Monnin, P. et al. 2016. Physics in Medicine and Biology 61:2083–108.)
The DQE curves of most energy integrating-type detectors used for medical imaging generally exhibit an exposure dependency. This is a direct consequence of the presence of noise sources other than X-ray quantum noise in the X-ray detector/images and, hence, DQE should be characterized as a function of exposure/image at the X-ray input. Figure 24.17a shows DQE at 1.0 mm⁻¹ for four mammography detectors, measured as a function of DAK/image. If we consider ∼100 µGy/image as a standard operating point for these detectors, a strong reduction in DQE is seen for the a-Se DR detector, consistent with the presence of (additive) electronic noise (see Section 24.5.9). No such reduction is seen for the CR devices, where photomultiplier electronic noise is known to be low (Rowlands 2002). However, there is a gradual fall in DQE as the DAK/image is increased, indicating the presence of (multiplicative) structured noise (see Section 24.5.9). This has the effect of reducing DQE at higher detector exposures for the CR detectors, where flat-field correction is generally not performed. Figure 24.17b illustrates the behavior of the NNPS at 1.0 mm⁻¹ for the powder CR and a-Se DR detectors, compared to a detector with pure quantum noise. This figure clearly shows the increase in NNPS (relative to X-ray noise) for the a-Se DR detector at a low DAK/image, and the tendency to quantum noise limited behavior at a DAK/image above ∼90 µGy/image. Conversely, the CR detector is quantum noise limited at a low DAK/image (∼no electronic noise), but structure noise pushes the detector away from quantum limited behavior at a high DAK/image. Note that a discrimination threshold can be set for photon-counting detectors, whereby electronic noise is rejected. Taken together with flat-fielding, this means that photon-counting detectors are likely to be quantum noise limited across a wide exposure range (Monnin et al. 2007).

Given that DQE depends on the energy absorption efficiency or interaction probability of the primary photo receptor, then some energy dependence in DQE is to be expected. Figure 24.18 shows DQE measured at mean X-ray beam energies ranging from ∼18.0 to ∼23.4 keV. This relatively small 5 keV change

![Graphs and images showing DQE and NNPS curves for mammography detectors.](image-url)
in mean energy leads to a strong reduction (∼20%) in DQE for a dual plate CR system (Figure 24.18a). For an a-Se based FP detector, a similar 5 keV change in mean energy has only a small influence on the DQE (Figure 24.18b) (Marshall 2009). These evaluations were made using typical X-ray spectra in combination with PMMA blocks at the X-ray tube exit port; these are non-standard radiation qualities in terms of IEC reference, but are clinically realistic. Diagnostic X-ray detectors are used over a greater range of energies (Table 24.1) and, hence, characterization at the different RQA spectra is essential.

### 24.7.2 Uses of Physical Image Quality Measurements

Measurement of the response functions, MTF, NNPS, and DQE, have become standard means of characterizing detectors used for both medical imaging and non-destructive testing, thanks largely to the development and adoption of the IEC protocols (IEC 2003, 2005). Although initially developed within the framework of manufacturer standardization, these protocols are now widely used within the scientific community when assessing new detector technology. Given the objective and repeatable nature of these
measurements, there have been moves to implement these parameters as part of routine QC programs (Cunningham 2008, IPEM 2010). The short-term repeatability and longer-term stability (i.e., over months/years) of MTF, NNPS, and DQE measurements has already been demonstrated in the context of routine QC measurements (Marshall 2007; Cunningham 2008); furthermore, MTF and NNPS have been shown to be sensitive to changes in X-ray detector performance under fault conditions (Marshall 2006b). Hence, these parameters give a clear and direct picture of detector performance and any changes that may have occurred over time. Access to “For Processing” image data needed for this type of analysis is well established for digital mammography systems, but remains problematic for many diagnostic X-ray detectors/systems. It is hoped that the recent NEMA initiative (XR27-2013) providing “For Processing” images for use in QC of X-ray equipment for interventional procedures is extended to general diagnostic imaging devices. Physical image quality parameters evaluated from these image types give considerable insight into

![Figure 24.17](image1)

**FIGURE 24.17** (a) DQE at 1.0 mm\(^{-1}\) measured as a function of DAK for four mammography detectors. (b) NNPS at 1.0 mm\(^{-1}\) plotted against DAK for the powder CR detector and the a-Se detector. (Adapted from Marshall, N.W., K. Lemmens and H. Bosmans. 2012. Medical Physics 39:811–824.)

![Figure 24.18](image2)

**FIGURE 24.18** DQE measured as a function of energy for two mammography detectors: (a) a powder CR detector, and (b) an a-Se detector.
detector performance. Furthermore, observer models bridge the gap between quantitative measures and simple task-based evaluations using CD methods (Rose 1948; Burger 1950; Aufrichtig 1999; Monnin et al. 2011).

24.7.3 From Detector to System Characterization

Performance evaluation of radiographic systems has primarily focused on the assessment of detector DQE. This parameter characterizes detector efficiency in transferring the SNR from the X-ray fluence to the image and, as such, the influence of magnification and focal spot blur is minimized by the choice of geometry (Section 24.3.2), even though these factors are present in the pre-sampling MTF. Furthermore, DQE evaluations are performed under conditions without scatter and scatter rejection technique. Extensions of detector DQE to system DQE that includes scatter and anti-scatter devices have, therefore, been developed by a number of authors.

24.7.3.1 Effective DQE (eDQE)/Generalized DQE (gDQE)/System DQE

Generalized metrics were developed as extensions of DQE to include beams with scatter and the anti-scatter device efficiency into a global imaging system efficiency metrics. The concept of scatter detective quantum efficiency (SDQE), proposed initially by Wagner et al. (1980) to describe grid efficiencies, was combined with detector DQE by Samei et al. (2009) into a figure of merit named effective DQE (eDQE). The eDQE defined in Equation 24.80 for the frequency, \( f' \), at the object plane depends on the primary transmission of the grid (\( T_p \)), the scatter fraction at the detector (SF), and requires an estimate of the primary fluence (\( P \)) at the grid input (corrected for the distance to the detector plane):

\[
eDQE(f') = \frac{\text{MTF}^2(f') \cdot (1 - \text{SF})^2}{\text{NNPS}(f') \cdot P} = T_p \cdot (1 - \text{SF}) \cdot \text{DQE}(f').
\]

(24.80)

The eDQE is maximal (and equal to 1.0) for a system with a perfect detector (DQE = 1.0) and perfect grid. The eDQE is always equal to or less than the DQE, and reverts to the detector DQE for a perfect grid.

The generalized DQE (gDQE) proposed by Kyprianou et al. (2005) defines a generalized MTF (gMTF) determined for different beams characterized by different SF. The gMTF incorporates blurring due to the detector, the focal spot, and scattered radiation. The gDQE, defined in Equation 24.81, is normalized by the total photon fluence including primary and scatter at the detector input (\( P + S \)), and differs on this point from the eDQE,

\[
gDQE(f') = \frac{\text{gMTF}^2(f')}{\text{NNPS}(f') \cdot (P + S)}.
\]

(24.81)

The gDQE reverts to the detector DQE for a perfect grid, but scales differently from eDQE with the scatter fraction.

More recently, SNR transfer through the scatter reduction device and detector were multiplied as cascaded efficiencies in the form of a global system efficiency metric named system DQE (DQE\(_{\text{sys}}\)). DQE\(_{\text{sys}}\) is the product of the anti-scatter device efficiency (DQE\(_{\text{ASD}}\)) and the standard DQE of the X-ray detector (DQE\(_{\text{D}}\)) (Monnin et al. 2017).

\[
\text{DQE}_{\text{sys}}(f) = \text{DQE}_{\text{ASD}} \cdot \text{DQE}_{\text{D}}(f) = \frac{T_p^2 \cdot \text{MTF}^2(f)}{T_l \cdot \text{NNPS}(f) \cdot (P + S)}.
\]

(24.82)

DQE\(_{\text{ASD}}\) depends on the primary (\( T_p \)) and total (\( T_l \)) transmissions of the anti-scatter device, as a function of the SF. The pre-sampling MTF is measured without scatter, e.g., using RQA beam whose effective energy (HVL) is adjusted via the aluminum thickness to the beam with scatter. In the absence of an ASD, DQE\(_{\text{sys}}\) reverts to detector DQE. Therefore, DQE\(_{\text{sys}}\) is higher or lower than DQE\(_{\text{D}}\), depending on the ability of the ASD to modify the SNR.
Figure 24.19 shows the detector DQE, eDQE, gDQE and DQE_{sys} for a mammography system with a beam characterized by a scatter fraction of 0.38 at the detector input.

### 24.8 Linking Physical Parameters and Object Detectability

#### 24.8.1 Pixel Signal to Noise Ratio (SNR) and Contrast to Noise Ratio (CNR)

The measure of PV standard deviation (σ) in a homogeneous ROI is the simplest means of image noise assessment. This measure of noise can be combined with the mean PV measured within the same ROI to give the pixel SNR:

\[
\text{SNR}_{\text{pixel}} = \frac{\bar{I}}{\sigma}.
\]  

(24.83)

A related parameter is the contrast-to-noise ratio (CNR), which, for QC purposes, is often measured for some target object (e.g., a thin aluminum square of dimension 10 × 10 mm [EC 2006]), imaged in a homogeneous background. CNR is defined as the measured object contrast (C_o) (using mean PV), divided by the standard deviation measured in an ROI positioned in the image background,

\[
\text{CNR} = \frac{C_o}{\sigma}.
\]  

(24.84)

While these parameters reflect changes in image noise (and, hence, signal detectability), pixel SNR has to be used with caution (Burgess 1999). Both SNR_{pixel} and CNR involve the use of standard deviation, which is expressed in pixel values and is accordingly dependent on PV units. The magnitude of these units may be arbitrary or specific to an imaging system/modality and, hence, pixel SNR and CNR must not be used for direct (scale) comparisons between different detector/systems. Figure 24.20 shows how the detection of an object signal in a noisy background is CNR dependent. Given their ease of measurement, SNR_{pixel} and CNR are often specified in QC programs (EC 2006), as CNR offers a quick means of assessing the automatic exposure control system set up (EC 2006). For example, relative changes in CNR due to changes in breast equivalent phantom thickness have been shown to follow changes in small detail detectability measured using a CD test object (Salvagnini et al. 2015).

#### 24.8.2 Rose Model as Simple Detection Model: Fundamental Importance of SNR

Pixel SNR can be considered as a kind of generic measure of the image SNR, but it is not referenced to a particular imaging task or even object size of interest. We know that signal detectability

![FIGURE 24.20 Profiles showing low (a) and high (b) CNR.](image-url)
in an image is linked to the object characteristics (contrast, shape, and size), and signal detection theory provides models that quantify the task-dependent detection performance of human observers. Rose (1948) was the first to investigate the simplest task of detecting an object signal in a homogeneous, noisy background, and laid the foundation for both the concept of DQE and for modeling low contrast threshold detection. A useful derivation is given by Burgess (1999), using a flat topped, sharp edged signal of area \( A \), in a uniform background. A vital step taken by Rose (1948) was considering the photon noise integrated over the same area as the object; this step provides an absolute scale for image fluctuation. The mean number of photons in area \( A \) is \( N_b \), and, by assuming Poisson statistics, this is also the variance, \( \sigma_n^2 \). Photon densities (mean number of photons/unit area) are used to describe the background \( n_b \) and the incremental change in the object \( \Delta n \), so the contrast, \( C = \Delta N/N_b = \Delta n/n_b \). The SNR was defined by Rose (1948) as:

\[
\text{SNR}_\text{Rose} = \frac{\text{mean signal}}{\sigma_n} = \frac{\Delta N}{\sqrt{N_b}} = \frac{A \Delta n}{\sqrt{An_b}}
\]

or

\[
\text{SNR}_\text{Rose} = \text{SNR} \sqrt{A}.
\]

The detection performance (called detectability index, \( d' \)) of the Rose model, therefore, scales with the contrast-to-noise ratio (CNR) or, more explicitly, the area of the object, the exposure at the detector, and object contrast. The contrast, \( C \), of thin discs of thickness, \( T \), and attenuation coefficient, \( \mu \), surrounded by a flat background (constant signal and noise) will be equal to \( 1 - e^{-\mu T} \geq 0.9T \) for \( T \ll 1 \). For discs of different diameters, \( d \), the contrast-detail curve corresponding to a threshold detectability, \( d' \), will satisfy the conditions of the Rose model for detection (SNR$_\text{Rose}$), and Equation 24.86 becomes:

\[
T \cdot d = \text{const}.
\]

Equation 24.87 gives a linear contrast-detail curve of gradient \( d' \) on a log-log graph whose axes represent the discs diameter and contrast. This simple shape of contrast-detail curve arises from the assumptions behind the model and, hence, applies to the range of disc diameters where resolution is not a limiting factor. While the Rose formula usefully models simple detection tasks—a typical scenario is a low contrast object in homogenous \( \sim \)white noise (Marshall 2006a)—the model starts to fail when the assumptions of low contrast signals, no blurring (correlation), and uncorrelated Poisson noise no longer apply (Burgess 1999).

### 24.8.3 Linking Contrast-Detail Curves to NEQ and Simple Tasks

Physical image quality parameters (MTF, NPS, and NEQ) and observer performance, as measured using simple tasks like the CD analysis described in Section 24.2, can be linked together via observer modeling derived from statistical decision theory. Physical parameters measured for the imaging system are input to some numerical or model observer that generates an output SNR (or detectability index \( d' \)) that is related to the detectability of simple objects in the image (Loo et al. 1984; ICRU 1996; Aufrichtig 1999; Barrett and Myers 2003). Only the basics of observer models will be presented in this section.

As discussed earlier, the Rose model provides a starting point when trying to predict observer performance for an image with a given SNR (noise level), but does not account for correlations that are likely present in the image. A model that takes into account the frequency correlations can be achieved by combining the NEQ with the object signal spectrum \( S \) in the frequency domain, as described in ICRU 54 (1997):

\[
d'_{\text{NPW}} = C \cdot \left( \int_0^{\infty} \text{NEQ}(f) \cdot S^2(f) f df \right)^{1/2},
\]

where \( f \) is the radial frequency, and \( C \) is the object contrast on the image. The model given in Equation 24.88 is the ideal observer or pre-whitening (PW) model; the term “pre-whitening” refers to the step taken by the observer to remove correlations present in the noise (i.e., “whiten” the noise) and, hence, apply the signal template in white (\( \sim \)Gaussian) noise (Wagner and Brown 1985). This observer model, therefore, uses all the image information for detection (ideal observer), and tends to outperform real observers. A more realistic non-pre-whitening matched filter (NPW) model was, therefore, developed (Wagner and Brown 1985; ICRU 1997) in which the observer uses the known signal as a template, but is not able to use information about noise correlations (i.e., behaves as if the noise were white, even when it may not be). The detectability index is given by Equation 24.89:

\[
d'_{\text{NPW}} = C \cdot \left( \int_0^{\infty} \text{NPSM TF}(f) \cdot MTF^2(f) \cdot S^2(f) f df \right)^{1/2},
\]

Burgess (1994) suggested a modification to the NPW model that improved performance in simple non-homogeneous backgrounds by adding an eye filter \( (E) \) that represents the contrast sensitivity function of the human visual system. The eye filter effectively acts as a bandpass filter that suppresses the influence of low spatial frequencies on the detectability index. The non-pre-whitening matched filter with eye filter (NPWE) model is given in Equation 24.90:

\[
d'_{\text{NPWE}} = C \cdot \left( \int_0^{\infty} \text{MTF}^2(f) \cdot S^2(f) \cdot E^2(f) f df \right)^{1/2}.
\]
24.8.3.1 NPWE Model from MTF and NPS for CDMAM

Considering constant MTF and NPS shapes, the NPWE observer model (Equation 24.90) can be approximated for discs of different diameters, \( d \), as the product between the contrast-to-noise ratio (\( C/\sigma \)) and a factor defined in the frequency domain (\( \kappa \)). The factor \( \kappa \) depends on the signal and noise bandwidths,

\[
d'_{\text{NPWE}} = \frac{C}{\sigma} \left( \int_0^{\infty} \frac{\text{MTF}^2(f) \cdot S^2(f) \cdot E^2(f) df}{\sigma^2 \left( \int_0^{\infty} \frac{\text{NPS}(f)}{\sigma^2} \cdot \text{MTF}^2(f) \cdot S^2(f) \cdot E^4(f) df \right)^{1/2}} \right)^{1/2}
\]

(24.91)

\[
d'_{\text{NPWE}} \cong \kappa \frac{C}{\sigma} d^n.
\]

(24.92)

The contrast-detail curves for the gold discs of diameters \( d \) in the CDMAM phantom satisfy the iso-\( d' \) values (\( d' = \text{const} \)),

\[
\frac{C}{\sigma} \cdot d^n = \text{const.}
\]

(24.93)

For a thin disc of thickness \( T \) and attenuation coefficient \( \mu \), an image with a primary flat background signal \( P \) will give an object contrast \( C \) equal to

\[
C = P(1 - e^{-\mu T}) \cong P\mu T.
\]

(24.94)

Equations 24.93 and 24.94 give:

\[
\frac{P\mu T}{\sigma} \cdot d^n = \text{const.}
\]

(24.95)

If the parameters \( P, \mu, \) and \( \sigma \) are constant across the image, the contrast-detail curves expressed in log-log graphs are simply linear with a negative slope \( \alpha \),

\[
\log T - \alpha \cdot \log d = \text{const.}
\]

(24.96)

Momnin et al. (2011) and Liu et al. (2014) have used the NPWE model to accurately predict the visibility of small details in a homogeneous background for mammography imaging systems. This has been extended to general radiography detectors (Van Peteghem et al. 2016), again for the simple CD imaging task.

24.8.4 More Advanced Models of Object Detectability

The NPW and NPWE detectability indices are derived from the NEQ (Equation 24.78), using Fourier based metrics to characterize image signal and noise and, hence, linear shift invariant images with stationary Gaussian noise are a requirement. Although medical X-ray systems fulfill these requirements to some (limited) extent, there has been considerable effort to generalize metrics of task performance (Barrett and Myers 2003). This has led to the formulation of the (channelized) Hotelling observer, a linear observer that maximizes SNR—defined as the difference between the mean values of a test statistic divided by the square root of its average variance (Barrett and Myers 2003). Calculation of the (channelized) Hotelling SNR is implemented in the spatial domain and uses an estimate of the average covariance matrix. This is more difficult to compute than the MTF and NNPS needed for the NEQ based detectability models, but the Hotelling model makes no assumptions of stationarity or shift invariance. This leads to a model observer that is more robust, with a wider applicability to a range of image quality evaluation scenarios and, thus, forms the basis of much of the image quality evaluation research undertaken over the past years (Barrett 1990; Barrett et al. 1995, 1998, 2015).

24.9 Summary

This section has described methods used in the evaluation of physical image quality in mammography and digital radiography, with the emphasis on X-ray detectors. Test objects, primarily in the form of threshold contrast-detail (CD) methods, remain at the heart of image quality assessment methods used by medical physicists for QC testing. Minimum or typical performance standards are often given in terms of some threshold detectability value and, hence, the use of these methods will continue for the foreseeable future. The assumption of linear, shift invariant detectors/imaging systems enables the implementation of transfer function analysis and the associated metrics of MTF, NPS, NEQ, and DQE. Such methods provide considerable insight into factors that influence the quality of images produced by mammography and general radiography detectors. Signal detection theory can be used to show the link between NEQ and detection of objects in a homogeneous background used in CD methods. However, this is just the starting point for task-based assessments of image quality using realistic targets, such as mammography soft tissue lesions in a realistic background, that are the true measure of image quality.

REFERENCES


