Chapter 3

Precast frame analysis

3.1 TYPES OF PRECAST CONCRETE STRUCTURES

This chapter introduces the basic principles and some of the design and analysis procedures, involved in the design of precast concrete skeletal structures, essentially a beam–column framework possibly braced using walls and/or cores, as well as briefly discusses precast portal frames and wall frames. Eurocodes EC0, EC1 and EC2 used to determine the combinations and arrangement of gravity and horizontal loads acting on floors, beams and structures are introduced. The design of reinforced and prestressed concrete elements, connections and structures will follow in later chapters.

Preliminary structural design, which many people refer to as the feasibility stage, is more often a recognition of the type of structural frame that is best suited to the form and function of a building than the structural design itself. The creation of a large ‘open plan’ accommodation giving the widest possible scope for room utilisation clearly calls for a column and slab structure, as shown in Figure 3.1, where internal partitions could be erected to suit any client’s needs. The type of structure used in this case is often referred to as ‘skeletal’ – resembling a skeleton of rather small but very strong components of columns, beams, floors, staircases, and sometimes structural (as opposed to partition) walls. Of course, a skeletal structure could be designed in cast in situ concrete and structural steelwork, but here we will consider only the precast concrete version.

The basis for the design of precast skeletal structures has been introduced in Figures 1.11 and 1.13. The major elements (the precast components) in the structure are shown in Figure 3.2. Note that the major connections between beams and floors are designed and constructed as ‘pinned joints’, and therefore the horizontal elements (slabs, staircases, beams) are all simply supported. They need not always be pinned (in seismic zones, the connections are made rigid and very ductile) but in terms of simplicity of design and construction it is still the preferred choice. Vertical elements (walls, columns) may be designed as continuous, but because the beam and slab connections are pinned there is no global frame action and no requirement for a frame stiffness analysis, apart from the distribution of some column moments arising from eccentric beam reactions. The stiff bracing elements such as walls are designed either as a storey height element, bracing each storey in turn, or as a continuous element bracing all floors as tall cantilevers.

In office and retail development, distances between columns and beams are usually in the range of 6–12 m (Figures 1.7 and 3.3) depending on the floor loading, method of stability and intended use. In multi-storey car parks, where the imposed loading (vehicle gross weight <30 kN according to the NA to BS EN 1991-1-1, Table NA.6) is 2.5 kN/m² it is around 16 m for floor spans × 7.2 m for beams, giving three parking bays between columns (Figure 1.6). The exterior of the frame – the building’s weatherproof envelope – could also be a skeletal structure, in which case the spaces between the columns would be clad in brickwork,
Precast concrete panels, sheeting, etc. Alternatively, the envelope might be constructed in solid precast bearing walls, which dispenses with the need for beams, and is referred to as a ‘wall frame’ (Figure 1.14).

Examples of residential buildings where a precast wall frame would be the obvious choice are shown in Figures 3.4 through 3.7 – the walls are all load-bearing and they support one-way spanning floor slabs. There is less architectural freedom compared to the skeletal frame, for example walls should (preferably) be arranged on a rectangular grid and of fixed modular distance, usually 300 mm, which is quite important.
Figure 3.3 Precast skeletal structure in Portugal. (Courtesy of Ergon, Belgium.)

Figure 3.4 Wall frames are best suited for apartments, hotels, schools, shopping units, as in this example at Rhodes, near Sydney, Australia.
A wall frame may be more economical and may often be faster to build, especially if the external walls are furnished with thermal insulation and a decorative finish at the factory. Figures 1.14 and 1.22 are good examples of this. Distances between walls may be around 6 m for hotels, schools, offices and domestic housing, and 10–15 m in commercial developments. Given this description, wall frames appear to be very simple in concept, but in fact are quite complicated to analyse because the walls have
very large in-plane rigidity whilst the connections between walls and floors are more flexible. Differential movement between wall panels and between walls and floors has resulted in major serviceability problems over a 25+ year life, often leading to a breakdown in the weatherproof envelope and the eventual condemnation of buildings, which are structurally adequate.

The third category of precast building is the ‘portal frame’ used for industrial buildings and warehouses where clear spans of some 25–40 m I-section or T-section prestressed rafters are necessary; Figures 3.8 and 3.9. Although portal frames are nearly always used for single-storey buildings, they may actually be used to form the roof structure to a skeletal frame, and as this book is concerned with multi-storey structures it gives us a reason to mention them. The portal frame looks simple enough and in fact is quite rudimentary in design, providing that the flexural rotations at the end of the main rafters, which we can assume will always cause cracking damage to the bearing ledge, are catered for by inserting a flexible pad (e.g. neoprene) at the bearing. As mentioned before, pinned connections between the rafter and column are the preferred choice – they are easy to design and construct. But the columns must be designed as moment-resisting cantilevers, which might cause a problem in some structures as explained later in Section 3.6.2. A moment-resisting connection is equally possible allowing some moment continuity into the column at the eaves. However, unless the columns are particularly tall, say more than about 8 m, it is not worth the extra effort.

Precast portal frames with flat (or shallow inclination) roof structures comprising prestressed or reinforced beams of 6–8 m span supporting long-span precast folded plate roof elements, spanning around 20 m. This is a popular option for industrial buildings, and in the case of Figure 3.10 used in laboratory buildings. The overhang beam is an option for sun or rain shading.

Table 3.1 reviews the various types of precast structures with respect to their possible applications.
Precast Concrete Structures

Figure 3.9 Precast portal frame. (Courtesy David Fernandez-Ordoñez, Escuela Técnica Superior de Ingeniería Civil, Madrid, Spain.)
3.2 SIMPLIFIED FRAME ANALYSIS

One of the most frequently asked questions is ... how is a precast concrete structure analysed compared to a monolithic cast in situ one? The first response is to say that a precast concrete structure is not a cast in situ structure cut up into little pieces making it possible to transport and erect. It was mentioned in Chapter 1 that the passage of forces through the prefabricated and assembled components in a precast structure is quite different to a continuous (monolithic) structure. This is certainly true near to connections. It is therefore

Table 3.1 Application and types of precast concrete frames

<table>
<thead>
<tr>
<th>Use of building</th>
<th>Number of storeys</th>
<th>Interior spans (m)</th>
<th>Skeletal frame</th>
<th>Wall frame</th>
<th>Portal frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office</td>
<td>2–0</td>
<td>6–15</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2–50</td>
<td>6–15</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Retail, shopping</td>
<td>2–10</td>
<td>6–10</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>complex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cultural</td>
<td>2–10</td>
<td>6–10</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>2–5</td>
<td>6–10</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Car parking</td>
<td>2–10</td>
<td>15–20</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stadia</td>
<td>2–4</td>
<td>6–8</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hotel</td>
<td>2–30</td>
<td>6–8</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hospital</td>
<td>2–10</td>
<td>6–10</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residential</td>
<td>1–40</td>
<td>4–6</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial</td>
<td></td>
<td>25–40</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Warehouse</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with office</td>
<td>2–3</td>
<td>6–8</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>25–40</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

* Typical values, depending on the location, terrain, requirements, etc.
possible to begin a global analysis by first considering the behaviour of a continuous frame and identifying the positions where suitable connections in a precast frame may be made. A two-dimensional in-the-plane simplification is appropriate in the first instance. This is defined in Figure 3.11 where there are no structural frame components, only simply supported floor units, connecting the 2-D in-plane frames together.

Figure 3.12 shows the approximate bending moments and deflected shape in a three-storey continuous beam and column frame subject to vertical (gravity) patch loads and horizontal (wind) pressure. Call this frame F₁. The beam-column connections have equal strength and stiffness as the members. The stability of F₁ is achieved through the combined action of the beams, columns and beam-column connections in bending, shear and axial. This is called an ‘unbraced’ frame. There are points of zero moment (‘contraflexure’) in F₁, which depend on the relative intensity of the two load cases. If gravity loads are dominant, beam contraflexure is near to the beam-column connection, typically 0.1 times the span of the beam as shown in Figure 3.13; but if the horizontal load is dominant (more rare), contraflexure is at mid-span, with the final location for combined loading at about 0.15 × span. In the column, contraflexure is always at mid-storey height, and this is a good place to make a pinned (notionally = small moment capacity) connection between two precast columns.

Now, if the strength and stiffness of the connection at the end of the beam are reduced to zero, whilst the column and the foundation are untouched, the resulting moments and deflections in this frame, called F₂, are as shown in Figure 3.14. The columns alone achieve the stability of F₂ – the beams transfer no moments, only axial forces and shear. The foundations must be moment-resisting (rigid). This is the principle of a pinned jointed unbraced skeletal frame. In taller structures, > three storeys or about 10 m, the large sizes of the columns...
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become impractical and uneconomic leading to bracing. The bracing may be used in the full height, called a ‘fully braced’ frame, or up to or from a certain level, called a ‘partially braced’ frame. The differences are explained in Figure 3.15. The bracing could be located in the upper storeys, providing the columns in the unbraced part below the first floor are sufficiently stable to carry horizontal forces and any second-order moments resulting from slenderness.

Pinned connections may be formed at other locations. Referring back to frame F1, if the flexural stiffness of the members at the lower end of a column is greater than that at the upper end, the point of contraflexure will be near to the lower (stiffer) end of the column. If the strength and stiffness of the lower end of the column are reduced to zero, whilst the beam and beam–column connections are untouched, the resulting moments and deflections in this frame, called F3, are as shown in Figure 3.16a. The stability of F3 is achieved by the portal frame action of inverted U frames – clearly not a practical solution for factory cast

Figure 3.12 Deformation and bending moment distribution in a continuous structure subjected to (a) gravity loads and (b) horizontal sway load.
large spans so that this method is used for repetitious site casting. Therefore, a practical solution is to prefabricate a series of L-frames as shown in Figure 3.16b for long-span beams and small-storey height columns in a parking structure. Foundations to F3 may be pinned, although most contractors prefer to use a fixed base for safety and immediate stability.

The so-called H-frame is a variation on F3. Referring back to frame F1, if pinned connections are made at the points of column contraflexure, structural behaviour is similar

![Figure 3.13 Beam half-joints at 0.1× span close to points of contraflexure in a continuous beam.](image)

Figure 3.14 Deformation and bending moment distribution in a pinned jointed structure subjected to (a) gravity loads. (Continued)
to a continuous frame as explained in Figure 3.17. Connections between frames are made at mid-storey height positions. Although in theory the connection is classed as pinned, in reality there will be some need for moment transfer, however small. Therefore, H-frame connections are designed with finite moment capacity, this also gives safety and stability to the H-frames, which by their nature tend to be massive. The foundation to half-storey height ground floor columns must be rigid. The connection at the upper end of the column may be pinned if it is located at a point of contraflexure. If not the connection must possess flexural strength as shown in Figure 3.17, where the H-frame has been used in a number of multi-storey grandstands.

Figure 3.14 (Continued) Deformation and bending moment distribution in a pinned jointed structure subjected to (b) horizontal sway load.

Figure 3.15 Partially braced structures.
3.3 SUBSTRUCTURING METHODS

3.3.1 Two-dimensional plane frames

The object of analysis of a structure is to determine bending moments, shear and axial forces throughout the structure. Monolithic two-dimensional plane frames are analysed using either rigorous elastic analysis, for example moment distribution or stiffness method, either manually or using a computer program. Moment redistribution may be included in the analysis if appropriate. However, often it is only required to determine the moments and forces in one beam or one column, so codes of practice allow simplified substructuring.
techniques to be used to obtain these values. Figure 3.18 gives one such substructure, called a ‘subframe’ – refer to (Bhatt et al. 2014) for further details. If the frame is fairly regular, that is spans and loads are within 15% of each other, substructuring gives 90%–95% agreement with full frame analysis.

Substructuring is also carried out in precast frame analysis, except that, where pinned connections are used, no moment distribution or redistribution is permitted. Figure 3.19 shows

Figure 3.17 H-frames (a) structural system, (b) deformation and bending moments.
Figure 3.18 Substructuring method for internal beam in a continuous frame.

Figure 3.19 Substructuring methods for internal beam and columns in a pinned jointed frame (a) internal beam. (Continued)
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For rigid connections, refer to Figure 3.18. Horizontal wind loads and sway forces due to imperfections are not considered in subframes because the bending moments due to horizontal loads in an unbraced frame (there are no column moments due to horizontal loads in a braced frame) are additive to those derived from subframes. Elastic analysis is used to determine moments, forces and deflections, but a plastic (ultimate) section analysis is used for the design of the components. Clearly, some inaccuracies must be accepted, but according to ‘Designer’s Guide to EN 1992-1-1 and EN 1992-1-2’ (Narayanan and Beeby 2005), a design using the partial safety factors (PSFs) and methodologies in the Eurocodes (design philosophy and materials) is “likely to lead to a structure with a reliability index greater than the target value of 3.8 stated in the code for a 50-year reference period.”

### 3.3.2 Design loads on beams and frames

The primary Eurocodes used in the design of precast concrete structures are:

- **Eurocode 0** ‘Basis of design’ (BS EN 1990 2002).
- **Eurocode 1** ‘Actions on Structures – Part 1-1: General Actions – Densities, self-weight, imposed loads for buildings’ (BS EN 1991-1-1 2002) plus other parts dealing with fire, snow, wind, thermal, execution (during construction) and accidental (explosion, impact, etc.) actions.
- **Eurocode 2** ‘Design of Concrete Structures – General rules and rules for buildings’ (BS EN 1992-1-1 2004) including coefficients for sway loads due to imperfections (lack of plumb, built-in curvature, etc.).
Each pan-European document is accompanied by national annexes (NAs) appropriate to national working practices, regional conditions and established/historical precedence, for example stability ties for robustness are the same as in the British code BS 8110: 1997. This book will refer to the UK NAs to Eurocodes EC0 (NA to BS EN 1990 2002), EC1 (NA to BS EN 1991-1-1 2002), EC2 (NA to BS EN 1992-1-1 2004) and briefly to the NA to Eurocode 3 where steelwork, inserts, welding, etc. is required (NA to BS EN 1993-1-1 2005). Appendix 3A (at the end of this chapter) summarises the content of Eurocodes EC2 Parts 1-1 and 1-2, together with the specific clauses related to precast and prestressed concrete elements in the NA to BS EN 1992-1-1. Reference will also be made to the UK’s Published Document PD 6687-1 (PD 6687-1 2010) that gives guidance on some specific items that were not published in the concrete Eurocodes or were in need of additional or noncontradictory additional information. The main items in the PD relating to the design of precast concrete structures are listed in Appendix 3B.

These documents give the magnitude and combinations of loads, loading patterns, and PSFs γf (in BS EN 1990) for gravity and horizontal loads in frames and beams. Four conditions are considered, each with their own values of γf follows

a. Serviceability limit state (stress, cracking, deflection, dynamic, fatigue)

b. Ultimate limit state (ULS) (strength, buckling)

c. Instability limit state (for over-turning)

d. Accidental limit state (fire, robustness, progressive collapse)

However, each condition varies depending on the nature of the loads. These are called ‘actions’ in the Eurocodes, and those applicable to the super-structure are as follows:

i. Permanent actions: self-weight, dead loads of toppings, finishes, services, permanent walls Gk; prestressing forces P; settlement of supports, sway loads due to imperfection.

ii. Variable actions: imposed floor live loads; demountable partitions, snow loads Qk; wind loads Wk; temperature effects.

iii. Accidental actions: fire, impact, loss of support, collapse, explosion, Ad, etc.

Dead, live and wind loads are based on the 95% characteristic value for uniformly distributed load (UDL) known as gk, qk and wk [kN/m2] and for line/beam loads and point loads as Gk, Qk and Wk [kN/m or kN].

The self-weight of plain concrete made with normal-weight aggregates (approx. 2600 kg/m3) is taken as 24 kN/m3, according to BS EN 1991-1-1, Table A.1, unless it is shown by the manufacturer that the characteristic self-weight of elements is different. An additional 1 kN/m3 is made for reinforcement and prestressing tendons, although it is unlikely that tendons will add this amount, for example 10 no. 9.3 mm strands in a 1200 × 150 deep solid slab add only 0.22 kN/m3. The density of wet concrete is taken as 25 kN/m3. The densities or self-weight of other building materials and stored materials in warehouses, etc. are given in BS EN 1991-1-1, Tables A.2 through A.12. Note that the self-weight of masonry units are given in BS EN 771 (BS EN 771 2011) and not in the masonry code (BS EN 1996-1-1 2005).

The design values of actions for each of the limit states depend on the nature of the load (i) to (iii), the use of the floor slabs (e.g. residential, parking, storage) and the number and location of the variable loads. Statistically, it is improbable that all imposed loads will be acting at their characteristic value Q1k, Q2k … Qki and at the same time, that is full live loads will not act at all floor levels in a multi-storey building, or live, wind and snow
loads will not act at the same time. Exceptions to this obviously apply and the designer must be aware of the certain simultaneous combination, such as full live loads acting on a staircase and landing at the same time, in which case the characteristic load will be taken for both elements.

### 3.3.2.1 Serviceability limit state

Historically, the ‘characteristic’ imposed (live) load \( Q_k \) was used in all serviceability calculations of service stresses in prestressed concrete, crack spacing and widths, and short- and long-term deflections using viscoelastic deformations due to creep and other effects such as the relative shrinkage between concrete cast at different times. The Eurocodes consider this too severe for long-term effects of cracking and deflection, and, with the exception of calculating service stresses in prestressed concrete in order to avoid sudden rupture after cracking, reduced values of \( Q_k \) are permitted as shown in Figure 3.20. This is an illustration of the representative values for the characteristic \( Q_k \), combination \( \psi_0 Q_k \), frequent \( \psi_1 Q_k \), and quasi-permanent \( \psi_2 Q_k \) values of imposed loading over a period of time, which can of course be extended to the whole life of the structure. In fact, clause A1.4.2 of EN 1990 allows the serviceability criteria to be specified for each project and agreed with the client, but the actual circumstantial definitions recommended to be used with particular serviceability requirements in clause A1.4.2 of the NA to BS EN 1990 are

a. for function and damage to structural and non-structural elements (e.g. partition walls, etc.), the *characteristic* combination, for example stress, strength  
b. for comfort to user, use of machinery, avoiding ponding of water, etc. the *frequent* combination  
c. for appearance of the structure, the *quasi-permanent* combination, for example deformation, deflections
These are according to Expressions 6.14b, 6.15b and 6.16b of EN 1990 as follows.

The design service moment $M_s$, shear force $V_s$ and end reaction $F_s$ are based on the design service load = characteristic load $\times$ set of load factors $\psi$ as follows:

1. ‘Characteristic’ combination according to BS EN 1990, Exp. 6.14b.

$$G_{kj} \text{ "+" } P \text{ "+" } Q_{k,1} \text{ "+" } \psi_0 Q_{k,1} \text{ with } j \geq 1 \text{ and } i > 1 \quad (3.1)$$

where $\psi_0$ is the characteristic load factor according to the UK NA to BS EN 1991, reproduced here in Table 3.2, for example for domestic usage, $\psi_0 = 0.7$.

The symbol “+” means in combination with. For example referring to Figure 3.21, if the stresses at the bottom of a prestressed beam due to prestress = +12.0 N/mm$^2$ (compression), due to dead load = -5.0 N/mm$^2$ (tension), and due to two independent live loads = -8.0 and -4.0 N/mm$^2$, the characteristic stress $f_b = +12.0 - 5.0 - 8.0 - 0.7 \times 4.0 = -3.8$ N/mm$^2$ (tension).

2. ‘Frequent’ combination according to BS EN 1990, Exp. 6.15b.

$$G_{kj} \text{ "+" } P \text{ "+" } \psi_1 Q_{k,1} \text{ "+" } \psi_2 Q_{k,1} \text{ with } j \geq 1 \text{ and } i > 1 \quad (3.2)$$

### Table 3.2 Partial load factors for live loads

<table>
<thead>
<tr>
<th>Floor usage</th>
<th>$\psi_0$</th>
<th>$\psi_1$</th>
<th>$\psi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic, residential, offices</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Shopping, congregation</td>
<td>0.7</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Storage</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Traffic area &lt; 3t vehicle weight</td>
<td>0.7</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Traffic area &gt; 3t vehicle weight</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Roof</td>
<td>0.7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Snow at altitude &gt; 1000 m</td>
<td>0.7</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Snow at altitude &lt;= 1000 m</td>
<td>0.5</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>Wind pressure</td>
<td>0.5</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: From NA to BS EN 1990, Table NA.A1.1.

$\psi_0$ is used for load combinations of ultimate strength, $\psi_1$ is used for checking crack widths and decompression stresses for certain durability requirements, $\psi_2$ is used for calculating deflections.

Figure 3.21 Example of the characteristic, frequent and quasi-permanent combinations of service stresses in a fictitious prestressed concrete section.
where $\psi_1$ and $\psi_2$ are the frequent and quasi-permanent load factors in Table 3.2, for example for domestic usage, $\psi_1 = 0.5$ and $\psi_2 = 0.3$.

Continuing the example mentioned earlier, the final frequent stress $f_b = +12.0 - 5.0 - 0.5 \times 8.0 - 0.3 \times 4.0 = +1.8$ N/mm² (effectively remains in compression).

3. ‘Quasi-permanent’ combination according to BS EN 1990, Exp. 6.16b

$$G_{ki} \ "+" \ P \ "+" \ \psi_2 \ Q_{ki} \text{ with } j \geq 1 \text{ and } i \geq 1 \quad (3.3)$$

Continuing the example mentioned earlier would not be meaningful as the ‘quasi-permanent’ combination is used for calculating deflections, but for completeness $f_b = +3.4$ N/mm² (compression). It is clear from these three examples that the conditions of stress are less onerous with each successive combination, and this is a reflection of the diminishing effect of viscoelastic deformations according to the use of buildings and the effect of specific creep. Note that in Table 3.2 for storage the $\psi$ factors are between 0.8 and 1.0, indicating a higher specific creep.

### 3.3.2.2 Ultimate limit state

This limit state is known as ‘structure STR’. The design ultimate moment $M_{Ed}$, shear force $V_{Ed}$ and end reaction $F_{Ed}$ are based on the design ultimate load $E_d$ = characteristic load x set of PSFs $\gamma$. Loads are called ‘favourable’ or ‘unfavourable’ in creating the worst possible effects in an element, frame or subframe. The ultimate load combination is according to BS EN 1990, Exp. 6.10, or for STR limit state the least favourable (greater) of Exp. 6.10a and 6.10b, which will always be less than Exp. 6.10. The NA to BS EN 1990, Table NA.A1.2 (B) – Design values of actions (STR) (Set B) give the PSF values. The combinations are

$$6.10a \ \ \ \gamma_{G,i} \ G_{k,i} \ "+" \ \gamma_P \ P \ "+" \ \Sigma \ \gamma_{Q,i} \ \psi_{0,i} \ Q_{k,i} \quad (3.4)$$

$$6.10b \ \ \ \xi \ \gamma_{G,i} \ G_{k,i} \ "+" \ \gamma_P \ P \ "+" \ \gamma_{Q,i} \ Q_{k,i} \ "+" \ \Sigma \ \gamma_{Q,i} \ \psi_{0,i} \ Q_{k,i} \quad (3.5)$$

with $j \geq 1$ and $i > 1$

where $\gamma_{G,i} = 1.35$ unfavourable, $\gamma_{G,i} = 1.0$ favourable, $\gamma_{Q,i} = 1.5$, $\xi = 0.925$

for prestress $\gamma_P = 0.9$ (used for ultimate shear capacity)

NA to BS EN 1990, Table NA.A1.2 (B) notes: “Either expression 6.10, or expression 6.10a together with and 6.10b may be made, as desired. The characteristic values of all permanent actions from one source are multiplied by $\gamma_{G,mp} = 1.35$, if the total resulting action effect is unfavourable and $\gamma_{G,inf} = 1.0$, if the total resulting action effect is favourable. For example all actions originating from the self-weight of the structure may be considered as coming from one source; this also applies if different materials are involved. When variable actions are favourable $Q_k$ should be taken as 0.” In other words, in frame or subframe analysis $1.35 \ G_{k,j} + 1.5 \ \psi_0 \ Q_k$ (6.10a or b) is carried on one element and 1.0 $G_{k,j}$ on the adjacent element. Each live load is taken as the ‘dominant’ in turn, with all of the others as ‘accompanying’ in turn, until the maximum combination is found. Therefore, if there are two live loads present there will be three load combinations, that is 6.10a; 6.10b $Q_{k2}$ dominant; and 6.10b $Q_{k2}$ dominant.

If the building is domestic with $\psi_0 = 0.7$, use the greater of

$$6.10a \ \ 1.35 \ G_{k,j} + 1.5 \times 0.7 \ Q_{k,1}$$

$$6.10b \ \ 0.925 \times 1.35 \ G_{k,j} + 1.5 \ Q_{k,1} + 1.05 \ Q_{k2}$$

$$6.10b \ \ 0.925 \times 1.35 \ G_{k,j} + 1.05 \ Q_{k1} + 1.5 \ Q_{k2}$$
To satisfy the ULS design, the three load combinations must be used to determine the maximum end reactions, and bending moments and shear forces at all points along the span.

### 3.3.2.2.1 Effective span

Finally, it is necessary to define the effective span of floor slabs (simply supported, cantilevers) and beams (on dry bearings or mechanical connectors). The clear span of floors $l_n = \text{distance between beam centres} - \text{sum of half breadth of beams}$. Referring to BS EN 1992-1-1,

$$l_{\text{eff}} = l_n + \min\{h/2; L_{b1}/2\} + \min\{h/2; L_{b2}/2\}$$

where $h$ is the depth of slab (including topping) $L_{b2}$ and $L_{b1}$ are the bearing lengths at either end

For cantilevers, $L_{b2}$ is also the width of the support. For continuous elements after completion of the continuity (i.e. stage 2 loading), $l_{\text{eff}} = \text{distance between beam centres}$. If a bearing medium (pad, plate) is provided, $l_{\text{eff}}$ is to the centre of the pad, and this is also the case for beams supported on steel inserts, cleats and plates, etc.

**Example 3.1**

Calculate the maximum ultimate end reaction $F_{Ed}$ in a simply supported beam of effective span 6.0 m subjected to $G_k = 30 \text{ kN/m}$, $Q_{k1} = 20 \text{ kN/m}$, and $Q_{k2} = 40 \text{ kN point load at mid-span}$. $\psi_0 = 0.7$.

**Solution**

$$w_{Ed} = 1.35 \times 30 + (0.7 \times 1.5) \times 20 = 61.5 \text{ kN/m}$$

$$P_{Ed} = 1.05 \times 40 = 42 \text{ kN}.$$ 

Then $F_{Ed} = 61.5 \times 6.0/2 + 42/2 = 205.5 \text{ kN}$.

Exp. 6.10b $= 1.25 \times 30 + 1.5 \times 20 = 67.5 \text{ kN/m} + 42 \text{ kN (UDL dominant)}$

Then $F_{Ed} = 67.5 \times 6.0/2 + 42/2 = 223.5 \text{ kN. Maximum.}$

Exp. 6.10b $= 1.25 \times 30 + 1.05 \times 20 = 58.5 \text{ kN/m} + 1.5 \times 40 = 60 \text{ kN (point load dominant)}$

Then $F_{Ed} = 58.5 \times 6.0/2 + 60/2 = 205.5 \text{ kN.}$

### 3.3.2.3 Instability limit state

This limit state, known as ‘equilibrium EQU’, is used to check uplift in the back-span of cantilevers, and over-turning of frames including the effect of horizontal wind pressure or other forces. The equilibrium load combination is according to BS EN 1990, Exp. 6.10. The PSFs are given in NA to BS EN 1990, Table NA.A1.2 (A) – Design values of actions (EQU) (Set A) Exp. 6.10 as follows:

$$\gamma_{G,j} \frac{G_{ki}}{G_{k,i}} + \frac{Q_{k1}}{Q_{k,i}} \frac{\psi_{0,i}}{\psi_{0}} + \frac{Q_{k2}}{Q_{k,i}}$$

with $j \geq 1$ and $i \geq 1$

$\gamma_{G,j} = 1.1$ (unfavourable) and 0.9 (favourable)

$\gamma_Q = 1.5$ (unfavourable) and 1.5 $\psi_0$ accompanying
Precast frame analysis

\[ \gamma_Q = 0 \] (favourable)

At installation refer to BS EN 1990, Table A2.4(A) as follows:
\[ \gamma_{G,i} = 1.05 \] (unfavourable) and 0.95 (favourable)
\[ \gamma_{Q,i} = 1.35 \] (unfavourable) and 0 (favourable)

Example 3.2

Calculate the minimum end reaction \( F_{Ed} \) in a simply supported beam of effective span 6.0 m with a 3.0 m span overhanging cantilever at one end subjected to \( G_k = 30 \text{ kN/m} \) and \( Q_k = 20 \text{ kN/m} \).

Solution

Exp. 6.10. Main span = \( 0.9 \times 30 = 27 \text{ kN/m} \)

Exp. 6.10. Cantilever span = \( 1.1 \times 30 + 1.5 \times 20 = 63 \text{ kN/m} \)

Over-turning moment at cantilever = \( 63 \times 3.0^2/2 = 283.5 \text{ kN} \)

Then \( F_{Ed} = 27 \times 6.0/2 - 283.5/6.0 = 81.0 - 47.25 = 33.75 \text{ kN} > 0 \). No uplift.

3.3.2.4 Accidental limit state

The accidental load combination is according to BS EN 1990, Exp. 6.11. The PSFs are all \( \gamma = 1.0 \). \( \psi \) values are given in NA to BS EN 1990, Table NA.A1.3 – Design values of actions for use in accidental combinations of actions, Exp. 6.11 as follows:

\[
6.11 \quad G_{k,i} \ “+” P “+” A_d “+” \psi_{1,1} Q_{k,1} “+” \Sigma \gamma_{Q,i} \psi_{2,i} Q_{k,i}
\]  

(3.8)

with \( j \geq 1 \) and \( i > 1 \)

where \( A_d \) is the value of the accidental action. \( \psi_{1,1} \) is applied to the dominant action and \( \psi_{2,i} \) to the others. However, in accidental situations, it may not be obvious which is which and therefore \( \psi_{1} \) is applied to all.

3.3.3 Gravity and horizontal ultimate loads on frames

3.3.3.1 Permanent, variable and wind actions

In frame and subframe analysis without sway, the critical gravity load combinations with their associated PSFs \( \gamma_G \) and \( \gamma_Q \) are

1. All spans loaded with the maximum ultimate load \( w_{Ed,\text{max}} = \gamma_G G_k + \gamma_Q Q_k \) (for BS EN 1990, Exp. 6.10a or b).
2. Alternate (‘pattern’) spans loaded with \( w_{Ed,\text{max}} \) on one span and the minimum \( w_{Ed,\text{min}} = 1.0 G_k \) on the adjacent span.

For frame analysis with sway, horizontal loads \( W_k \) are combined with gravity \( G_k \) and \( Q_k \) load combinations 6.10a and 6.10b for three situations:

1. Permanent “+” imposed actions (gravity dead + live)
2. Permanent “+” wind actions (gravity dead + wind)
3. Permanent “+” imposed “+” wind actions (all)
The fundamental PSF for wind load (notation used here $\gamma_W$) is as NA to BS EN 1990, Table NA.A1.2 (B) – Design values of actions (STR) (Set B) $\gamma_W = 1.5$, and is modified by $\psi_0 = 0.5$ (see Table 3.2) in the same way as for gravity loads. The values for $\gamma$ and $\psi$ are summarised in Table 3.3.

### Example 3.3

Calculate the maximum ultimate bending moment $M_{Ed}$ at the lower end of the columns of height $h = 4.0$ m in Figure 3.22. The beam–column connections are pinned, and the foundation is rigid. The distance from the edge of the column to the centre of the beam end reaction is 100 mm. Characteristic beam loading is $G_k = 40$ kN/m and $Q_k = 30$ kN/m, and the wind pressure equates to a horizontal load $W_k = 12$ kN. The carry-over moment at the lower end of the column is equal to 50% of the upper end moment due to beam eccentricity. Let $\psi_0$ (gravity load) = 0.7.

**Solution**

Eccentricity of beam reaction $R$ from the centre of column $e = 300/2 + 100 = 250$ mm.

Moment at the lower end of column due to $R_{Ed}$, $M_{Ed} = 0.5 \times R_{Ed} \times e$

![Figure 3.22 Detail to Example 3.3.](image-url)
Moment at the lower end of each column due to wind load, \( M_{Ed} = W_{Ed} h/2 \) (because there are two columns)

Ultimate load combinations and moments are summarised in the following table.

<table>
<thead>
<tr>
<th>Load combination</th>
<th>( w_{Ed} ) (kN/m)</th>
<th>( R_{Ed} ) (kN)</th>
<th>( M_{Ed} = 0.5 R_{Ed,e} ) (kNm)</th>
<th>( W_{Ed} ) (kN)</th>
<th>( M_{Ed} = W_{Ed} ) h/2 (kNm)</th>
<th>Total ( M_{Ed} ) (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.10a P + I</td>
<td>85.5</td>
<td>342.0</td>
<td>42.75</td>
<td>—</td>
<td>—</td>
<td>42.75</td>
</tr>
<tr>
<td>P + W</td>
<td>54.0</td>
<td>216.0</td>
<td>27.0</td>
<td>9.0</td>
<td>18.0</td>
<td>45.0</td>
</tr>
<tr>
<td>All</td>
<td>85.5</td>
<td>342.0</td>
<td>42.75</td>
<td>9.0</td>
<td>18.0</td>
<td>60.75</td>
</tr>
<tr>
<td>6.10b P + I</td>
<td>95.0</td>
<td>380.0</td>
<td>47.5</td>
<td>—</td>
<td>—</td>
<td>47.5</td>
</tr>
<tr>
<td>P + W</td>
<td>50.0</td>
<td>200.0</td>
<td>25.0</td>
<td>18.0</td>
<td>36.0</td>
<td>61.0</td>
</tr>
<tr>
<td>All</td>
<td>95.0</td>
<td>380.0</td>
<td>47.5</td>
<td>9.0</td>
<td>18.0</td>
<td>65.5</td>
</tr>
</tbody>
</table>

\( P \), permanent (dead load); \( I \), imposed (live load); \( W \), wind load.

Then \( M_{Ed,max} = 65.5 \text{ kNm} \) using Exp. 6.10b for all loads (it is interesting to note that according to BS 8110 values for all loads, \( w_{ult} = 1.2 \times 70 = 84 \text{ kN/m} \), \( M_u \) (gravity) = 42 kNm, \( W_u = 1.2 \times 12 = 14.4 \text{ kN} \), \( M_u \) (wind) = 28.8 kNm. Total \( M_u = 70.8 \text{ kNm} \).

### 3.3.3.2 Horizontal forces due to imperfections

All buildings, including precast concrete structures built with the greatest practical accuracy, will contain imperfections due to construction methods, errors or natural effects. Some of these are unavoidable, for example over-turning moments due to balconies, hanging façade panels, etc. resulting in horizontal deflection and curvature in columns and walls. The reactions to the precast frame from the inclined staircase shown in Figure 3.23 are not

**Figure 3.23** Inclined staircase imposing horizontal forces to the structure at Bella Sky Hotel, Denmark. (Courtesy Ramboll, Denmark.)
exactly imperfections but demonstrate the point of transferring inclined gravity loads into horizontal forces.

BS EN 1992-1-1 defines imperfections as ‘possible deviations’ in geometry and ‘positions’ of loads in Section 5.2 and as quantified by code Exp. 5.1 through 5.4 as an ultimate horizontal force \( H_i = N \theta \) due the inclination \( \theta \) according to the height \( l \) and number \( m \) the elements contributing to the imperfection, recognising the improbability that imperfection will be the same in all elements. The value attributed to \( \theta \) is related to ‘Class 1 execution deviations’ according to BS EN 13670 (BS EN 13670 2009) and is taken into account at the ULS and accidental design situations, but not at serviceability. Deviations in cross section dimensions are taken into account in material safety factors. Imperfections and deviations should not be included in structural analysis as their effect is additional to first-order bending moments and shear forces, but not when checking deflections.

BS EN 1992-1-1 distinguishes between (a) whole braced or unbraced structures, known as ‘global analysis’, (b) isolated columns in braced or unbraced structures, and (c) floor and roof diaphragm action (see Chapter 8) as follows:

(a) For structures, the horizontal force \( H_i \) is applied to the bracing system, for example cores or shear walls, at each floor level as the horizontal component of the total ultimate gravity load at that floor level. Referring to Figure 3.24a (adapted from BS EN 1992-1-1, Fig. 5.1b).

\[
H_i = \theta_i (N_b - N_a) = \theta \sum V_{Ed} \text{ at the floor level connections} \tag{3.9}
\]

where \( \theta_i = \theta_0 \alpha_b \alpha_m \) \tag{3.10}

\( \theta_0 = 1/200 \) (NA BS EN 1992-1-1)

![Figure 3.24](https://example.com/figure3.24.png)

*Figure 3.24 Examples of the effect of geometric imperfections. (a) Bracing system and (b) isolated column in unbraced structure. (Adapted from BS EN 1992-1-1. 2004, Eurocode 2: Design of Concrete Structures – Part 1-1: General rules and rules for buildings, BSI, London, February 2014, Fig. 5.1a1 and b.)*
\[ \alpha_b = \frac{2}{3} \leq \left( \frac{2}{\sqrt{l}} \right) \leq 1 \]  
\[ l = \text{total height of braced structure (m)} \]  
\[ \alpha_m = \sqrt{0.5(1 + 1/m)} \]  
\[ m = \text{number of vertical members contributing to the horizontal force on the bracing system.} \]

(b) For isolated columns, \( H_i \) is applied to unbraced columns as an eccentricity \( e_i \) as shown in Figure 3.24b (adapted from BS EN 1992-1-1, Fig. 5.1a and Exp. 5.2). This is used because columns in a precast structure are statically determinate and there is no moment continuity between the rows of columns.

\[ e_i = 0; \frac{l_0}{2} \]  
\[ \text{where } l_0 \text{ is effective length (m) of the column at the floor level that } e_i \text{ is considered, that is at the second floor level } l_0 \text{ is based on the height to the second floor, etc.} \]
\[ \alpha_b = \frac{2}{3} \leq \left( \frac{2}{\sqrt{l}} \right) \leq 1, \text{where } l = \text{actual length of column at the level that } e_i \text{ is acting (m) and } m = \alpha_m = 1 \]

(c) For isolated columns (and minor axis of walls) in a braced structure \( \alpha_b \) is simplified such that \( \alpha_b = m = \alpha_m = 1 \)

\[ e_i = \frac{l_0}{400} \]  
\[ \text{Example 3.4} \]

Calculate the horizontal forces at each roof and floor level and the over-turning moment at the foundation due to imperfection in the braced skeletal structure shown in Figure 3.25. The number of columns in each line is six, and there are five rows of column. There are two sets of shear walls in each of the external rows of columns. The total ultimate gravity load per floor = 15,000 kN and at the roof = 7,000 kN.

Figure 3.25 Detail to Example 3.4 (braced) and Example 3.5 (unbraced).
Solution

\[ l = 10.5 \text{ m}, \text{ then } \alpha_h = 2/\sqrt{10.5} = 0.617 \text{ use } 2/3 \]

\[ m = 30 \text{ columns. } \alpha_m = \sqrt{0.5(1 + 1/30)} = 0.719 \]

\[ \theta_i = (1/200) \times (2/3) \times 0.719 = 0.002396 \text{ (or 1 in 417)} \]

At roof level, \( H_{i,\text{roof}} = \theta_i \times \frac{N}{2} \times \text{sets of shear walls} = 0.002396 \times 7000/2 = 8.38 \text{ kN per wall} \)

At floor level, \( H_{i,\text{floor}} = 0.002396 \times 15,000/2 = 17.97 \text{ kN per wall} \)

The over-turning moment \( M_i \), due to \( H_i = 8.38 \times 10.50 + 17.97 \times (7.25 + 4.00) = 290.1 \text{ kNm per wall} \).

Example 3.5

Calculate the horizontal forces at each roof and floor level and the over-turning moment at the foundation due to imperfection in one of the internal columns, if the same structure shown in Figure 3.25 is unbraced. The effective length factor for the columns may (in this example) be taken as 2.2. The total ultimate gravity load per column per floor = 900 kN and at the roof = 500 kN.

Solution

At the roof level, \( l = 10.5 \text{ m}, \text{ then } \alpha_h = 2/\sqrt{10.5} = 0.617 \text{ use } 2/3. I_0 = 2.2 \times 10.5 = 23.1 \text{ m} ^2 \)

\[ e_i = (1/200) \times (2/3) \times 23.1/2 = 0.039 \text{ m} \]

Then \( M_i \) due to \( e_i = 500 \times 0.039 = 19.5 \text{ kNm} \) which equates to \( H_i = 19.5/10.5 = 1.85 \text{ kN} \)

At the second floor level, \( l = 7.25 \text{ m}, \text{ then } \alpha_h = 2/\sqrt{7.25} = 0.742. I_0 = 2.2 \times 7.25 = 15.95 \text{ m} ^2 \)

\[ e_i = (1/200) \times 0.742 \times 15.95/2 = 0.030 \text{ m} \]

Then \( M_i \) due to \( e_i = 900 \times 0.030 = 27.0 \text{ kNm} \) which equates to \( H_i = 27.0/7.25 = 3.72 \text{ kN} \)

At the first floor level, \( l = 4.00 \text{ m}, \text{ then } \alpha_h = 2/\sqrt{4.00} = 1. I_0 = 2.2 \times 4.00 = 8.80 \text{ m} ^2 \)

\[ e_i = (1/200) \times 1 \times 8.80/2 = 0.022 \text{ m} \]

Then \( M_i \) due to \( e_i = 900 \times 0.022 = 19.8 \text{ kNm} \) which equates to \( H_i = 19.8/4.00 = 4.95 \text{ kN} \)

Total \( M_i = 19.5 + 27.0 + 19.8 = 66.3 \text{ kNm per column} \).

(Note that \( M_i \) for isolated columns is greater per column than if the total \( M_i \) for the walls was divided over the total number of columns).

3.3.4 Beam subframe

Figure 3.19a. The subframe consists of the beam to be designed of span \( L_1 \), and half of the adjacent beams of span \( L_2 \) and \( L_3 \). The eccentricity of the beam end reaction from the centroidal axis of the column is \( e \). Alternate pattern loading is used. The height of the column above and below the beam is actually of no consequence to beam. It is assumed that the cross section and flexural stiffness of the column is constant.
The maximum moment in the beam is: 
\[ M_1 = w_{Ed,\max} (L_2 - 2e)^2 / 8 \]  
(3.15)

The beam end reaction is 
\[ R_1 = w_{Ed,\max} L_2 / 2 \]  
(3.16)

(Note the shear force in the beam is 
\[ V_{Ed} = w_{Ed,\max} (L_2 - 2e) / 2 \].)

End reactions in the adjacent beams are 
\[ R_1 = w_{Ed,\min} L_1 / 2 \] and 
\[ R_3 = w_{Ed,\min} L_3 / 2 \]  
(3.17)

The resulting maximum bending moment in the column is given by 
\[ M_{col} = (R_2 - R_1) e b_3 / (b_2 + b_3) \]  
(3.18)

assuming that \( R_1 < R_3 \) and \( b_1 > b_3 \). Figure 3.26a shows the final moments.

Figure 3.26 Bending moments in a pinned jointed frame for (a) internal beams, (b) upper floor columns.

(Continued)
3.3.5 Upper floor column subframe

Figure 3.19b. The subframe consists of the column to be designed of height (distance between centres of beam bearing) \( h_2 \), and half the adjacent columns of heights \( h_1 \) and \( h_3 \). Because the column is continuous, the cross section and flexural stiffness \( EI \) of each part of the column is considered as shown in the figure. The beams are pattern loaded as mentioned earlier, of span \( L_4/2 \) and \( L_5/2 \), and the eccentricity of each beam end reaction from the centroidal axis of the column is \( e_4 \) and \( e_5 \), respectively. The moment at the upper end of the designed column is given by

\[
M_{col, upper} = \left( R_4 e_4 - R_5 e_5 \right) \frac{EI_2}{h_2} + \frac{EI_3}{h_3}
\]

and at the lower end is

\[
M_{col, upper} = \left( R_4 e_4 - R_5 e_5 \right) \frac{EI_1}{h_1} + \frac{EI_2}{h_2}
\]

where \( R_4 \) and \( R_5 \) are given in Equations 3.2 and 3.3. Figure 3.26b shows the final moments. Note that patch loading produces single curvature in the columns.

3.3.6 Ground floor column subframe

Figure 3.19c The subframe consists of the column to be designed of height (distance between the centre of first floor beam bearing and 50 mm below top of foundation (see section 9.4)) \( h_1 \), and half the adjacent column of height \( h_2 \). All other details are as before. If the foundation
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is rigid (moment resisting), the moment at the upper end of the designed column is given by Equation 3.5 with appropriate notation. The carry-over moment at the lower end is equal to 50% of the upper end moment. If the foundation is pinned, the upper end moment is given by

\[
M_{\text{col,upper}} = \left( R_4 e_4 - R_5 e_5 \right) \frac{0.75 E I_1}{b_1} \left( \frac{0.75 E I_1 + E I_2}{b_1 + b_2} \right)
\]

(3.20)

and the lower end moment is zero. Figure 3.26c shows the final moments. Patch loads produce single curvature in the columns.

Example 3.6

Determine, using substructuring techniques, the bending moments in the beam X and columns Y and Z identified in Figure 3.27. The beam–column connections are pinned, and the foundation is rigid. The distance from the edge of the column to the centre of the beam end reaction is 100 mm. Characteristic beam loading is \( G_k = 40 \, \text{kN/m} \) and \( Q_k = 30 \, \text{kN/m} \).

Solution

\( w_{\text{Ed, max}} = \max\{1.35 \times 40 + 1.05 \times 30; 1.25 \times 40 + 1.5 \times 30\} = \max\{85.5; 95.0\} = 95.0 \, \text{kN/m} \); \( w_{\text{Ed, min}} = 40 \, \text{kN/m} \).

Beam subframe

\( e = \frac{450}{2} + 100 = 325 \, \text{mm} \)

Equation 3.1. \( M_1 = 95.0 \times (8.000 - 2 \times 0.325)^2/8 = 641.5 \, \text{kNm} \)

Column Y subframe

Beam end reactions \( R_1 = 95.0 \times 8.000/2 = 380.0 \, \text{kN} \); \( R_2 = 40 \times 6.000/2 = 240.0 \, \text{kN} \)

\( e_1 = e_2 = 300/2 + 100 = 250 \, \text{mm} \)

Figure 3.27 Detail to Example 3.6.
but \( \frac{E_1}{h_1} = \frac{E_1}{h_2} \)

Equation 3.5. At upper and lower ends, \( M_{col} = (380.0 - 240.0) \times 0.250 \times 0.5 = 17.5 \text{ kNm} \)

**Column Z subframe**

Beam end reactions as before. \( e_1 = e_2 = 450/2 + 100 = 325 \text{ mm} \)

Given that \( E \) is constant

\[
\frac{I_1}{h_1} = \frac{300 \times 450^3}{12 \times 5050} = 451 \times 10^3 \text{mm}^2
\]

\[
\frac{I_2}{h_2} = \frac{300 \times 300^3}{12 \times 3200} = 211 \times 10^3 \text{mm}^2
\]

Equation 3.5. At upper end, \( M_{col, upper} = (380 - 240) \times 0.325 \times \frac{451}{451 + 211} = 31.0 \text{ kNm} \)

At lower end, \( M_{col, lower} = 50\% \times 31.0 = 15.5 \text{ kNm} \).

### 3.4 CONNECTION DESIGN

Connections form the vital part of precast concrete design and construction. They alone can dictate the type of precast frame, the limitations of that frame, and the erection progress. It is said that in a load-bearing wall frame the rigidity of the connections can be as little as 1/100 of the rigidity of the wall panels − 200 N/mm² per mm length for concrete panels versus 2.7–15.0 N/mm² per mm length for joints (Straman 1990). Moreover, the deformity of the bedding joint, that is the invisible interface where the panel is wet bedded onto a mortar, between upper and lower wall panels can be 10 times greater than that of the panel.

The previous paragraph contained the words **connections** and **joints** to describe very similar things. Connections are sometimes called ‘joints’ – the terminology is loose and often interposed. The definition adopted in this book is as follows:

**Connection**: is the total construction between two (or more) connected components: it includes a part of the precast component itself and may comprise several joints.

**Joint**: is the part of a connection at individual boundaries between two elements (the elements can be precast components, *in situ* concrete, mortar bedding, mastic sealant, etc.)

For example in the beam–column assembly shown in Figure 3.28, a bearing joint is made between the beam and column corbel, a shear joint is made between the dowel and the angle, and a bolted joint is made between the angle and column. When the assembly is completed by the use of *in situ* mortar/grout, the entire construction is called a connection. This is because the overall behaviour of the assembly includes the behaviour of the precast components plus all of the interface joints between them. Engineers prove the capacity of the entire connection by assessing the behaviour of the individual joints.

Structurally, joints are required to transfer all types of forces − the most common of these being not only compression and shear, but also tension, bending and occasionally torsion. The combinations of forces at a connection can be resolved into components of compressive, tensile and shear stress, and these can be assessed according to limit state design. Steel (or other materials) inserts may be included if the concrete stresses are greater than permissible
values. The effects of localised stress concentrations near to inserts and geometric discontinuities can be assessed and proven at individual joints. However, connection design is much more important than that because of the sensitivity of connection behaviour to manufacturing tolerances, erection methods and workmanship.

It is necessary to determine the force paths through connections in order to be able to check the adequacy of the various joints within. Compared with cast in situ construction, there are a number of forces which are unique to precast connections, namely frictional forces due to relative movement caused by shrinkage, etc. pretensioning stresses in the concrete and steel, handling and self-weight stresses. In the example shown in Figure 3.29, a reinforced concrete column and corbel support a pretensioned concrete beam. The figure shows that there are 10 different force vectors in this connection as follows:

1. A: diagonal compression strut in corbel
2. B: horizontal component reaction to force at A
3. C: vertical component reaction to force at A
4. D: internal diagonal resultant to forces B and C
5. E: diagonal compression strut in beam
6. F & G: horizontal component reaction to force E
7. H: tension field reaction to forces E & F
8. J: horizontal friction force caused by the relative movement of beam and corbel
9. K: horizontal membrane reaction to beam rotation due to eccentric prestressing

The structural behaviour of the frame can be controlled by the appropriate design of connections. In achieving the various structural systems in Section 3.3.2, it may be necessary to design and construct either/bboth rigid and/or pinned connections. Rigid monolithic connections can only truly be made at the time of casting, although it is possible to site cast connections that have been shown to behave as monolithic, for example cast in situ filling of prefabricated soffit
beams before and after casting as shown in Figure 3.30a and b. The advantages lost to *in situ* concreting work (cold climates in particular), the delayed maturity, the increase in structural cross section, and the reliance on correct workmanship, etc. detract this solution in favour of bolted or welded mechanical devices. Rigid connections may be made at the foundation where there is less restriction on space as shown in Figure 3.31. In very simple terms, a bending moment is generated by the provision of a force couple in rigid embedment, that is no slippage when the force is generated. Pinned connections are designed by an absence of this couple, although many connectors designed in this way inadvertently contain a force couple, giving rise to spurious moments which often cause cracking in a region of flexural tension.

To gain an overview of the various types, Figure 3.32 and Table 3.4 show the locations, classification and basic construction of connections in a precast structure.

In theory, no connection is fully rigid or pinned – they all behave in a semi-rigid manner, especially after the onset of flexural cracking. Using a ‘beam-line’ analysis, Figure 3.33, we can assess the structural classification of a connection. Although the beam-line approach was developed for structural steelwork in c.1936, research carried out since 1990 has shown that the method is appropriate to precast connections (Elliott et al. 1998, Elliott et al. 2003a,b, Ferriera et al. 2003, Elliott and Jolly 2013).

The moment–rotation ($M$-$\theta$) diagram in Figure 3.33 is constructed by considering the two extremes in the right hand part of Figure 3.33. The hogging moment of resistance of the beam at the support is given by $M_{Rd} > wL^2/12$, and the rotation of a pin ended beam subjected to a UDL of $w$ is $\theta = wL^3/24EI$. The gradient of the beam line is $2EI/L$. The $M$-$\theta$ plot for plots 1 and 2 give the monolithic and pinned connections, respectively. In reality, the behaviour of a connection in precast concrete will follow plots 3, 4 or 5, etc. If the $M$-$\theta$ plot for the connection fails to pass through the beam-line, that is plot 5, the connection is deemed not to possess sufficient ductility and should be considered in design as ‘pinned’. Furthermore, its inherent
stiffness (given by the gradient of the $M-\theta$ plot) is ignored. Conversely, if the $M-\theta$ behaviour follows plot 3 (the gradient must lie in the shaded zone and the failure takes place outside the shaded zone), the effect of the connection will not differ from a monolithic by more than 5%.

3.5 STABILISING METHODS

Structural stability and safety are necessary considerations at all times during the erection of precast concrete frames. The structural components will not form a stabilising system until the connections are completed – in some cases, this can involve several hours of maturity of cast in situ concrete/grout joints, and several days if structural cast in situ toppings
are used to transfer horizontal forces. A stabilising system must comprise two things as shown in Figure 3.34:

1. A horizontal system, often called a ‘floor diaphragm’ because it is extremely thin in relation to its plan area
2. A vertical system in which the reactions from the horizontal system are transferred to the ground (or other substructure)
Table 3.4 Types of connections in precast frames

<table>
<thead>
<tr>
<th>Connection type</th>
<th>Location in Figure 3.32</th>
<th>Classification</th>
<th>Method of jointing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam – column head</td>
<td>1</td>
<td>Pinned</td>
<td>Dowel</td>
</tr>
<tr>
<td>Beam – column head</td>
<td>2</td>
<td>Rigid</td>
<td>Dowel plus continuity top steel</td>
</tr>
<tr>
<td>Rafter – column head</td>
<td>3</td>
<td>Pinned</td>
<td>Dowel</td>
</tr>
<tr>
<td>Rafter – column head</td>
<td>4</td>
<td>Rigid</td>
<td>Bolts (couple)</td>
</tr>
<tr>
<td>Column splice</td>
<td>5</td>
<td>Pinned</td>
<td>Bars in grouted sleeve (couple)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rigid</td>
<td>Threaded couplers</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Steel shoes</td>
</tr>
<tr>
<td>Beam – column face</td>
<td>6</td>
<td>Pinned</td>
<td>Bolts</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Welded plates</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Notched plates</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dowels</td>
</tr>
<tr>
<td>Beam – column corbel</td>
<td>7</td>
<td>Pinned</td>
<td>Dowel</td>
</tr>
<tr>
<td>Beam – column corbel</td>
<td>8</td>
<td>Rigid</td>
<td>Dowel plus continuity top steel</td>
</tr>
<tr>
<td>Beam – beam</td>
<td>9</td>
<td>Pinned</td>
<td>Bolts</td>
</tr>
<tr>
<td>Slab – beam</td>
<td>10</td>
<td>Pinned</td>
<td>Tie bars</td>
</tr>
<tr>
<td>Slab – wall</td>
<td>11</td>
<td>Pinned</td>
<td>Tie bars</td>
</tr>
<tr>
<td>Column – foundation</td>
<td>12</td>
<td>Pinned</td>
<td>Bolts</td>
</tr>
<tr>
<td>Column – cast in situ beam or retaining wall</td>
<td>13</td>
<td>Pinned or rigid*</td>
<td>Bolts</td>
</tr>
</tbody>
</table>

* Depending on the design of the cast in situ substructure.

\[
\theta_R = \frac{wL^3}{24EI}
\]

\[
M_R = \frac{wL^2}{12}
\]

**Figure 3.33** Definition of moment–rotation characteristics.
The horizontal system is considered in detail in Chapter 8 where reference is also made to the many code regulations on this topic. When subjected to horizontal wind or lack-of-plumb forces, the floor slab acts as a deep beam and is subjected to bending moments $M_h$ and shear forces $V_h$ ($h$ being the subscript used for horizontal diaphragms). The design method is shown in Figure 3.35. The design is a three-stage approach:

1. The floor diaphragm is analysed as a long, deep beam which is supported by a number of shear walls, shear cores, deep columns (wind posts), or other kinds of bracing such as steel cross bracing. Figure 3.35a.

2. If there are only two supports (bracing), the analysis is statically determinate and $M_{Ed,h}$ and $V_{Ed,h}$ may be calculated directly. If there are more than two supports, irrespective of where they are positioned, the analysis is statically indeterminate. The support reactions must first be found by a technique which considers the relative stiffness and position of each support, and the horizontal (e.g. wind load) pressure distribution. The derivation is given in Section 8.1 after which $M_{Ed,h}$ and $V_{Ed,h}$ may be calculated.

3. The area of reinforcement required to resist $M_{Ed,h}$ and $V_{Ed,h}$ is determined as follows:

$$A_{sh} = M_{Ed,h} \gamma_m / 0.8 \ B \ f_{yk}$$

(3.21)

where

- $0.8 \ B$ is the assumed lever arm between the compression zone and the tie steel (the assumption is known to be conservative)
- $f_{yk} / \gamma_m$ is the design stress in the tie steel with $\gamma_m = 1.15$

High tensile rebar with $f_{yk} = 500 \text{ N/mm}^2$ or standard helical strand with $f_{pk} = 1770 \text{ N/mm}^2$ is used (super strand or Dyform tend to be too stiff to handle) – the reasons are given in Section 8.4.

$$A_{shd} = V_{Ed,h} \gamma_m / 0.6 \ \mu \ f_{yk}$$

(3.22)
where $\mu$ is the coefficient of friction as given in BS EN 1992-1-1, clause 6.2.5(2). Hollow core slabs are considered as being untreated and smooth, then $\mu = 0.6$ with no special, that is ex-factory, edge preparation (see Section 8.2). For ex-steel mould with smooth surfaces, $\mu = 0.5$.

4. The tie steel $A_{shd}$ must be placed everywhere moments occur. The tie steel $A_{svh}$ must be placed only where the shear force is greater than a certain value. This is found by checking that the interface shear stress $\tau_{Ed,i} = V_{Ed,i} / B (D - 30 \text{ mm})$ does not exceed $\tau_R \leq 0.15 \text{ N/mm}^2$ for smooth and rough surfaces (as in the case of machine cast hollow core units and as-cast precast planks) or $\tau_R \leq 0.10 \text{ N/mm}^2$ for very smooth surfaces cast against steel moulds, according to BS EN 1992-1-1, clause 10.9.3(12). (The reason for the deduction of 30 mm is explained in Section 8.4.1.)

---

**Figure 3.35** Diaphragm floor action. (a) Deep beam analogy. (b) Reinforced structural topping in double-tee floors. (Continued)
Diaphragms may be reinforced in several ways. In Figure 3.35b, a reinforced cast in situ topping transfers all horizontal forces to the vertical system – the precast floor plays no part but for restraining the topping against buckling. In Figure 3.35c, there is no cast in situ topping. Perimeter and internal tie steel resists the chord forces resulting from horizontal moments. Coupling bars are inserted into the ends of the floor units, and together with the perimeter steel provides the means for shear friction generated in the concrete-filled longitudinal joints between the units.

Example 3.7

Determine the shear wall reactions and diaphragm reinforcement in the floor shown in Figure 3.36a. The precast units are 150 mm deep hollow cored and have an ex-factory edge finish. The characteristic wind pressure on the floor \( w_k = 3 \text{ kN/m} \). Tie steel is high tensile ribbed bar \( f_{sk} = 500 \text{ N/mm}^2 \). Suggest some reinforcement details.
Precast frame analysis

Solution

NA to BS EN 1990, Table NA.A1.2 (B) – Design values of actions (STR) (Set B) \( \gamma_W = 1.5 \)

Design ultimate wind load = \( 1.5 \times 3.0 = 4.5 \text{kN/m} \).

From Figure 3.36b, support reaction \( R_1 = 50.62 \text{kN} \); \( R_2 = 84.38 \text{kN} \).

Shear span from LHS (distance to zero shear and hence point of maximum moment) = \( 50.62/4.5 = 11.25 \text{ m} \).

\( M_{Ed,h,\text{max}} = 50.62 \times 11.25/2 = 284.74 \text{kNm} \); \( V_{Ed,h,\text{max}} = 57.38 \text{kN} \) at RHS of 24 m span.

\( A_{s,h} = 284.74 \times 10^6/0.8 \times 5000 \times (500/1.15) = 164 \text{ mm}^2 \). Use 2 no. H12 bars (226).

Interface shear stress \( v_{Edi} = 57.38 \times 10^3/5000 \times (150 - 30) = 0.096 \text{N/mm}^2 \) \(< 0.15 \text{N/mm}^2 \) allowed. No shear reinforcement needed.

Vertical stabilising systems are dictated by the necessary actions of the structural system, that is skeletal, wall or portal frame. Column effective lengths depend on the type and direction of the bracing. However, there is a broad classification as the structure is

1. Unbraced frame, Figure 3.37, where horizontal force resistance is provided either by moment resisting frame action, cantilever action of columns, or cantilever action of wind posts (deep columns)
2. Braced frame, Figure 3.38, where horizontal force resistance is provided either by cantilever action of walls or cores, in-plane panel action of shear walls or cores, infill walls, cross bracing, etc. or
3. Partially braced frame, Figure 3.39, which is some combination of (1) and (2)
The type of stabilising system may be different in other directions. The floor plan arrangement and the availability of shear walls/cores will dictate the solution. The simplest case is a long narrow rectangular plan where, as shown in Figure 3.40a, shear walls brace the frame in the y direction only, the x direction being unbraced. In other layouts, shown for example in Figure 3.40b, it is nearly always possible to find bracing positions. Precast skeletal frames
Figure 3.39 Alternative partial height bracing mechanisms and resulting column effective length factors.

Figure 3.40 Positions of shear walls and cores in alternative floor plan layouts. (a) Positions of shear walls, (b) positions of shear cores or walls around stairs and lift shafts. (Continued)
of three or more storeys in height are mostly braced or partially braced. This is to avoid having to use deep columns to cater for sway deflections, which give rise to large second-order bending moments. Section 6.2.6 refers in more detail.

It is not wise to use different stabilising systems acting in the same direction in different parts of a structure. The relative stiffness of the braced part is likely to be much greater than in the unbraced part, giving rise to torsional effects due to the large eccentricity between the centre of external pressure and the centroid of the stabilising system, as explained in Figure 3.40c. The different stabilising systems should be structurally isolated – Figure 3.40d.

In calculating the position of the centroid of a stabilising system, the stiffness of each component of thickness $t$ and length $L$ is given by $E_{cm,long} I$, where $E_{cm,long}$ = long-term Young’s modulus (usually taken as 15 kN/mm$^2$) and $I = tL^3/12$. First moments of stiffness are used to calculate the centroid as explained in Example 3.8.

Example 3.8
Propose stabilising systems for the five-storey skeletal frame shown in Figure 3.41a. The beam–column connections are all pinned, and the columns should be the minimum possible cross section to cater for gravity loads. Wind loading may be assumed to be uniform over the entire façade. Use only shear walls for bracing.

Hint: the grid dimensions around the stairwell may be taken as 4 m × 3 m, and at the lift shaft 3 m × 3 m.

Solution
A braced frame is required up to the fourth floor, after which a one-storey unbraced frame may be used. It would not otherwise be possible to satisfy the requirement of minimum column sizes for gravity loads. To avoid torsional effects (see Figure 3.40c), the centroid of the stabilising system should be as close as possible to the centre of external pressure, that is at $x \approx 24$ m and $y \approx 16$ m. It is necessary to first consider the two orthogonal directions.

Stability in $y$-direction
The centroid of the stability walls $x' \approx 24$ m.
Select walls as shown in Figure 3.41b. On the assumption that the material and construction of all walls is the same, Young’s modulus and thickness of wall are common to all walls and need not be used in the calculation.

Centroid of stiffness \( x' \) is calculated as:

\[
x' = \frac{(4^3 \times 0) + (4^2 \times 3) + (3^3 \times 33) + (3^2 \times 36) + (4^2 \times 45) + (4^1 \times 48)}{(4 \times 4^1) + (2 \times 3^3)} = 25.0 \text{m},
\]

which is sufficiently close to the required point to eliminate significant torsional effects.
Stability in x-direction
The centroid of the stability walls \( y' \approx 32/2 = 16 \text{ m} \).
Centroid of stiffness
\[
y' = \frac{(3^3 \times 0) + (3^3 \times 16) + (3^3 \times 32)}{3 \times 3^3} = 16.0 \text{ m}, \text{ which is at the correct point.}
\]

3.6 COMPARISON OF STANDARD DESIGNS TO BS 8110 AND EUROCODES

To assist the transition between the British code BS 8110:1997 and the Eurocodes EC0, EC1 and EC2, the following precast reinforced and prestressed concrete elements are designed:

1. Reinforced concrete rectangular beam
2. Reinforced concrete rectangular column
3. Prestressed concrete solid floor slab

Clauses and tables in the codes are indicated within ‘{ }’.

3.6.1 Reinforced concrete rectangular beam

A 600 mm deep \( \times 300 \) mm wide r.c. beam carries uniformly distributed dead and live loading of 40 and 30 kN/m over a simply supported clear span of 5.85 m with bearing lengths of 150 mm. The beam carries office loading. The exposure is internal, and the fire resistance is 60 min. The design life is 50 years.

Design the main reinforcement at mid-span and the shear reinforcement at the support and position where nominal links are required. Calculate the crack width and the long-term deflection using the appropriate creep factor and check the span/depth ratio for the area of rebars designed. Use \( f_{ck}/f_{cu} = 32/40 \), high tensile main bars and links grade \( f_{yk}/f_y = 500 \text{ N/mm}^2 \), and normal-weight concrete with a 20 mm coarse aggregate.

<table>
<thead>
<tr>
<th>BS 8110 solution</th>
<th>Eurocodes solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durability.</strong> BS 8500-1. Table A.4 for 50 years</td>
<td>Exposure XCl. ( C_{nom} = 15 + \Delta C_{dev} )</td>
</tr>
<tr>
<td>Exposure XCl. Cover ( c = 15 + \Delta c )</td>
<td>{4.4.1.3(3)} ( \Delta C_{dev} = 5 \text{ mm} )</td>
</tr>
<tr>
<td>{7.3} ( \Delta c = 5 \text{ mm} )</td>
<td>Cover to links = 20 mm</td>
</tr>
<tr>
<td>Cover to links ( c = 20 \text{ mm} )</td>
<td>Fire. R60. BS EN 1992-1-2</td>
</tr>
<tr>
<td>Fire. 1 h</td>
<td>{Table 5.5} for ( b = 300 \text{ mm}, axis a = 25 \text{ mm} )</td>
</tr>
<tr>
<td>{Table 3.4} ( c = 20 \text{ mm} )</td>
<td>( \vdash a = \max(20 + 8 = 28 \text{ mm}; 25 \text{ mm}) )</td>
</tr>
<tr>
<td><strong>Section properties.</strong> Let links ( \phi = 8 \text{ mm} )</td>
<td>( b = 300 \text{ mm}; d = 600 – 20 – 8 – 16 = 556 \text{ mm} )</td>
</tr>
<tr>
<td>Assume main bars ( \phi = 32 \text{ mm} )</td>
<td>( b = 300 \text{ mm}; d = 600 – 28 – 16 = 556 \text{ mm} )</td>
</tr>
<tr>
<td>( b = 300 \text{ mm}; d = 600 – 20 – 8 – 16 = 556 \text{ mm} )</td>
<td></td>
</tr>
<tr>
<td><strong>Flexural design</strong></td>
<td>Self-weight ( {\text{BS EN 1991-1-1, Table A1.1}} )</td>
</tr>
<tr>
<td><strong>Self-weight</strong></td>
<td>( = 0.6 \times 0.3 \times 24 = 4.32 \text{ kN/m} )</td>
</tr>
<tr>
<td>( = 0.6 \times 0.3 \times 25 = 4.5 \text{ kN/m} )</td>
<td>( {\text{BS EN 1990, Table A1.2(B) and Table A1.1}} \psi_0 = 0.7 )</td>
</tr>
</tbody>
</table>

(Continued)
Precast frame analysis

BS 8110 solution

\[ w_u = 1.4 \times 44.32 + 1.6 \times 30 = 110.1 \text{ kN/m} \]

Effective span (3.4.1.2)

\[ l_e = \min(5.85 + 0.15; 5.85 + 0.46) = 6.0 \text{ m} \]
\[ M = 110.1 \times 6.0^2/8 = 495.5 \text{ kNm} \]
\[ \{3.4.4.4\} K = 495.5 \times 10^3/40 \times 300 \times 556^2 = 0.134 < 0.156 \text{ for } x/d \leq 0.5 \]
\[ z/d = 0.5 + \sqrt{0.25 - K/0.9} = 0.82 < 0.95 \]
\[ A_x = 495.5 \times 10^3/455 \times 0.87 \times 500 = 2503 \text{ mm}^2 \]

Eurocodes solution

\{Exp. 6.10a; 6.10b\} \[ w_{Ed} = \max\{1.35 \times 44.5 + 0 \times 1.5 \times 30; 1.25 \times 44.5 + 1.5 \times 30\} = 100.7 \text{ kN/m} \]

\[ w_{Ed} = 1.4 \times 44.32 + 1.6 \times 30 = 110.1 \text{ kN/m} \]
\[ \{3.3.2.2, \text{Fig. 5.4a}\} \]
\[ l_{eff} = \min(5.85 + 0.15; 5.85 + 0.46) = 6.0 \text{ m} \]
\[ M_{Ed} = 100.7 \times 6.0^2/8 = 453.1 \text{ kNm} \]
\[ \{3.1.7(3)\} K = 453.1 \times 10^3/32 \times 300 \times 556^2 = 0.153 < 0.206 \text{ for } x/d \leq 0.6 \]
\[ z/d = 0.5 + \sqrt{0.25 - K/1.133} = 0.84 < 0.95 \]
\[ A_x = 453.1 \times 10^3/467 \times 0.87 \times 500 = 2230 \text{ mm}^2 \]

Although compression steel is not required (Table 3.25) \[ A_v = 0.2% = 360 \text{ mm}^2 \]

Use 2 no. H32 \[ \{9.2.1.1(1)\} \]

Shear design

\{3.4.5.3\} \[ V = 110.1 \times 6.0^2/2 = 261.0 \text{ kN} \]
\[ \{6.3.2. Exp. 6.9\} V_{bd,\text{max}} = v_i b z f_{cd} 0.5 \sin 2\theta \]
\[ v_i = 0.6 (1 - f_{cd}/250), z = 0.9d \text{ and } f_{cd} = f_{ck}/1.5 \]
\[ \theta = 0.5 \sin^{-1} (238,700/(0.5 \times 0.523 \times 300 \times 501) = (32/1.5)) = 8.2^\circ < 22.5^\circ \]
\[ \therefore \text{use } \cot \theta = 2.5 \]
\[ \{6.3.2. Exp. 6.8\} \]

Nominal where \( v_c = 0.4 = 1.24 \text{ N/mm}^2 \)

or where shear force

\[ V_{\text{nom}} = 1.24 \times 300 \times 556 \times 10^{-3} = 207 \text{ kN} \]

at 1,530 mm from centre of support.

Comments. EC2 requires 11% less area of rebar.

Deflection

Short-term Young's modulus

\{2.5.4\} \[ E_s = 200 \text{ kN/mm}^2 \]
\[ (\text{Part 2.7.2}) E_s = 20 + 0.2 \times 40 = 28 \text{ kN/mm}^2 \]
\[ \alpha = 200/28 = 7.14 \]

Long-term Young's modulus

Bottom and sides exposed \( h_o = 2A_u = 360,000/(300 + 2 \times 600) = 240 \text{ mm} \)

\[ \{3.2.7(4)\} E_c = 200 \text{ kN/mm}^2 \]
\[ (\text{Table 3.1}) E_{\text{con}} = 22 (40/10)^{0.3} = 33.34 \text{ kN/mm}^2 \]
\[ \alpha = 200/33.34 = 6.00 \]

(Continued)
Age at loading = 28 days. Indoor exposure
RH = 50%

\{Part 2, Fig. 7.1\} \ \phi = 2.45
\{Part 2, 3.6\} \ E_{\text{long}} = 28/3.45 = 8.11 \text{kN/mm}^2
\vdash \alpha_e = 200/8.11 = 24.64
\alpha_e - 1 \ (\text{uncracked concrete}) = 23.64

Uncracked section properties
Not required in BS 8110

BS 8110 solution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_{\text{long}}</td>
<td>8.11 kN/mm²</td>
</tr>
<tr>
<td>\alpha_e</td>
<td>24.64</td>
</tr>
<tr>
<td>\alpha_e - 1</td>
<td>23.64</td>
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Eurocodes solution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>\phi</td>
<td>2.4</td>
</tr>
<tr>
<td>E_{\text{ef}}</td>
<td>33.34/3.4 = 9.80 kN/mm²</td>
</tr>
<tr>
<td>\alpha_e</td>
<td>200/9.80 = 20.39</td>
</tr>
<tr>
<td>\alpha_e - 1</td>
<td>19.39</td>
</tr>
</tbody>
</table>

Uncracked transformed section

\[ bh + \sum (\alpha_e - 1) A_i = 234,587 \text{ mm}^2 \]
\[ x_u = \frac{300 \times 600^2}{2} + 19.39 \times (2590 \times 566 + 402 \times 36)/234,587 = 342.3 \text{ mm from top} \]
\[ I_u = 8,590 \times 10^6 \text{ mm}^4 \]
\[ Z_u = 8,590/257.5 = 33.36 \times 10^6 \text{ mm}^4 \]
\[ M_{\text{cr}} = 33.36 \times 3.02 = 100.8 \text{ kNm} \]
\[ f_{\text{ctm}} = 0.3 \times 322/3 = 3.02 \text{ N/mm}^2 \]
\[ M_{\text{QP}} = (44.5 + 0.3 \times 30) \times 6.02/8 = 240.7 \text{ kNm} \]

Cracked section properties

\{Part 2, 3.6\} Instantaneous value

Solving first m.o.a. \( b x_c^2/2 + (\alpha - 1) A_i (x_c - d') = \alpha A_i (d - x) \)
\[ x_c = 202.2 \text{ mm} \]
\[ I_{\text{perc}} = 3211 \times 10^6 \text{ mm}^4 \]

Long-term value

Solving first m.o.a. \( b x_c^2/2 + (\alpha_e \times 1) A_i (x_c - d') = \alpha_e A_i (d - x) \)
\[ x_c = 302.0 \text{ mm} \]
\[ I_{\text{perc}} = 7545 \times 10^6 \text{ mm}^4 \]

Curvature \( 1/r_b = M/EI \)
\{Part 2, 3.3.3\} \ \psi = 0.25 offices

Instantaneous curvature \( (\times 10^{-6} \text{ mm}^{-1} \text{ units}) \)
\[ w_o = 44.32 + 0.25 \times 30 = 51.82 \text{ kN/m} \]
\[ M_{\text{total}} = 51.82 \times 6.0/8 = 323.2 \text{ kNm} \]
\[ 1/r_{b,\text{total}} = 233.2/(28,000 \times 3,211) = 2.59 \]
\[ M_{\text{cr}} = 44.32 \times 6.0/8 = 199.4 \text{ kNm (dead)} \]
\[ 1/r_{b,\text{cr}} = 199.4/(28,000 \times 3,211) = 2.22 \]

Long-term curvature
\[ 1/r_{b,\text{cr}} = 199.4(8,116 \times 7,545) = 3.26 \]
\[ 1/r_b = 3.26 + (2.59 - 2.22) = 3.63 \]
\{Part 2, 3.7.2\} Deflection \( \delta = K l_o^2 \left( 1/r_b \right) \)
\{Part 2, Table 3.1\} \ K = 0.104
\[ \delta = 0.104 \times 6,000^2 \times 3.63 \times 10^{-6} = 13.6 \text{ mm} \]
\{3.4.6\} \( l_o/\delta = 440 > 250 \text{ OK} \)

Span/depth ratio
\[ 1/D = 20 \times 0.744 = 14.87 \]

Long-term curvature \( (\times 10^{-6} \text{ mm}^{-1} \text{ units}) \)
\[ 1/r_{b,\text{cr}} = M_{\text{cr}}/E_{\text{ef}} l_{\text{ef}} \]
\[ 1/r_b = 240.7/9,806 \times 6,599 = 3.72 \]
\[ \delta = K l_{\text{ef}}^2 \left( 1/r_b \right) \]
\[ \delta = 0.104 \times 6,000^2 \times 3.72 \times 10^{-6} = 13.9 \text{ mm} \]
\{7.4.1(4)\} \( l_{\text{ef}}/\delta = 431 > 250 \text{ OK} \)

Span/depth ratio
\[ (7.4.2(2)) l/\delta = 11 + 1.5 \sqrt{32} \times 0.432 = 14.67 \]
3.6.2 Reinforced concrete rectangular column

A two-storey 300 mm × 300 mm edge column supports beams on one side only in an unbraced sway frame as shown in Figure 3.42a. The exposure, fire resistance, design life and beam end reactions are as given in 3.6.1, except that the roof beam may be taken as 60% of the floor beam reactions. The beam reactions act at 80 mm from the face of the column. The flexural stiffness of the beam-to-column connection may (in this exercise) be taken as 1/10 of that of the column. The characteristic horizontal wind load is shown in Figure 3.42a.

Design the main reinforcement and specify the shear links. Use $f_{ck}/f_{cu} = 40/50$, $f_{yd}/f_y = 500 \text{ N/mm}^2$, and normal-weight concrete. Effective creep factor $q_{ef} = 1$ (used in EC2). Moment distribution factors at the first floor (upper end of column is pinned, lower end at foundation is fixed) = $4EI/3.5 / (4EI/3.5 + 3EI/3.0) = 54\%$ with 50% carryover (c/o) to the foundation.
Pinned joints with 1/10 column stiffness

$W_k = 7 \text{kN}$

$W_k = 14 \text{kN}$

Clear 6100

Clear 2900

All columns 300 $\times$ 300

Floor beams designed in 3.6.1

Figure 3.42 Detail to code comparison for reinforced concrete column. (a) Frame arrangement, (b) bending moments due to beam eccentricity, horizontal loads (wind and imperfections) and second-order deflections.

<table>
<thead>
<tr>
<th>BS 8110 solution</th>
<th>Eurocodes solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durability</strong> is the same as for the beam, $\therefore$ cover to links 20 mm</td>
<td>Fire. R60. BS EN 1992-1-2</td>
</tr>
<tr>
<td>Fire. 1 h</td>
<td>(BS EN 1990, Table A1.1) In fire $\psi_2 = 0.3$ for offices beam load and $\psi_2 = 0$ for wind load</td>
</tr>
<tr>
<td>${\text{Table 3.4}} c = 20 \text{ mm}$</td>
<td></td>
</tr>
</tbody>
</table>

*(Continued)*
Precast frame analysis

**BS 8110 solution**

- $G_k = 44.5 \times 6.0/2 = 133.5$ kN per beam
- $Q_k = 30 \times 6.0/2 = 90$ kN per beam
- $V_{Ed,fi,\max} = 133.5 + 0.3 \times 90 = 160.5$ kN
- Per floor plus 60% at roof = 96.3 kN
- Self-weight = $0.3 \times 0.3 \times 25 \times 3 = 6.75$ kN
- $N_{Ed,fi}$ at first floor level = 424 kN
- Eccentricity of reaction = 150 + 80 = 230 mm
- $M_{Ed,fi} = 160.5 \times 0.23 = 36.9$ kNm × 54% as distributed = 19.9 kNm
- $\varepsilon_{Ed,fi} = 19.9/424 = 0.047$ m = 47 mm
- $\lambda_0 > 3.0$ m
- $e/h = 47/300 = 0.16 < 0.25$
- Then $n = N_{Ed,fi}/(0.7(A_f c_{fd} + A_s f_{yd}))$
- Try 4 H25 bars = 1963 mm²
- $n = 424/(0.7 (300^2 \times 26.67 +1963 \times 435) \times 10^{-3}$
- $= 0.19$ and $\omega = 1963 \times 435/300^2 \times 26.67 = 0.36$
- $\beta = (1 + 0.1/1.1) \times (1 + 10/11) = 2.08$
- $l_0 = 3500 – 600 = 2900$ mm
- $l_e = 2.08 \times 2900 = 6032$ mm. $\lambda = l_0/i = 69.6$

**Effective height factors**

- Assumed main bars $\phi = 25$ mm
- $\beta = 2 + 0.3 \alpha_{c,min}$ where $\alpha_c = 1.0$ for foundation and 10 given for beam
- $\beta = 2 + 0.3 \times 1.0 = 2.3$
- Clear first floor $l_b = 3500 – 600 = 2900$ mm
- $l_e = 2.3 \times 2.900 = 6670$ mm. $l_e/h = 22.2$

**Section properties.**

Assume main bars $\phi = 20$ mm

- $b = 300$ mm; $d = 300 – 20 – 8 – 12 = 260$ mm
- $d/h = 0.87$. Use design chart in Figure 3.43a
- Self-weight = $0.3 \times 0.3 \times 24 \times 6.5 = 14.0$ kN
- $\beta = 2 + 0.3 \alpha_{c,min}$ where $\alpha_c = 1.0$ for foundation and 10 given for beam
- $\beta = 2 + 0.3 \times 1.0 = 2.3$
- Clear first floor $l_b = 3500 – 600 = 2900$ mm
- $l_e = 2.08 \times 2900 = 6032$ mm. $\lambda = l_0/i = 69.6$
- where $i = 300/\sqrt{12} = 87$ mm
- $\lambda > \lambda_{min} = 20 ABC/\sqrt{\pi} = 33.4$
- where $A = 1/(1 + 0.2\varphi_{ef}) = 1/1.2 = 0.83$
- $B = \sqrt{1 + 2\omega} = 1.31$ where $\omega = 0.36$
- $C = 1.7 – r_m = 0.7$ where $r_m = +1$ for unbraced
- $n = 502 \times 10^3/300^2 \times 26.67 = 0.209$
- $N_{Ed} = 502$ kN (see case 1 in the following)
- $\lambda > \lambda_{min} = 33.4$. slender
- $\varepsilon_2 = 19.4 \times 10^{-6} \times 6032^2/10 = 71$ mm
- $\varepsilon_{Ed,fi} = 19.4 \times 10^{-6} \times 6032^2/10 = 71$ mm
- $\lambda = 1/r = K_s f_{yd}/0.45 d E_1 = 1.05 \times 435/0.45 \times 260 \times 200,000 = 19.4 \times 10^{-6}$ mm⁻¹
- $N_{Ed}/f_{cd}$ $bh < 0.25$ see the following)

(Continued)
BS 8110 solution

Clear height to roof $l_0 = 6,500 - 400 = 6,100$ m

$$l_i = 2.3 \times 6,100 = 14,030$$ mm

(3.3.1) $\phi_u = 46.7 \times 300/2,000 = 327$K

(3.3.8) All columns have equal stiffness

(3.8.6) Column deflection checked as $l_i/h > 30$

Horizontal loads and moments per column

Refer to Figure 3.42b

$M_{wind} = (7.0 \times 6.5 + 14.0 \times 3.5)/3 = 31.5$ kNm

(3.1.4.2) Total $G$, for floor beams + self-weight

$G_{floor} = 44.32 \times 12 + 3 \times 14 = 574$ kN

$H_{floor} = 1.5\% G_i = 0.015 \times 574 = 8.6$ kN

$<14$ kN wind load

At roof $H_{roof} = 0.6 \times 8.6 = 5.2$ kN

$<7$ kN wind load

Ultimate loads and moments at foundation

Case 1. Gravity dead + live

$V_{floor} = 110.1 \times 6.0/2 = 330.3$ kN

$V_{roof} = 0.6 \times 330.3 = 198.2$ kN

$N = 330.3 + 198.2 + 1.4 \times 14.0 = 548$ kN

Refer to Figure 3.42b

Node moment $M_0 = V_e$, where $e = 150 + 80$

$= 230$ mm from the centre of column

$M_0 = 330.3 \times 0.23 = 76.0$ kNm $\times 54\%$

as distributed $50\% c/o = 20.5$ kNm

(Eq. 35) $M_{dud} = N \phi_u = 330.3 \times 0.0739K$

$+ 198.2 \times 0.327K = 89.2$ kNm

$M = 20.5 + 89.2 = 109.7$ kNm

(3.8.2.4) $M_{min} = 548 \times 0.020 = 11.0$ kNm

$N/f_{cu} bh = 548 \times 10^3/50 \times 300^2 = 0.12$

$M/f_{cu} bh^2 = 109.7 \times 10^3/50 \times 300^2 = 0.081$

Figure 3.43a gives $K = 1$

$A_{ac} = 0.12 \times 50 \times 300 \times 300/500 = 1,080$ mm$^2$

Case 2. Dead + live + wind

$\gamma_f = 1.2$

$V_{floor} = 1.2 \times 74.32 \times 6.0/2 = 267.5$ kN

$V_{roof} = 0.6 \times 267.5 = 160.5$ kN

$N = 267.5 + 160.5 + 1.2 \times 14.0 = 445$ kN

Eurocodes solution

$$\{5.8.8.3(4)} K_p = 1 + (0.35 + f_{cf}/250 - \lambda/150) \phi_{ef}$$

$= 1 + (0.35 + 0.16 - 69.6/150) \times 1 = 1.05$

$$l_0 = 2.08 \times 6,100 = 12,688$$ mm

$e_2 = 18.6 \times 10^{-6} \times 12,688^2/10 = 299$ mm

where $K_p = 1$ and $1/r = 18.6 \times 10^{-6}$ mm$^{-1}$

$F_{Ed,roof} = 0.6 \times 302.1 = 181.2$ kN

$N = 267.5 + 160.5 + 1.2 \times 14.0 = 445$ kN

(Continued)
Precast frame analysis

**BS 8110 solution**

\[
M_0 = 267.5 \times 0.23 \times 54\% \times 50\% = 16.6 \text{ kNm}
\]

\[
M_{l0} = 267.5 \times 0.0739K + 160.5 \times 0.327K = 126.6 \text{ kNm}
\]

\[
M_{\text{wind}} = 1.2 \times 31.5 = 37.8 \text{ kNm}
\]

\[
M = 16.6 + 72.2 + 37.8 = 126.6 \text{ kNm}
\]

\[
M_{\text{Ed},w} = 0.75 \times 31.5 = 23.6 \text{ kNm}
\]

\[
M_{\text{Ed}} = 103.6 + 23.6 = 127.2 \text{ kNm}
\]

\[
M_{l0} = 133.0 \times 0.0739K + 79.8 \times 0.327K = 35.9 \text{ kNm}
\]

\[
M_{\text{Ed,i}} = 133.5 \times 0.015 + 80.1 \times 0.025 = 4.0 \text{ kNm}
\]

\[
M_{\text{wind}} = 1.4 \times 31.5 = 44.1 \text{ kNm}
\]

\[
M_{\text{Ed},w} = 1.5 \times 31.5 = 47.2 \text{ kNm}
\]

\[
N/f_{cu} bh = 0.10 \text{ and } M/f_{cu} bh^2 = 0.094
\]

\[
N_{\text{Ed}}/f_{ck} bh = 0.14 \text{ and } M_{\text{Ed}}/f_{ck} bh^2 = 0.118
\]

\[
A_s = 0.19 \times 50 \times 300 \times 300/500 = 1710 \text{ mm}^2
\]

\[
A = 0.21 \times 40 \times 300 \times 300/500 = 1512 \text{ mm}^2
\]

**Eurocodes solution**

\[
M_{l0} = 0.75 \times 31.5 = 23.6 \text{ kNm}
\]

\[
M_{\text{Ed}} = 103.6 + 23.6 = 127.2 \text{ kNm}
\]

\[
N_{\text{Ed}}/f_{ck} bh = 0.14 \text{ and } M_{\text{Ed}}/f_{ck} bh^2 = 0.118
\]

Case 3. Dead + wind

\[
\gamma_f = 1.0 \text{ and } 1.4
\]

\[
V_{\text{floor}} = 1.0 \times 44.32 \times 6.0/2 = 133.0 \text{ kN}
\]

\[
F_{\text{Ed,roof}} = 44.5 \times 6.0/2 = 133.5 \text{ kN}
\]

\[
V_{\text{roof}} = 0.6 \times 133.0 = 79.8 \text{ kN}
\]

\[
F_{\text{Ed,roof}} = 0.6 \times 133.5 = 80.1 \text{ kN}
\]

\[
N = 133.0 + 79.8 + 14.0 = 226.8 \text{ kN}
\]

\[
N_{\text{Ed}} = 133.5 + 80.1 + 14.6 = 228.2 \text{ kN}
\]

Comments

The Eurocodes require 12% less \(A_s\) due mainly to reduced \(\gamma_f\); (1.25 & 1.5 versus 1.4 & 1.6) which reduces \(N\) by 9%. First- and second-order bending moments are roughly equal.

### 3.6.3 Prestressed concrete slab

Calculate the service and ultimate moment of resistance, and the ultimate shear capacity of a 1200 mm wide x 200 mm deep prestressed concrete solid floor slab. The slab is pretensioned using 6 no. 12.5 mm plus 6 no. 9.3 mm diameter standard 7-wire helical strands at 30 mm bottom cover, plus 4 no. 5 mm diameter wires at 25 mm top cover. The tendons are Class 2 relaxation and stressed initially at 70%. The exposure is internal with relative humidity = 50%, fire resistance is 60 minutes, and design life is 50 years. Bearing length is 100 mm. The slab is designed as a Class 2 member for permissible tension to BS 8110. The floor carries office loading for characteristic dead loads of 1.5 kN/m² finishes and 0.6 kN/m² services/ceiling, a superimposed live load of 4.0 kN/m² and demountable partitions of 1.0 kN/m².

The section properties are cross-sectional area \(A_s = 232,040 \text{ mm}^2\); second m.o.a \(I_{x,y} = 779 \times 10^6 \text{ mm}^4\); depth = 200 mm; centroid from bottom \(y_b = 99.4 \text{ mm}\); breadth at top, bottom and at centroid = 1154, 1197 and 1134 mm, respectively. Height to the centroid of all tendons \(y_s = 47.0 \text{ mm}\). Height to the centroid of tendons in tension zone \(y_T = 35.7 \text{ mm}\).
Use $f_{ck}/f_{cu} = 45/55$, at transfer $f_{ck}(t)/f_{ci} = 30/35$, $f_{pu}/f_{pu} = 1770 \text{ N/mm}^2$, cement CEM I class 52.5R and a 10 mm coarse gravel aggregate. The self-weight of the precast concrete is determined by the manufacturer as 24.5 kN/m$^3$ giving a self-weight of 4.90 kN/m$^2$. In this exercise, ignore the reduced compression acting at the level of the strands due to self-weight and dead loads in the calculation of creep losses.

Figure 3.43 (a) Reinforced concrete column design chart to BS 8110. (b) Reinforced concrete column design chart to BS EN 1992-1-1.
Precast frame analysis

BS 8110 solution

<table>
<thead>
<tr>
<th>BS 8110 solution</th>
<th>Eurocodes solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durability, BS 8500-1. Table A.4 for 50 years</td>
<td>Durability, BS 8500-1. Table A.4 for 50 years</td>
</tr>
<tr>
<td>Exposure XC1. Cover c = 15 + Δc</td>
<td>Exposure XC1. ( C_{nom} = 15 + ΔC_{dev} )</td>
</tr>
<tr>
<td>( {7.3} \Delta c = 5 \text{ mm} )</td>
<td>( {4.4.1.3(3)} ) Cover controlled ( ΔC_{ct} = 5 \text{ mm} )</td>
</tr>
<tr>
<td>Cover to tendons ( c \geq 20 \text{ mm} )</td>
<td>Cover to tendons ( C_{nom} \geq 20 \text{ mm} )</td>
</tr>
<tr>
<td>Fire, 1 h</td>
<td>Fire, R60. BS EN 1992-1-2</td>
</tr>
<tr>
<td>( {\text{Table 3.4}} \ c = 20 \text{ mm} )</td>
<td>( {\text{Table 5.8}} ) Depth ( h \geq 80 \text{ mm} \leq 200 \text{ mm} )</td>
</tr>
<tr>
<td>( {5.2(5)} ) Axis ( a = 25 + 15 - \Delta a = 29 \text{ mm} )</td>
<td>( {5.2} \ k_p(\theta_{cr}) = 805/1,770 = 0.455 )</td>
</tr>
<tr>
<td>( {\exp. 5.3} \ Δa = 0.1(500 - \theta_{cr}) = 10.7 \text{ mm} )</td>
<td>with ( G_s = 4.9 + 1.5 + 0.6 = 7.0 \text{ kN/m}^2 ) and ( Q_s = 5.0 \text{ kN/m}^2 )</td>
</tr>
<tr>
<td>( {\exp. 5.2} )</td>
<td>Centroid to steel tendons in fire zone, ( a = y_f = 35.7 \text{ mm} &gt; 29 \text{ mm} )</td>
</tr>
<tr>
<td>( {\exp. 5.3} \</td>
<td>Δa = 0.1(500 - \theta_{cr}) = 10.7 \text{ mm} )</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>( {\text{Part 2, 7.2}} \ E_c = 20 + 0.2 \times 55 = 31 \text{ kN/mm}^2 )</td>
</tr>
<tr>
<td>( {4.8.3.1} \ E_c = 20 + 0.2 \times 35 = 27 \text{ kN/mm}^2 )</td>
<td>( {\text{Table 3.1}} \ E_c = 22 (53/10)^{0.3} = 36.3 \text{ kN/mm}^2 )</td>
</tr>
<tr>
<td>( {\text{BS 5896, Table 6}} \ E_s = 195 \text{ kN/mm}^2 \text{ strand} )</td>
<td>( {3.3.6(3)} \ E_p = 195,000 \text{ N/mm}^2 \text{ strand} )</td>
</tr>
<tr>
<td>Section properties</td>
<td>Although ( E_s ) for wire = 205 \text{ kN/mm}^2 use same as for strand</td>
</tr>
<tr>
<td>( Z_b = 779 \times 10^6/99.4 = 7.836 \times 10^6 \text{ mm}^3 )</td>
<td>( m = 195/27 = 7.22, m = 195/31 = 6.29 )</td>
</tr>
<tr>
<td>( Z_t = 779 \times 10^6/101.6 = 7.744 \times 10^6 \text{ mm}^3 )</td>
<td>Section properties</td>
</tr>
<tr>
<td>( e = 99.4 - 47.0 = 52.4 \text{ mm} )</td>
<td>Flexural capacity – service limit of stress</td>
</tr>
<tr>
<td>( y_{x,co} = 98.3 \text{ mm}; I_{x,co} = 799.6 \times 10^4 \text{ mm}^4 )</td>
<td>( m(\text{t}) = 195/32.8 = 5.94, m = 195/36.3 = 5.37 )</td>
</tr>
<tr>
<td>( Z_{x,co} = 8.134 \times 10^4 \text{ mm}^3; Z_{co} = 7.862 \times 10^4 \text{ mm}^3 )</td>
<td>( m - 1 = 4.37 )</td>
</tr>
<tr>
<td>( Z_{x,co} = 8.082 \times 10^4 \text{ mm}^3; Z_{co} = 7.842 \times 10^4 \text{ mm}^3 )</td>
<td>Flexural capacity – service limit of stress</td>
</tr>
<tr>
<td>( A_{ps} = 6 \times 52 + 6 \times 93 + 4 \times 19.6 = 948.5 \text{ mm}^2 )</td>
<td>( \sigma_p = 0.7 \times 1770 = 1,239 \text{ N/mm}^2 )</td>
</tr>
<tr>
<td>Initial ( \sigma_i = 0.7 \times 1770 = 1,239 \text{ N/mm}^2 )</td>
<td>( \rho_{ps} = 1239 \times 948.5 = 1,175,241 \text{ N} )</td>
</tr>
<tr>
<td>( {4.8.2.1} ) relaxation loss = ( 1.2 \times 2% = 0.024 )</td>
<td>( {3.3.2(7)} ) Relaxation at ( t = 20 \text{ h} )</td>
</tr>
<tr>
<td>( f_e ) after relaxation loss = +8.99 \text{ N/mm}^2 )</td>
<td>( {\exp. 3.29} ) for Class 2, ( \mu = 0.7; \rho_{1000} = 2.5% )</td>
</tr>
<tr>
<td>Shortening loss = 8.99 \times 7.22/1,239 = 0.0524</td>
<td>Loss ( \Delta\sigma_p = 1239 \times 0.66 \times 2.5 \times e^{(0.1 \times 0.7)} )</td>
</tr>
<tr>
<td>Transfer ( R_t = 1 - 0.024 - 0.0524 = 0.924 )</td>
<td>( (20/1000)^{0.75(1 - 0.7)} = 4.95 \text{ N/mm}^2 )</td>
</tr>
<tr>
<td>Transfer ( P_t = 0.924 \times 1,175,241 = 1,085,467 \text{ N} )</td>
<td>( \sigma_a ) after relaxation loss = +9.17 \text{ N/mm}^2</td>
</tr>
<tr>
<td>(Continued)</td>
<td>( {\exp. 3.29} ) ( \Delta\sigma_p = 9.17 \times 5.94 = 54.46 \text{ N/mm}^2 )</td>
</tr>
</tbody>
</table>

(Continued)
Precast Concrete Structures

BS 8110 solution  
\{4.3.5.2\} \( f_{ca} = 0.45\sqrt{35} = 2.66 \text{ N/mm}^2 \)
\( f_{ca} = +11.94 \text{ N/mm}^2 < 0.5 \times 35 = 17.5 \text{ OK} \)
\( f_{ca} = -2.67 \text{ N/mm}^2 > -2.66 \text{ say OK} \)
\{4.8.5.2\} Creep \( \varphi = 1.8 \)

Eurocodes solution  
\{Table 3.1\} \( f_{cm} = 0.3 \times 45^{2/3} = 3.80 \text{ N/mm}^2 \)
\{Exp. 3.4\} \( f_{cm(t)} = (38/53) \times 3.80 = 2.72 \text{ N/mm}^2 \)
\( \sigma_r(t) = +12.31 \text{ N/mm}^2 < 0.6 \times 30 = 18.0 \text{ OK} \)
\( \sigma_r(t) = -2.75 \text{ N/mm}^2 > -2.72 \text{ say OK} \)

\( f_{ctm} = 0.3 \times 45^{2/3} = 3.80 \text{ N/mm}^2 \)
\{4.3.5.2\} \( f_{cti} = 0.45\sqrt{35} = 2.66 \text{ N/mm}^2 \)  
\{Exp. 3.4\} \( f_{ctm(t)} = (38/53) \times 3.80 = 2.72 \text{ N/mm}^2 \)

\( f_{ctm(t)} = (38/53) \times 3.80 = 2.72 \text{ N/mm}^2 \)
\( f_{cti} = -2.67 \text{ N/mm}^2 > -2.66 \text{ say OK} \)
\( \sigma_{bc(t)} = +11.94 \text{ N/mm}^2 < 0.5 \times 35 = 17.5 \text{ OK} \)
\( \sigma_{bc(t)} = +12.31 \text{ N/mm}^2 < 0.6 \times 30 = 18.0 \text{ OK} \)
\( \sigma_{bc(t)} = -2.75 \text{ N/mm}^2 > -2.72 \text{ say OK} \)

\( \varphi_{(t,t)} = 2.31 \times 1.39 \times 0.62 \times 0.99 = 1.98 \)
\{Exp. B.4\} \( \beta(f_{cm}) = 2.31 \)
\{Exp. B.3b/B.8c\} \( \varphi_{RH} = 1.39, \text{ with } h_o = 400 \text{ mm} \)
\{Exp. B.5\} \( \beta(t_i) = 0.62, \text{ where } t_i = 7.6 \text{ days at } 50^\circ \text{C curing for } 20 \text{ h} \)
\{Exp. B.7\} \( \beta_c(t,t_i) = 0.99 \text{ for } t = 20,833 \text{ days and } \beta_{H(d)} = 803 \text{ days} \)
\{Exp. 3.12\} \( \epsilon_{cd,o} = 0.85 \times (220 + 110 \times 6) \times e^{-(0.11 \times 53/10)} \times 1.356 = 566 \times 10^{-6} \)
\( \epsilon_{ca} = 2.5 \times (45 - 10) \times 10^{-6} = 88 \times 10^{-6} \)
\( \epsilon_{cd} = 0.985 \times 0.73 \times 566 \times 10^{-6} = 404 \times 10^{-6} \)
\( \beta_{ds}(t,t_s) = 0.985 \text{ with } t \text{ as earlier, } t_s = 1 \text{ day} \)
\{Table 3.3\} \( k_n = 0.73 \text{ for } h_o = 400 \text{ mm} \)
\{Exp. 5.46\} \( \Delta \sigma_{p,c} = 1.98 \times 8.77 \times 5.37/1.103 = 84.40 \text{ N/mm}^2 \)
\{Exp. 3.9\} \( \epsilon_{cd} = 0.985 \times 0.73 \times 566 \times 10^{-6} = 404 \times 10^{-6} \)
\{Exp. 5.46\} \( \Delta \sigma_{p,sh} = 492 \times 10^{-6} \times 195,000/1.103 = 87.0 \text{ N/mm}^2 \)
\{5.10.6(1b)\} \( R_{wk} = 0.924 − 0.087 − 0.0472 = 0.789 \text{ (21.1\%)} \)
\( \frac{f_{pc}}{f_{pc}} = 0.798 \times 1,175,241 = 927,621 \text{ N} \)
\( \frac{R_{po}}{R_{po}} = 0.791 \times 1,175,241 = 929,076 \text{ N} \)
\( R_{wk} = 979.5/1,239 = 0.7905 \text{ (20.95\%)} \)
\( P_{po} = 0.791 \times 1,175,241 = 929,076 \text{ N} \)

Prestress at transfer  
\( \{4.3.7.1\} \) Ulitmate moment of resistance Mur
Refer to this book first ed. for full analysis
Refer to Section 4.3.4 for full analysis
\( f_c = 0.45 \times f_{ca} = 24.75 \text{ N/mm}^2 \) and \( \lambda = 0.9 \)
Precast frame analysis

**BS 8110 solution**

- $A_{ps}$ in tension zone = 870 mm$^2$; $d = 164.3$ mm
- Prestrain after losses $\varepsilon_{p0} = 0.005015$
- Refer to stress versus strain diagrams in the following

| Strain $\varepsilon_p = 0.012970$; stress $f_p = 1539$ N/mm$^2$ |
|-----------------------------|-----------------------------|
| $X = 50.1$ mm; $z = 141.8$ mm |
| $M_{ur} = 870 \times 1539 \times 141.8 \times 10^{-6} = 189.9$ kNm |

| Strain $\varepsilon_p = 0.004946$; stress $f_p = 1442$ N/mm$^2$ |
|-----------------------------|-----------------------------|
| $X = 51.2$ mm; $z = 143.8$ mm |
| $M_{rd} = 870 \times 1442 \times 143.8 \times 10^{-6} = 180.5$ kNm |

\[\sigma_s = 195000 \quad \varepsilon_s = 0.02 \quad \sigma_{cp} = 3.60 \text{ N/mm}^2\]

\[\alpha_i = l/l_{p2} = 0.451 \quad \left(\sigma_{pc} = 9.9 \times 929,076/232,040 = 3.60 \text{ N/mm}^2\right)\]

\[f_{ctd}(t) = 0.7 \times 2.72/1.5 = 1.27 \text{ N/mm}^2\]

\[\gamma_{p,fav} = 0.9 \quad f_{pt} = 0.24\sqrt{55} = 1.78 \text{ N/mm}^2\]

\[f_{ctd} = 0.3 \times 452/3 \times 0.7/1.5 = 1.77 \text{ N/mm}^2\]

\[x/l_p = 199.4/442 = 0.451 \quad \left(x/l_p = 0.9 \times 232,040 = 200 \times \sqrt{(1.78^2 + 0.8 \times 2.79 \times 1.78)} = 406.1 \text{ kN}\right)\]

**Eurocodes solution**

- $\varepsilon_{p0} = 0.004946$
- $\varepsilon_p = 0.012700$; stress $f_p = 1442$ N/mm$^2$
- $X = 51.2$ mm; $z = 143.8$ mm
- $M_{rd} = 870 \times 1442 \times 143.8 \times 10^{-6} = 180.5$ kNm

\[\sigma_s = 1385 \quad \varepsilon_s = 0.02 \quad \left(\sigma_{sp} = 929,076/232,040 = 4.06 \text{ N/mm}^2\right)\]

\[\alpha_i = l/l_{p2} = 0.276 \quad \left(\sigma_{pc} = 99.4 \times 929,076/232,040 = 3.60 \text{ N/mm}^2\right)\]

\[f_{ctd} = 0.3 \times 452/3 \times 0.7/1.5 = 1.77 \text{ N/mm}^2\]

\[\gamma_{p,fav} = 0.9 \quad f_{pt} = 0.24\sqrt{55} = 1.78 \text{ N/mm}^2\]

\[\sigma_{sp} = 3.60 \text{ N/mm}^2 \quad \alpha_i = 0.276 \quad \sigma_s = 1385 \text{ N/mm}^2\]

**Comments**

1. $M_{ur}$ to the Eurocodes is greater than Class 2 BS 8110 because $f_{ctm}$ is 13% greater than $f_{ct}$. However, there is no greater stress than $f_{ctm}$ allowed in the Eurocodes, such as Class 3 (0.2) in BS 8110.

2. $M_{rd} < M_{ur}$ because $f_p$ is not allowed to reach maximum design stress in the Eurocodes stress versus strain idealisation.

3. $V_{rdc} < V_{co}$ because $l_{p2}$ is 63% greater than $l_p$, and the build-up of prestress is linear in the Eurocodes rather than parabolic in BS 8110.

*See Section 4.3.3 for the application of $M_{ur}$ in the top surface in design.*
3A APPENDIX A: SUMMARY OF EUROCODE EC2: DESIGN OF CONCRETE STRUCTURES – GENERAL RULES AND RULES FOR BUILDINGS, BS EN 1992, PART 1-1

This code effectively replaces BS 8110, Parts 1 to 3, although the execution of work (tolerances, setting out, etc.) is found in BS EN 13670:2009, Execution of concrete structures. The division between commonplace and special design work separated in BS 8110 Parts 1 and 2 no longer exists, and there are no N-M interaction charts for column design. The last point reflects the fact that EC2 is a limit state code of principles rather than methods. The current amendment was published in February 2014. The UK Technical Committee B/525 (sub-committee 2) is currently engaged in a revision of the code.

Precast concrete is not treated as a separate design and construction method although, as with BS 8110, there are certain aspects of design, such as bearings, anchorage at supports, bursting, floor systems, compression/tension/shear joints, connections, pocket foundations, and corbels, collected in a separate section, in this case Section 10.

The format of BS EN 1992-1-1, as with all material based on the Eurocodes, is as follows:

Section 1 Scope – references; assumptions; definitions; symbols. Note that symbols are often only defined here and not in the text
Section 2 Basis of design – requirement related to BS EN 1990, Annex B; requirements related to BS EN 1991-1; material properties; PSFs \( \gamma_c \) and \( \gamma_s \), load combinations and equilibrium
Section 3 Materials – (concrete, rebar, tendons) strength, stress – strain models, deformation, shrinkage and creep; fatigue; anchorage; prestressing
Section 4 Durability – environmental and exposure classes; cover to reinforcement
Section 5 Structural analysis – load cases; imperfections, sway; structural models; linear elastic, plastic and non-linear analysis; redistribution; second-order effects with axial load (columns, walls); prestressing – stressing; forces; losses; service and ultimate; fatigue
Section 6 ULS – bending, shear, torsion and punching shear; strut-and-tie models; anchorage and laps; partially loaded areas (localised bearings); fatigue
Section 7 Serviceability limit state – crack control, spacing and crack width, deflections
Section 8 Detailing in general – rebars – bar spacing, anchorage, laps, links details; prestressing tendons – anchorage, transmission length, development length
Section 9 Detailing in particular – maximum and minimum areas; anchorage at supports; shear; torsion and surface reinforcement; solid and flat slabs, columns and walls, deep beams and stability ties
Section 10 Precast concrete elements and structures – materials; losses of prestress; bearings; anchorage at supports; bursting; floor systems; compression/tension/shear joints; half joints; pocket foundations; corbels
Section 12 Plain and lightly reinforced concrete – reduction factors for strength; precast walls and infill shear walls, construction joints, strip and pad footings

Informative annexes – (A) improved PSFs; (B) creep and shrinkage strains in detail; (C) reinforcement properties; (D) prestressing tendons losses; (E) strength classes for durability; (F) tensile stresses in rebars in biaxial and shear stress fields; (G) soil-structure; (H) second-order effects; (I) flat slab and shear walls; (J) regions of discontinuity

The code is not prescriptive, and it is necessary to turn to calculation methodology given in documents published for example by The Concrete Centre, for example calculation of area of flexural and shear reinforcement in beams and N–M charts for r.c. columns.
The main issues relating to the design of precast concrete structures in NA to BS EN 1992-1-1 are

2.4.2.2(1) Partial factor for prestress at ULS $\gamma_{ps} = 0.9$.

3.1.2(2)P Value of $c_{max}$, Shear strength of concrete classes higher than C50/60 should be determined by tests, etc.

3.1.6(3)P Value of $\alpha_{cc} = 0.85$ for compression in flexure and axial loading and 1.0 for other phenomena, that is in bending $f_{cd} = 0.85 f_{ck}/1.5 = 0.567 f_{ck}$, but in shear $f_{cd} = 0.667 f_{ck}$.

4.4.1.3(3) $\Delta c_{adv}$, under controlled conditions, such as steel mounts known as soldier, may be reduced to 10 mm $> \Delta c_{adv} > 5$ mm.

5.1.3(1)P Simplified load arrangements. Consider the two following arrangements for ‘all spans’ and alternate spans: (i) all spans carrying $\gamma_c G_k + \gamma_Q Q_k + P_i$; and (ii) alternate spans carrying $\gamma_c G_k + \gamma_Q Q_k + P_i$, other spans carrying only $\gamma_c G_k + P_i$; the same value of $\gamma_c$ should be used throughout the structure. For one-way spanning slabs, use the ‘all spans’ loaded if (i) area of each bay $> 30$ m$^2$; (ii) $Q_k/G_k \leq 1.25$; and (iii) $Q_k < 5$ kN/m$^2$ excluding partitions.

5.5(4) Moment redistribution formula: values for steels with $f_{ck} \leq 500$ N/mm$^2$, $k_1 = 0.4$, $k_2 = 0.6 + 0.0014 f_{ck}$. Then if $e_{cos} = 0.0035$, $k_1 = 1$. Code Exp. 5.10a for $f_{ck} \leq 50$ N/mm$^2$, $\delta = k_1 + k_2$ x/l/d for zero moment redistribution $\delta = 1 = 0.4 + x/l/d$, then $\leq 0.6$, that is for the balanced section, the limiting depth of the neutral axis is 0.6d.

5.10.9(1)P For pre-tensioning, $\tau_{sup} = 1.0$ and $\tau_{inf} = 1.0$, that is there are no modifications to the action of prestress.

6.2.3(3) Values of $\nu_1 = \nu$ unless the design stress of the shear reinforcement $< 0.8 f_{ps}$, $\nu_1$ is modified.

7.2 The different limits of compressive stress in service depending on durability requirements and the avoidance of non-linear creep in prestressed sections in flexure.

7.3.1(5) Limitations of crack width $w_{max}$. Use Table National Annex NA4. This reduces $w_{max}$ in r.c. sections to 0.3 mm and, in prestressed sections, the limiting permissible tension in service to zero for exposure class greater than XC1, although the value of the imposed live load may be reduced.

7.4.2(2) Values of basic span/depth ratios. Use Table NA5 which gives additional information and limits.

8.3(2) Minimum mandrel diameter $\Phi_{min}$ used in Table NA6a and Table NA6b, which contain additional information regarding scheduling reinforcement.

8.8(1) Additional rules for large diameter bars $\Phi_{large} > 40$ mm.

9.5.2(1) Minimum diameter of longitudinal reinforcement in columns $\Phi_{min} = 10$ mm.

9.5.2(3) Maximum area of longitudinal reinforcement in columns. The designer should consider the practical upper limit taking into account the ability to place the concrete around the rebar, that is when casting columns horizontally mould, the maximum area is often around 8 to 10% of the area of concrete. This issue is considered further in the PD 6687-1:2010 (PD 6687-1:2010).

9.7(1) Minimum area of distribution reinforcement in deep beams $= 0.2\%$ in each face.

9.10.2.2(2) Force to be resisted by peripheral tie $q_i = (20 + 4n_0)$ where $n_0$ is the number of storeys; $q_2 = 60$ kN.

9.10.2.3(3) Minimum tensile force internal tie $F_{tie,int} = [(g_c + g_s)/7.5](l/5)(F_t) > F_t$, kN/m, with full definitions. Maximum spacing of internal ties $= 1.5l/l$.

9.10.2.4(3) Internal ties on floors without screed. $F_{tie} = [(g_c + g_s)/7.5](l/5)F_t$ kN/m.

9.10.2.4(2) Horizontal ties to external columns and/or walls at each floor level: $F_{tie,ed} = \text{the greater of } 2F_t$, $l/2.5l$, and 3% $N_{ed}$ at that level.

12.3.1(1) Values of $\alpha_{cc,pl} = 0.6$ and $\alpha_{ct,pl} = 0.6$ (plain concrete).


This code provides the following two alternatives for designing r.c. and prestressed concrete elements and structures for the actions of fire:

1. Performance-based design procedures in Section 2 to 4
2. Prescriptive rules, that is design aids such as tables and diagrams in Section 5.

The design procedure gives an analytical procedure taking into account the behaviour of the structural system at elevated temperatures, the potential heat exposure and the beneficial effects of active and passive fire protection systems, together with the consequences of failure. The main text, together with informative annexes A to E, includes most of the principal concepts and rules necessary for structural fire design of concrete structures.

### Section 1 Scope – references; assumptions; definitions; symbols

### Section 2 Basis of design – requirements; actions; material properties; verification methods

### Section 3 Material properties – at elevated temperatures; concrete with siliceous and calcareous aggregates; thermal elongation of bars and tendons

### Section 4 Design procedures – simplified and advanced calculation methods; shear, torsion and anchorage; spalling; joints; protective layers

### Section 5 Tabulated data – columns, walls, beams and slabs; fire thickness and axis distance to bars

### Section 6 High-strength concrete – calculation models and tabulated data for columns, walls, beams and slabs

### Informative annexes

(A) Temperature profiles; (B) Simplified calculation methods; (C) Buckling of columns under fire conditions; (D) Calculations for shear, torsion and anchorage; (E) Simplified calculations for beams and slabs

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Section 1 Scope – references; assumptions; definitions; symbols.

The main issues relating to the design of precast concrete structures in NA to BS EN 1992-1-2 are

3.2.3(5) Values for the parameters of the stress–strain relationship of reinforcing steel at elevated temperatures. Use Class N (Table 3.2a).

3.2.4(2) Ditto cold worked (wires and strands) prestressing steel at elevated temperatures. Use Class A.

5.6.1(1) Web thickness. Use dimensions for Class WA.

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2B APPENDIX B: SUMMARY OF RELEVANT ITEMS IN PD 6687-1:2010

This PD gives guidance on some specific items that were not published in the concrete Eurocodes or were in need of additional or noncontradictory additional information. Background research is cited in many cases. It is not to be regarded as a British standard. This PD gives noncontradictory complementary information for use with BS BS EN 1992 Parts 1-1 and 1-2 and their UK NAs.
The main items in the PD relating to the design of precast concrete structures are

2.5 Bond stress for mild steel cl. 8.4.2(2). The PD gives \( f_{bd} = \eta_1 \eta_2 (0.36f_{ck}) / \gamma_c \).

2.11 Calculation of effective length of columns, cl. 5.8.3.1, 5.8.3.2 (4) and (5). In the calculation of the flexibility at the ends of the column, the stiffness of the beam(s) attached to the column is taken as \( 2 (E/l)_{beam} \) to allow for the effects of cracking.

2.11.3 Calculation of limiting slenderness ratio, \( \lambda_{lim} \), where adjacent spans of beams do not differ by more than 15%, columns may be assumed to be in double curvature bending for the calculation of \( \lambda_{lim} \) (i.e. value of moment ratio \( r_m < 0 \)).

2.12 Design moment in columns, cl. 5.8.7.3 and 5.8.8.2. For braced structures, \( M_{Ed} = \) maximum of \( (M_{01} + M_2) \), \( (M_{01} + 0.5 M_2) \).

2.14 Design shear – point loads close to support, cl. 6.2.2 (6). Point loads close to support will need to be considered in conjunction with other loads on the member. Design shear \( V_{Ed} \) between the point load and the support is \( V_{Ed} = V_{Ed,other} + \beta V_{Ed,point-load} \) and therefore the reduction factor \( \beta \) cannot be applied to the total \( V_{Ed} \). The clause explains how to deal with this situation.

2.20 Stress limitation in serviceability limit state, cl. 7.2(5). A modular ratio of 15 should be used when calculating tensile stresses in rebars (\( \leq 0.8 f_yk \) and tendons \( \leq 0.75 f_p k \)) under the characteristic combination of loads.

2.21 Control of cracking without direct calculation, cl. 7.3.3. Where the assumptions relating to Table 7.2N and Table 7.3N are not met, crack width is verified using the calculation procedure.

2.21.2 Calculation of crack widths, cl. 7.3.4. Values for \( h_{cr,eff} \) from Fig. 6 of the PD are proposed. It is unsure how Fig. 6a is interpreted.

2.22 Crack widths for non-rectangular tension zones and irregular bar layouts. Based on BS 8110, the recommendation is \( w_c = 3a_c e_m [1 + 2(a_c - c)/(h - x)] \).

2.23.2 Span/depth ratio. Exp. 7.17 in cl. 7.4.2(2). Values of \( (A_{s,prov}/A_{s,req}) \) or \( (310/\sigma_s) \) should be limited to 1.5.

2.23.3 The value for \( \zeta \) in Exp. 7.18 in cl. 7.4.3, and hence the value of \( \sigma_s \) or \( M_s \), should be based on the frequent (not quasi-permanent) combination of loading.

2.26 Anchorage of precast floor and roof units and stair members. BS EN 1992-1-1:2004 or BS EN 1991-1-7 does not cover this. All precast floor, roof and stair members should be effectively anchored whether or not such members are used to provide other ties required in BS EN 1992-1-1:2004, cl. 9.10.2. The anchorage should be capable of carrying the dead weight of the member to that part of the structure that contains the ties.

2.28 Detailing rules for particular situations, Annex J. NA to BS EN 1992-1-1:2004 declares that this is not applicable in the United Kingdom. Alternative versions for frame corners and corbels are given in Annex B of this PD.

3 BS EN 1992-1-2:2004, Structural fire design. The tabular methods for assessing the fire resistance of columns are limited to braced structures. However, at the discretion of the designer, the methods given in BS EN 1992-1-2:2004 for columns may be used for the initial design of unbraced structures. In critical cases, the chosen column sizes should be verified using BS EN 1992-1-2:2004, Annex B.
REFERENCES


Ferreira, M. A., El Debs, M. K. and Elliott, K. S. 2003. Analysis of multi-storey precast frames with semi-rigid connections. 45th Brazilian Concrete Congress, Brazilian Conference on Concrete, IBRACON 2003, Brazilian Concrete Institute – IBRACON, Vitoria, Brazil.

