Transients on Cable Systems

3

3.1 Introduction

This chapter focuses on transient phenomena specifically related to cables. Transients on cable systems are characterized by the large charging capacities of cables and the presence of a metallic sheath around the phase conductor. Temporary overvoltages (TOVs), such as the overvoltage caused by system islanding and the resonance overvoltage observed on the cable system, contain low-frequency components due to large charging capacities. Because of the low frequency, namely low damping, these TOVs can be sustained for extended durations, posing challenges to the insulation performance of related equipment. This chapter introduces examples of such studies.

Other issues, such as the zero-missing phenomenon, the leading current interruption, and the cable discharge, also stem from the large charging capacities of cables. The effects of these issues on the cable system design are discussed in Section 3.5. The discussion includes countermeasures for the problems and suggestions for equipment selection.

Sheath bonding and grounding is another important issue in cable system design. Sheath overvoltage requires careful study not only to avoid failures of sheath voltage limiters (SVLs) and sheath interrupts, but also to ensure the safety of maintenance crews. Sections 3.2 and 3.3 cover all the major aspects of sheath bonding and grounding, thus providing a wide variety of information from fundamentals to applications. In addition, impedance calculations, wave propagation characteristics, and transient voltage behaviors discussed in these sections provide grounding for the transient phenomena discussed in the later sections.
3.2 Impedance and Admittance of Cable Systems

3.2.1 Single-Phase Cable

3.2.1.1 Cable Structure

The most significant difference between a cable and an overhead line is that a cable is generally composed of two conductors for one phase, while an overhead line is generally composed of one conductor. Thus, a three-phase cable consists of six conductors, while a three-phase overhead line consists of three conductors.

Figure 3.1 illustrates the cross section of a typical coaxial cable. The core conductor carries current in the same way the phase conductor of an overhead line does. The metallic sheath is grounded at both ends of the cable in order to shield the core current. Thus, the metallic sheath is often called the “shield.”

3.2.1.2 Impedance and Admittance

The impedance and admittance of a single-phase cable are presented in matrix form because the cable contains two conductors:

\[ [Z_i] = \begin{bmatrix} Z_{cc} & Z_{cs} \\ Z_{cs} & Z_{ss} \end{bmatrix}, \quad [Y_i] = \begin{bmatrix} Y_c & -Y_c \\ -Y_c & Y_s \end{bmatrix} \text{ for phase } i \tag{3.1} \]

Each element of the impedance matrix is composed of the cable internal impedance and the cable outer media (earth-return) impedance, as explained in Chapter 1 of this volume. In the overhead line case, the conductor internal impedance is composed of only one impedance (i.e., the outer surface

![Diagram of a typical coaxial cable](image-url)
impedance of a conductor). The cable internal impedance consists of the following six components [1]:

1. Core outer surface impedance (same as the internal impedance of an overhead line)
2. Core-to-sheath insulator impedance
3. Sheath inner surface impedance
4. Mutual impedance between the sheath’s inner and outer surfaces
5. Sheath outer surface impedance
6. Sheath outer insulator (the outer cover shown in Figure 3.1) impedance

It is quite clear that cable impedance is far more complicated than overhead line impedance [1,2].

The admittance matrix is expressed in the following form using the potential coefficient matrix:

\[ [Y] = j\omega[C] = j\omega[P]^{-1} \quad \text{for one phase} \quad (3.2) \]

where

\[ [P] = \begin{bmatrix} P_c & P_s \\ P_s & P_s \end{bmatrix} = \begin{bmatrix} P_{12} + P_{23} & P_{23} \\ P_{23} & P_{23} \end{bmatrix} \]

\[ P_{12} = \frac{1}{2\pi\epsilon_1} \ln \frac{R_3}{R_2} \]

\[ P_{23} = \frac{1}{2\pi\epsilon_2} \ln \frac{R_3}{R_4} \]

### 3.2.2 Sheath Bonding

Before we discuss the impedance and admittance of a three-phase cable, it is necessary to learn about sheath bonding. Underground cables that are longer than 2 km normally adopt cross-bonding to reduce sheath currents and to suppress sheath voltages at the same time [3]. Figure 3.2 shows a representative cross-bonding diagram of a cable. In the figure, one of the three sheath circuits is highlighted with a dotted line. Starting from the left termination, the sheath circuit goes along the phase \(a\) conductor in the first minor section, the phase \(b\) conductor in the second minor section, and the phase \(c\) conductor in the third minor section. Theoretically, the vector sum of the induced voltage of the sheath circuit in these three minor sections becomes zero when three phase currents in the phase conductors are balanced and the three minor sections are of the same length. This is why cross-bonding can reduce sheath currents and suppress sheath voltages at the same time.
If the lengths of the three minor sections are different, an imbalance in the induced voltages will result that causes sheath currents. However, when there are more than a few major sections, it is a common practice to design cross-bonding after considering the best balance for the induced voltage. This results in the so-called homogeneous nature of cable impedance [4,5].

For submarine cables, it is more common to adopt solid bonding due to the difficulty in constructing joints offshore, as shown in Figure 3.3. Hence, submarine cables have higher sheath currents compared to underground cables that are normally cross-bonded. In order to reduce the loss caused by higher sheath currents, the sheath conductors of submarine cables often have a lower resistance (i.e., a larger cross section).

Single-point bonding has an advantage in terms of reducing the sheath currents. The sheath current loss can be reduced virtually to zero by applying single-point bonding, as shown in Figure 3.4. However, it can only be applied to short cables or short cable sections due to a limitation in the acceptable sheath voltage. In order to prevent the sheath voltage from exceeding the limitation, SVLs are installed at the unearthed end of the sheath circuit. (The installation of SVLs is discussed in greater detail in Section 3.3.) Additionally, installing an earth continuity cable (ECC) is highly recommended in order to suppress sheath overvoltage.
Figure 3.5 shows an example in which single-point bonding is employed in a long cable. As discussed earlier, cross-bonding is adopted for the long cable. In the figure, the first three minor sections from the left termination compose one major section of cross-bonding. Since the number of minor sections is four, which is not a multiple of three, the fourth minor section from the left termination cannot become a part of cross-bonding. In this situation, single-point bonding is applied to the remaining minor section (as shown in Figure 3.5) as long as the sheath voltage allows it.

**FIGURE 3.5**
Single-point bonding as part of a cross-bonded cable.
This situation is often observed in actual installations, as the number of minor sections is not determined by the cross-bonding. Rather, it is determined to reduce the number of joints as much as possible as an aspect of cost consideration.

The joint labeled EJ/SSJ functions both as an earthing joint (EJ) and as a sheath-sectionalizing joint (SSJ). The left side of the joint is solidly grounded as in an earthing joint. The left and right sides of the joint are insulated as in a sheath-sectionalizing joint, and the right side of the joint is unearthed. Since the grounding resistance at the EJ/SSJ is normally much higher than the resistance at the termination (substation), this addition of the single-point-bonding section may significantly increase the zero-sequence impedance of the cable without the ECC.

### 3.2.3 Homogeneous Model of a Cross-Bonded Cable

#### 3.2.3.1 Homogeneous Impedance and Admittance

Section 3.2.1 addressed the impedance and admittance of a single-phase cable. This section addresses the impedance and admittance of a cross-bonded three-phase cable and how $6 \times 6$ impedance and admittance matrices can be reduced to $4 \times 4$ matrices.

Figure 3.6 illustrates a major section of a cross-bonded cable. The bold solid line and the broken line express the core and sheath, respectively. The sheaths are grounded through grounding impedance $Z_g$ at both sides of the major section. The core and sheath voltages $V_k$ and $V'_k$ and currents $I_k$ and $I'_k$ at the $k$th cross-bonded node are related as in the following equations:

\[
\begin{align*}
(V'_k) &= [R](V_k) \\
(I'_k) &= [R](I_k)
\end{align*}
\] (3.3)
\[ (V_k) = \begin{pmatrix} (V_{kc}) \\ (V_{ks}) \end{pmatrix}, \quad (V_{kc}) = \begin{pmatrix} V_{ka} \\ V_{kb} \\ V_{kc} \end{pmatrix}, \quad (V_{ks}) = \begin{pmatrix} V_{ka} \\ V_{kb} \\ V_{ks} \end{pmatrix} \]  

(3.4)

The second subscripts \( c \) and \( s \) denote the core and sheath, respectively, and the third subscripts, \( a \), \( b \), and \( c \), express the phases. The other voltage and current vectors \((V'_k)\), \((I'_k)\), and \((I'_s)\) have the same form as \((V_k)\).

The sheath-sectionalizing joint is mathematically expressed by a rotation matrix \([R]\):

\[
[R] = \begin{bmatrix} \text{[U]} & \text{[0]} \\ \text{[0]} & \text{[R]} \end{bmatrix}
\]

(3.5)

\[ [R]_3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \]

where \([0]\) and \([U]\) denote a \(3 \times 3\) null and unit matrix, respectively.

The rotation matrix has the following characteristics:

\[ [R]^3 = [U], \quad [R]^2 = [R] = [R]^{-1} \]  

(3.6)

where the subscript \( t \) represents the transposed matrix.

When defining the voltage difference \( \Delta V_{k-1} \) between nodes \( k-1 \) and \( k' \), the following equation is used:

\[ (V_k) = (V_{k-1'}) + (\Delta V_k) \]  

(3.7)

The voltage difference between the major section \( \Delta V \) (between nodes \( 0 \) and \( 3 \)) is represented by

\[ (\Delta V) = (V_3) - (V_0) \]  

(3.8)

From Equations 3.3 to 3.8, \( \Delta V \) is expressed by \( \Delta V_k \) \((k = 1, 2, 3)\) in the following form:

\[
\Delta V = (V_3) - (V_0) = (V_2) + (\Delta V_3) - (V_0) = [R](V_2) + (\Delta V_3) - (V_0) \\
= [R]\left\{ (V'_1) + (\Delta V_1) \right\} + (\Delta V_3) - (V_0) = [R]\left\{ [R](V'_1) + (\Delta V_1) \right\} + (\Delta V_3) - (V_0) \\
= [R][\left\{ [R]\left\{ (V'_0) + (\Delta V_1) \right\} + (\Delta V_2) \right\} + (\Delta V_3) - (V_0) \\
= [R]^2(V'_0) + [R]^2(\Delta V_1) + [R](\Delta V_2) + (\Delta V_3) - (V_0) \]  

(3.9)

The second subscripts \( c \) and \( s \) denote the core and sheath, respectively, and the third subscripts, \( a \), \( b \), and \( c \), express the phases. The other voltage and current vectors \((V'_k)\), \((I'_k)\), and \((I'_s)\) have the same form as \((V_k)\).
Voltage and current deviations are expressed by using the cable impedance \([Z]\) and admittance \([Y]\) in the following example:

\[
(\Delta V_k) = -[Z]I_k(I_k), \quad (\Delta I_k) = -[Y]I_k(V_k)
\] (3.10)

where \(l_k\) is the length of the \(k\)th minor section.

The voltage difference between the terminals of the major section \((\Delta V)\) gives an equivalent impedance of a cross-bonded cable. It is obtained (3.13) by applying the following relations:

\[
(I_{k-1'}) = (I_k) = [R](I_{k-1})
\] (3.11)

\[
(V_0) = [R]^2(V''), \quad \therefore V_{0sa} = V_{0sc} = V_{0'sa} = V_{0'sc} = V_{0'sc}
\] (3.12)

\[
(\Delta V) = -[R]^2[Z]I_1(I_1) - [R][Z]I_2(I_2) - [Z]I_3(I_3)
\]

\[
= -[R]^2[Z]I_1[R][I''] - [R][Z]I_2[R]^2(I'') - [Z]I_3[R]^2(I'')
\]

\[
= -\left\{[R]^2[Z]I_1[R] + [R][Z]I_2[R]^2 + [Z]I_3\right\}(I'')
\] (3.13)

If the lengths of the minor sections are identical \((l_k = l)\), an equivalent series impedance \([Z']\) can be obtained:

\[
(\Delta V) = -[Z']3I(I'')
\]

\[
[Z'] = \frac{1}{3}([R]^2[Z][R] + [R][Z][R]^2 + [Z])
\]

\[
= \frac{1}{3}([R][Z][R] + [R][Z][R] + [Z])
\]

\[
= \begin{bmatrix}
Z_{cc}'
& Z_{cs}'
Z_{cs}'
& Z_{ss}'
\end{bmatrix}
\] (3.14)

The physical meaning of this equation can be explained using the following calculation:

\[
[R][Z][R] = \begin{bmatrix}
[U] & [0]
[0] & [R_{33}]
\end{bmatrix}
\begin{bmatrix}
[Z_{cc}]
& [Z_{cs}]
[Z_{cs}]
& [Z_{ss}]
\end{bmatrix}
\begin{bmatrix}
[U]
[0]
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(Z_{cc})
& (Z_{cs})
(Z_{cs})
& (Z_{ss})
\end{bmatrix}
\begin{bmatrix}
[U]
[0]
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(Z_{cc})
& (Z_{cs})\begin{bmatrix}
[R_{33}]
\end{bmatrix}
(Z_{cs})
& (Z_{ss})\begin{bmatrix}
[R_{33}]
\end{bmatrix}
\end{bmatrix}
\] (3.15)
\[
[R][Z][R] = \begin{bmatrix}
[Z_{cc}] & [Z_{cs}] & [R_{cc}] \\
[R_{cc}] & [Z_{ss}] & [R_{cc}] \\
[R_{cc}] & [R_{ss}] & [R_{cc}]
\end{bmatrix}
\] (3.16)

where \([Z_{cc}], [Z_{cs}], \text{ and } [Z_{ss}]\) are the submatrices of the cable impedance matrix.

The submatrix for the cores \([Z_{cc}]\) remains unchanged based on the operation shown in Equation 3.14.

\[ [Z'_{cc}] = [Z_{cc}] \] (3.17)

Equations 3.18 and 3.19 show that the operation to the submatrix for the mutual impedance between the cores and sheaths \([Z_{cs}]\) is averaging within the rows:

\[ [Z'_{cs}] = \frac{1}{3}([Z_{cs}] [R_{cc}] + [Z_{cs}] [R_{ss}] + [Z_{cs}] R_{ss}) = \frac{1}{3} [Z_{cs}](R_{ss}) + R_{ss} + [U]) \] (3.18)

\[ \frac{1}{3} (R_{ss}) + R_{ss} + [U]) = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \] (3.19)

The diagonal and mutual element \((Z'_{ss} \text{ and } Z'_{ssss})\) of the submatrix for sheaths \([Z_{ss}]\) is the mean of the self- and mutual impedance of the sheath. The shape of the matrix is identical to that of a transposed overhead line:

\[
\frac{1}{3} [R_{cc}] [Z_{ss}] [R_{ss}] = \frac{1}{3} \begin{bmatrix} 0 & 1 & 0 & Z_{ss11} & Z_{ss12} & Z_{ss13} & 0 & 0 & 1 \\ 0 & 0 & 1 & Z_{ss12} & Z_{ss22} & Z_{ss23} & 1 & 0 & 0 \\ 1 & 0 & 0 & Z_{ss13} & Z_{ss23} & Z_{ss23} & 0 & 1 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} Z_{ss11} & Z_{ss12} & Z_{ss13} & 0 & 0 & 1 \\ Z_{ss12} & Z_{ss22} & Z_{ss23} & 1 & 0 & 0 \\ Z_{ss13} & Z_{ss23} & Z_{ss33} & 0 & 1 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} Z_{ss22} & Z_{ss23} & Z_{ss12} \\ Z_{ss23} & Z_{ss33} & Z_{ss13} \\ Z_{ss12} & Z_{ss13} & Z_{ss11} \end{bmatrix}
\] (3.20)

\[
\frac{1}{3} [R_{cc}] [Z_{ss}] [R_{ss}] = \frac{1}{3} \begin{bmatrix} 0 & 0 & 1 & Z_{ss11} & Z_{ss12} & Z_{ss13} & 0 & 1 & 0 \\ 1 & 0 & 0 & Z_{ss12} & Z_{ss22} & Z_{ss23} & 0 & 0 & 1 \\ 1 & 0 & 0 & Z_{ss13} & Z_{ss23} & Z_{ss33} & 1 & 0 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} Z_{ss11} & Z_{ss12} & Z_{ss13} & 0 & 1 & 0 \\ Z_{ss12} & Z_{ss22} & Z_{ss23} & 0 & 0 & 1 \\ Z_{ss13} & Z_{ss23} & Z_{ss33} & 1 & 0 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} Z_{ss13} & Z_{ss23} & Z_{ss33} \\ Z_{ss11} & Z_{ss12} & Z_{ss13} \\ Z_{ss12} & Z_{ss22} & Z_{ss23} \end{bmatrix}
\] (3.21)
\[
[Z_{ss}'] = \frac{1}{3} ([R_{33}][Z_{ss}] [R_{33}] + [R_{33}'][Z_{ss}] [R_{33}] + [Z_{ss}]) =
\begin{bmatrix}
Z_{sxs} & Z_{sxs}' & Z_{sxs}' \\
Z_{sxs}' & Z_{sxs} & Z_{sxs}' \\
Z_{sxs}' & Z_{sxs}' & Z_{sxs}
\end{bmatrix}
\]

\[
[Z_{ss}'] = \frac{1}{3} \sum_{i=1}^{3} Z_{ssii}
\]

\[
[Z_{ssm}'] = \frac{1}{3} \sum_{i=1}^{3} \sum_{j=i+1}^{3} Z_{ssij}
\] (3.22)

In this same manner, the equivalent admittance of a cross-bonded cable can be obtained from the current difference \((\Delta I)\):

\[
(\Delta I) = -[Y']3l(V_0)
\]

\[
[Y'] = [R]^2[Y][R] + [R][Y][R]^2 + [Y] = [R][Y][R] + [R][Y][R] + [Y]
\] (3.23)

The admittance matrix of the cable can be expressed as follows:

\[
[Y] = \begin{bmatrix}
 [Y_{cc}] & -[Y_{cs}] \\
 -[Y_{cs}] & [Y_{ss}]
\end{bmatrix} = \begin{bmatrix}
 [Y_{cc}] & -[Y_{cs}] \\
 -[Y_{cs}] & [Y_{ss}]
\end{bmatrix}
\] (3.24)

\[
[Y_{cc}] = j\omega \begin{bmatrix}
 C_{c1} & 0 & 0 \\
 0 & C_{c2} & 0 \\
 0 & 0 & C_{c3}
\end{bmatrix}
\] (3.25)

\[
[Y_{ss}] = j\omega \begin{bmatrix}
 C_{ss1} + C_{sm12} + C_{sm13} & -C_{sm12} & -C_{sm13} \\
 -C_{sm12} & C_{ss2} + C_{sm12} + C_{sm23} & -C_{sm23} \\
 -C_{sm13} & -C_{sm23} & C_{ss3} + C_{sm23} + C_{sm13}
\end{bmatrix}
\] (3.26)

The core admittance submatrix \([Y_{cc}]\) is a diagonal matrix determined by the capacitances between the cores and the sheath, because a sheath encloses a core. The admittance submatrix of the cores for the cross-bonded cable is identical to the solidly bonded cable:

\[
[Y_{cc}'] = [Y_{cc}]
\] (3.27)

Equations 3.18 and 3.19 show that the operation sheaths to the submatrix for the mutual admittance between the cores and sheaths \([Y_{cs}]\) is averaging within the rows:
\[ [Y'_s] = \frac{1}{3} ([Y_c][R_{33}] + [Y_c][R_{33}] + [Y_c]) = \frac{1}{3} [Y_c]([R_{33}] + [R_{33}] + [U]) \]

\[ = j\omega \frac{1}{3} \begin{bmatrix} C_{c1} & C_{c1} & C_{c1} \\ C_{c2} & C_{c2} & C_{c2} \\ C_{c3} & C_{c3} & C_{c3} \end{bmatrix} \] (3.28)

The sheaths diagonal and mutual element \((Z'_{ss} \text{ and } Z'_{ssm})\) of the submatrix for the sheaths \([Z_{ss}]\) is the mean of the self- and mutual impedances of the sheath:

\[ [Y''_s] = \frac{1}{3} ([R_{33}] [Y_{ss}] [R_{33}] + [R_{33}] [Y_{ss}] [R_{33}] + [Y_{ss}]) = \begin{bmatrix} Y''_{ss} & Y''_{ssm} & Y''_{ssm} \\ Y''_{ssm} & Y''_{ss} & Y''_{ssm} \\ Y''_{ssm} & Y''_{ssm} & Y''_{ss} \end{bmatrix} \]

\[ Y''_{ss} = \frac{1}{3} \sum_{i=1}^{3} Y_{ssii} \]

\[ Y''_{ssm} = \frac{1}{3} \sum_{i=1}^{3} \sum_{j=i+1}^{3} Y_{ssij} \] (3.29)

### 3.2.3.2 Reduction of the Sheath

The lengths of the minor sections can have imbalances due to constraints on the locations of joints. The imbalances are designed to be as small as possible, since they increase sheath currents and raise sheath voltages. When a cable system has multiple major sections, the overall balance is considered for minimizing sheath currents. As a result, when a cable system has more than two major sections, sheath currents are generally balanced among three conductors, which allows for the reduction from three metallic sheaths to one conductor [4,5].

Since three-phase sheath conductors are short-circuited and grounded in every major section (as illustrated in Figure 3.6), the sheath voltages of the three phases are equal at each earthing joint. Assuming that the sheath currents are balanced among the three conductors, the sheath currents do not flow into the earth at each earthing joint:

\[ V_{1sa} = V_{1sb} = V_{1sc} \equiv V_{1s} \]
\[ V_{4sa} = V_{4sb} = V_{4sc} \equiv V_{4s} \]
\[ I_{1sa} + I_{1sb} + I_{1sc} \equiv I_{1s} \]
\[ I_{4sa} + I_{4sb} + I_{4sc} \equiv I_{4s} \] (3.30)

By applying connection matrix \([T]\), this equation can be rewritten as

\[ (V_k) = [T] (V'_k), \quad (I'_k) = [T] (I_k), \quad k = 0, 3 \] (3.31)
where

\[
[T] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix} = \begin{bmatrix} [U] & [0] \\ (0)_t & (1)_t \end{bmatrix}
\tag{3.32}
\]

\[
(0)_t = (0 \\ 0 \\ 0), \quad (1)_t = (1 \\ 1 \\ 1)
\]

\[
(V'_k) = \begin{bmatrix} (V_{kc}) \\ V_{ks} \end{bmatrix}, \quad (I'_k) = \begin{bmatrix} (I_{kc}) \\ I_{ks} \end{bmatrix}
\tag{3.33}
\]

From Equations 3.14, 3.23, and 3.31, the following relation is obtained:

\[
(\Delta V'') = -[Z'']3l(I'_0)
\]

\[
(\Delta I'') = -[Y'']3l(V'_0)
\tag{3.34}
\]

where

\[
[Z''] = (T)[Z'][T]_t^{-1}
\]

\[
[Y''] = (T)[Y'][T]_t
\tag{3.35}
\]

The impedance matrix \([Z'']\) and the admittance matrix \([Y'']\) are 4 × 4 matrices composed of three cores and a reduced single sheath.

Because the propagation mode of the cable can be expressed by both coaxial- and sheath-propagation mode, in a high-frequency region where skin depth is smaller than sheath thickness, the impedance matrix is composed of the following two submatrices:

\[
[Z_{cc}] = [Z_{cd}] + [Z_{ss}]
\tag{3.36}
\]

\[
[Z_{cs}] = [Z_{ss}]
\tag{3.37}
\]

where

\[
[Z_{cd}] = \begin{bmatrix}
Z_{cd} & 0 & 0 \\
0 & Z_{cd} & 0 \\
0 & 0 & Z_{cd}
\end{bmatrix}
\tag{3.38}
\]
The reduced impedance in a high-frequency region becomes:

\[
[Z''] = ([T][Z']^{-1}[T])^{-1} = \begin{bmatrix}
Z_{cc}'' & (Z_{cs}'') \\
(Z_{cs}'') & Z_{ss}''
\end{bmatrix}
\] (3.39)

The core impedances, including their mutual impedances, are identical to the original impedances:

\[
[Z''] = [Z_c'] = [Z_{cc}] = [Z_{cd}] + [Z_{ss}]
\] (3.40)

The mutual impedance between the \( k \)th core and the reduced sheath \( Z''_{kj} \) is the average of the impedances between the core and the three-phase sheaths:

\[
(Z_{cs}'' = \frac{1}{3} \sum_{j=1}^{3} Z_{skj}
\] (3.41)

Finally, the sheath impedance is the average of all the elements of the original sheath impedance matrix:

\[
Z_{ss}'' = \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} Z_{ssij}
\] (3.42)

In the same manner, the reduced admittance matrix becomes:

\[
[Y''] = [Y_c'] = [Y_{cc}]
\] (3.43)

\[
(Y_{cs}'' = \sum_{j=1}^{3} Y_{skj} = -j\omega C_{ck}
\] (3.44)

\[
Y_{ss}'' = \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} Y_{ssij} = j\omega \sum_{i=1}^{3} C_{ssi}
\] (3.45)

\[
[Y''] = [T][Y'][T]^\dagger = \begin{bmatrix} [Y_{cc}''] & (Y_{cs}'') \\
(Y_{cs}'') & Y_{ss}''
\end{bmatrix}
\] (3.46)
3.2.4 Theoretical Formula of Sequence Currents

The sequence impedance/current calculation of overhead lines is well-known and introduced in textbooks [2]. For underground cables, theoretical formulas are proposed for the cable itself [6–9]. However, in order to derive accurate theoretical formulas, it is necessary to consider the whole cable system, including sheath bonding, because the return current of an underground cable flows through both the metallic sheath and the ground. Until now, no formula existed for sequence impedances or currents that consider sheath bonding and sheath-grounding resistances at substations and earthing joints. As a result, it became a common practice to measure these sequence impedances or currents after installation, as it is considered difficult to predict these values in advance.

As mentioned earlier, it is a common practice for underground cable systems that are longer than approximately 2 km to cross-bond the metallic sheaths of three-phase cables to simultaneously reduce sheath currents and suppress sheath voltages [3]. Submarine cables, which are generally bonded solidly, are now gaining popularity due to the increase in offshore wind farms and cross-border transactions.

In this section, we derive theoretical formulas of the sequence currents for the majority of underground cable systems: that is, a cross-bonded cable that has more than two major sections. We also derive theoretical formulas for a solidly bonded cable considering the increased use of submarine cables.

3.2.4.1 Cross-Bonded Cable

3.2.4.1.1 Impedance Matrix

One cable system corresponds to six conductor systems composed of three cores and three metallic sheaths. As in the last section, the $6 \times 6$ impedance matrix of the cable system is represented by the following equation [1]:

$$
[Z] = 
\begin{bmatrix}
[Z_{cc}] & [Z_{cs}] \\
[Z_{cs}] & [Z_{ss}]
\end{bmatrix} = 
\begin{bmatrix}
[Z_{cc}] & [Z_{cs}] \\
[Z_{cs}] & [Z_{ss}]
\end{bmatrix} = 
\begin{bmatrix}
Z_{cc11} & Z_{cc12} & Z_{cc13} \\
Z_{cc12} & Z_{cc22} & Z_{cc23} \\
Z_{cc13} & Z_{cc23} & Z_{cc33}
\end{bmatrix},
$$

$$
[Z_{ss}] = 
\begin{bmatrix}
Z_{ss11} & Z_{ss12} & Z_{ss13} \\
Z_{ss12} & Z_{ss22} & Z_{ss23} \\
Z_{ss13} & Z_{ss23} & Z_{ss33}
\end{bmatrix},
$$

(3.47)
where
\[ c \] is the core
\[ s \] is the sheath
\[ t \] is the transpose

In Equation 3.47, cable phase \( a \) is assumed to be laid symmetrically to phase \( c \) and against phase \( b \). The flat configuration and the trefoil configuration, which are typically adopted, satisfy this assumption.

By reducing the sheath conductors, the six-conductor system is reduced to a four-conductor system composed of three cores and one equivalent metallic sheath, as shown in Figure 3.7. The \( 4 \times 4 \) reduced impedance matrix can be expressed as

\[
[Z''] = \begin{bmatrix}
Z_{cc11} & Z_{cc12} & Z_{cc13} & Z''_{14} \\
Z_{cc12} & Z_{cc22} & Z_{cc23} & Z''_{24} \\
Z_{cc13} & Z_{cc23} & Z_{cc33} & Z''_{34} \\
Z''_{14} & Z''_{24} & Z''_{34} & Z_{ss}
\end{bmatrix}
\]

(3.48)

**FIGURE 3.7**
Cross-bonded cable and its equivalent model: (a) a cross-bonded cable system with \( m \)-major sections and (b) an equivalent four-conductor system.
Here, \( Z''(4, j) = Z''(j, 4) \) can be calculated from the \( 6 \times 6 \) impedance matrix \( Z \) shown in Equation 3.41. \( Z_{14}'' = Z_{34}'' \) stands in the flat configuration and the trefoil configuration.

### 3.2.4.1.2 Zero-Sequence Current

The following equations are derived from Figure 3.8. Here, sheath grounding at earthing joints is ignored, but sheath grounding at substations can be considered through \( V_s' \):

\[
(V_1) = [Z'](I_1)
\]

where

\[
(V_1) = \begin{pmatrix} E & E & E & V_s' \end{pmatrix}^t
\]

\[
(I_1) = \begin{pmatrix} I_a & I_b & I_c & I_s' \end{pmatrix}^t
\]

![FIGURE 3.8](image-url)

Setup for measuring sequence currents for a cross-bonded cable: (a) zero-sequence current and (b) positive-sequence current.
Figure 3.8a shows the setup for measuring the zero-sequence current for a cross-bonded cable. Assuming the grounding resistance at substations $R_g$, the sheath voltage $V_s$ can be obtained by:

$$V_s = -2R_gI_s$$

(3.50)

The following equations can be obtained by solving Equations 3.49 and 3.50:

$$I_a = I_c = \frac{(Z_{22} - Z_{12})E}{\Delta_0}$$

$$I_b = \frac{(Z_{11} - Z_{21})E}{\Delta_0}$$

(3.51)

where

$$\Delta_0 = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$Z_{11} = Z_{cc11} + Z_{cc13} - \frac{2Z''_{14}^2}{Z''_{SR}}$$

$$Z_{22} = Z_{cc22} - \frac{Z''_{24}^2}{Z''_{SR}}$$

$$Z_{12} = Z_{cc12} - \frac{Z_{14}'Z_{24}''}{Z''_{SR}}, \quad Z_{21} = 2Z_{12}$$

$$Z''_{SR} = Z''_{ss} + 2R_g$$

The zero-sequence current can be obtained from Equation 3.51 as follows:

$$I_0 = \frac{2I_a + I_b}{3} = \frac{E}{3\Delta_0}(Z_{11} + 2Z_{22} - 2Z_{12} - Z_{21})$$

(3.52)

When three-phase cables are laid symmetrically to each other, the following equations are satisfied:

$$Z_{cc11} = Z_{cc22} = Z_c, \quad Z_{ss11} = Z_{ss22} = Z_s$$

$$Z_{cc12} = Z_{cc13} = Z_m, \quad Z_{14}'' = Z_{24}'' = Z_n$$

(3.53)

Using symmetrical impedances $Z_c, Z_m,$ and $Z_n$ in Equation 3.53, $Z_{11}, Z_{22},$ and $Z_{12}$ can be expressed as

$$Z_{11} = Z_c + Z_m - \frac{2Z_{14}''^2}{Z''_{SR}}$$

$$Z_{22} = Z_c - \frac{Z_n^2}{Z''_{SR}}$$

$$Z_{12} = Z_m - \frac{Z_n^2}{Z''_{SR}}$$

(3.54)
Substituting $Z_{11}$, $Z_{22}$, and $Z_{12}$ in Equations 3.51 and 3.52 by the symmetrical impedances will result in:

\[
I_a = I_b = I_c \approx \frac{E}{\Delta_1}, \quad I_s \approx \frac{-3Z_nE}{Z_{SR}^\prime \Delta_1},
\]

where $\Delta_1 = Z_e + 2Z_m - 3Z_n^2/Z_{SR}^\prime$.

### 3.2.4.1.3 Positive-Sequence Current

In Figure 3.8(b), the equation $I_{a1} + I_{b1} + I_{c1} = 0$ is satisfied at the end of the cable line. The following equations are obtained since $V_s = 0$:

\[
(V_1) = [E, \alpha^2E, \alpha E, 0]^T, \\
(I_1) = [I_a, I_b, I_c, I_s]^T
\]

where $\alpha = \exp(j2\pi/3)$.

Solving Equation 3.56 for $I_a$, $I_b$, and $I_c$ yields results in the following:

\[
\left(\begin{array}{c}
E \\
\alpha^2E \\
\alpha E
\end{array}\right) = 
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{12} & Z_{22} & Z_{12} \\
Z_{13} & Z_{12} & Z_{11}
\end{bmatrix}
\begin{pmatrix}
I_a \\
I_b \\
I_c
\end{pmatrix}
\]

\[
\therefore \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} = 
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{12} & Z_{22} & Z_{12} \\
Z_{13} & Z_{12} & Z_{11}
\end{bmatrix}^{-1}
\begin{bmatrix}
E \\
\alpha^2E \\
\alpha E
\end{bmatrix}
\]

\[
= \frac{1}{\Delta}
\begin{bmatrix}
Z_{11}Z_{22} - Z_{12}^2 & Z_{12}(Z_{13} - Z_{11}) & Z_{12}^2 - Z_{13}Z_{22} \\
Z_{12}(Z_{13} - Z_{11}) & Z_{11}^2 - Z_{13}^2 & Z_{12}(Z_{13} - Z_{11}) \\
Z_{12}^2 - Z_{13}Z_{22} & Z_{12}(Z_{13} - Z_{11}) & Z_{11}Z_{22} - Z_{12}^2
\end{bmatrix}
\begin{bmatrix}
E \\
\alpha^2E \\
\alpha E
\end{bmatrix}
\]

(3.57)

Here,

\[
Z_{11} = Z_{cc11} - \frac{Z_{n1}^2}{Z_{sn}} \\
Z_{22} = Z_{cc22} - \frac{Z_{n2}^2}{Z_{sn}} \\
Z_{12} = Z_{cc12} - \frac{Z_{n1}Z_{n2}}{Z_{sn}} \\
Z_{13} = Z_{cc12} - \frac{Z_{n1}Z_{n2}}{Z_{sn}}
\]
The positive-sequence current is derived from Equation 3.57:

\[ I_1 = \frac{1}{3} \left( I_a + \alpha I_b + \alpha^2 I_c \right) = \frac{E}{3\Delta_2} \left\{ (Z_{11} - Z_{13})(Z_{11} + Z_{13} + 2Z_{12}) + Z_{22}(2Z_{11} + Z_{13}) - 3Z_{12}^2 \right\} \] (3.58)

where \( \Delta_2 = (Z_{11} - Z_{13})\left\{ Z_{22}(Z_{11} + Z_{13}) - 2Z_{12}^2 \right\} \).

When three-phase cables are laid symmetrically to each other, Equation 3.58 can be further simplified using Equation 3.53:

\[ I_1 = \frac{E}{Z_c - Z_m} \] (3.59)

### 3.2.4.2 Solidly Bonded Cable

#### 3.2.4.2.1 Impedance Matrix

Figure 3.9 shows a sequence current measurement circuit for a solidly bonded cable. The following equations are given from the \( 6 \times 6 \) impedance matrix in Equation 3.47 and Figure 3.9:

\[ (E) = [Z_{cc}](I) + [Z_{cs}](I_s) \] (3.60)

\[ (V_s) = [Z_{ss}](I) + [Z_{ss}](I_s) = -2[R_s](I_s) \] (3.61)

![FIGURE 3.9](image)

Setup for measuring sequence currents for a solidly bonded cable: (a) zero-sequence current and (b) positive-sequence current.
Here, \((I) = (I_a, I_b, I_c)^t\) is the core current and \((I_s) = (I_{sa}, I_{sb}, I_{sc})^t\) is the sheath current:
\[
[R_s] = R_s \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
\]

From Equation 3.61, sheath current \((I_s)\) is found using:
\[
(I_s) = -([Z_{ss}] + 2[R_s])^{-1}[Z_{cs}] (I)
\] (3.62)

After eliminating the sheath current \((I_s)\) in Equation 3.60, core current \((I)\) can be derived as
\[
(I) = ([Z_{cc}] - [Z_{cs}]) ([Z_{ss}] + 2[R_s])^{-1} [Z_{cs}]^{-1}(E)
\] (3.63)

### 3.2.4.2.2 Zero-Sequence Current

From Figure 3.9a, \((E)\) and \((I)\) are expressed as
\[
(E) = [E\; E\; E], \quad (I) = [I_a\; I_b\; I_c]^t
\] (3.64)

The core current \((I)\) is obtained from Equations 3.63 and 3.64; and the zero-sequence current is calculated as \(I_0 = (I_a + I_b + I_c)/3\).

Since the relationship \([Z_{cs}] \approx [Z_{ss}]\) generally stands, Equations 3.60 and 3.61 can be simplified to Equation 3.65 using Equation 3.53:
\[
((E) - (V_s)) = ([Z_{cc}] - [Z_{cs}]) (I) = (Z_c - Z_s) [U] (I)
\] (3.65)

where \([U]\) is the \(3 \times 3\) unit (identity) matrix.

Hence,
\[
I_0 = I_a = I_b = I_c = \frac{1}{Z_c - Z_s} (E - V_s)
\] (3.66)

Using Equation 3.66, the core current \((I)\) in Equation 3.61 can be eliminated, which yields
\[
(V_s) = \frac{1}{Z_c - Z_s} [Z_{cs}] ((E) - (V_s)) - [Z_{cs}] (I_s)
\] (3.67)
After adding all three rows in Equation 3.67, the result is

\[
3V_s = 3 \frac{Z_s + 2Z_m}{Z_c - Z_s} (E - V_s) - \frac{Z_s + 2Z_m}{2R_g} V_s
\]

(3.68)

After solving Equation 3.68 for \(V_s\) and then eliminating \(V_s\) from Equation 3.66, the zero-sequence current becomes

\[
I_0 = \frac{6R_s + Z_s + 2Z_m}{6R_s(Z_c + 2Z_m) + (Z_c - Z_s)(Z_s + 2Z_m)} E
\]

(3.69)

### 3.2.4.2.3 Positive-Sequence Current

From Figure 3.9b, \((E)\) and \((l)\) are expressed as

\[
(E) = \begin{bmatrix} E \\ \alpha^2 E \\ \alpha E \end{bmatrix}, \quad (I) = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}\]

(3.70)

The core current \((l)\) is obtained from Equations 3.63 and 3.70. Once the core current is solved for, the positive-sequence current can be calculated as

\[
I_1 = (I_a + \alpha I_b + \alpha^2 I_c)/3.
\]

Using Equation 3.65, the theoretical formula of the positive-sequence current simplifies to

\[
I_1 = \frac{1}{3(Z_c - Z_s)} \left\{ (E - V_s) + \alpha \left( \alpha^2 E - V_s \right) + \alpha^2 (\alpha E - V_s) \right\} = \frac{E}{Z_c - Z_s}
\]

(3.71)

Equation 3.71 shows that the positive-sequence current can be approximated by the coaxial mode current. It also shows that, in a manner similar to that of a cross-bonded cable, the positive-sequence current remains unaffected by the substation-grounding resistance \(R_g\).

### 3.2.4.2.4 Example

Figure 3.10 shows the physical and electrical data of a 400-kV cable used for comparison. An existence of semiconducting layers introduces an error in the charging capacity of the cable. The relative permittivity of the insulation (XLPE) is converted from Equation 2.4 to 2.7, according to Equation 3.72, in order to correct the error and achieve a reasonable cable model [10]:

\[
\varepsilon_\text{r}' = \frac{\ln(R_3/R_2)}{\ln(R_{so}/R_{si})} \varepsilon_\text{r} = \frac{\ln(61.40/32.60)}{\ln(59.50/34.10)} = 2.4 = 2.729
\]

(3.72)
where

- \( R_{si} \) is the inner radius of the insulation
- \( R_{so} \) is the outer radius of the insulation

The total length of the cable is assumed to be 12 km. Figure 3.11 shows the layout of the cables. It is assumed that the cables are directly buried at a depth of 1.3 m with a separation of 0.5 m between the phases. Earth resistivity is set to 100 \( \Omega \) m.

The calculation process in the case of a cross-bonded cable using the proposed formulas is shown as follows (the 6 × 6 impedance matrix \( Z \) is obtained using cable constants [1,11,12]):

\[
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} \\
Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} \\
Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} \\
Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} \\
Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} \\
Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66}
\end{bmatrix}
\]

Zero-sequence current

\[
\Delta_0 = -3.9605900 + j12.489448
\]

\[
Z_{11} = 4.0641578 + j2.2224336
\]
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$$Z_{22} = 2.1959039 + j2.1450759$$
$$Z_{12} = 2.0193420 + j0.1372637$$
$$Z_{21} = 4.0386840 + j0.2745275$$
$$I_0 \text{ (rms)} = 81.814700 - j31.778479$$

Positive-sequence current
$$\Delta_2 = -4.8574998 - j1.8394591$$
$$Z_{11} = 0.3670612 + j1.8221556$$
$$Z_{22} = 0.3801952 + j1.4707768$$
$$Z_{12} = 0.2482475 - j0.5159115$$
$$Z_{13} = 0.2419636 - j0.8650940$$
$$I_1 \text{ (rms)} = 14.118637 - j251.86277$$

Table 3.1 shows zero- and positive-sequence currents derived by the proposed formulas. Grounding resistances at substations are assumed to be 1 Ω. In the calculations, the applied voltage is set to $E = 1\text{kV}/\sqrt{3}$ (angle: 0°) and the source impedance is not considered. Here, sequence currents are determined according to the setups for measuring sequence currents shown in

### TABLE 3.1
Comparison of Proposed Formulas with EMTP Simulations

<table>
<thead>
<tr>
<th></th>
<th>Zero Sequence</th>
<th>Positive Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude (A)</td>
<td>Angle (deg.)</td>
</tr>
<tr>
<td><strong>a. Cross-bonded cable</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed formulas, Equations 3.8/3.14</td>
<td>124.1</td>
<td>−21.23</td>
</tr>
<tr>
<td><strong>b. Solidly bonded cable</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed formulas, Equations 3.19, 3.20/3.26</td>
<td>124.8</td>
<td>−22.50</td>
</tr>
</tbody>
</table>

FIGURE 3.11
Layout of the cable.
Figures 3.8 and 3.9. The assumptions regarding the applied voltage and the source impedance match the condition in the actual setups for measuring sequence currents, since testing sets are generally used in such measurements.

The proposed formulas are known to have a satisfactory accuracy for planning and implementation studies. An acceptable level of error is introduced by the impedance matrix reduction discussed earlier. Owing to the matrix reduction, unbalanced sheath currents that flow into the earth at earthing joints are not considered in the proposed formulas.

Table 3.1 shows that the positive-sequence impedance is smaller for a solidly bonded cable than for a cross-bonded cable, and the positive-sequence current is larger for a solidly bonded cable. Because of this size differential, the return current flows only through the metallic sheath of the same cable and earth in the solidly bonded cable, whereas the return current flows through the metallic sheath of all three-phase cables in a cross-bonded cable ($Z_c - Z_m > Z_c - Z_o$).

The impedance calculation in IEC 60909-2 assumes solid bonding. As a result, if the positive-sequence impedance of a cross-bonded cable is derived based on IEC 60909-2, it might be smaller than the actual positive-sequence impedance.

The phase angle of the zero-sequence current mentioned in Table 3.1 demonstrates that grounding resistance at substations in both cross-bonded and solidly bonded cables significantly affects the zero-sequence current. As a result, there is little difference in the zero-sequence impedance of the cross-bonded cable and the solidly bonded cable. The results indicate the importance of obtaining an accurate grounding resistance at the substations to derive accurate zero-sequence impedances of cable systems.

### 3.3 Wave Propagation and Overvoltages

#### 3.3.1 Single-Phase Cable

##### 3.3.1.1 Propagation Constant

As explained in Chapter 1, the evaluation of wave propagation-related parameters necessitates eigenvalue/eigenvector calculations. Because of the coaxial structure of a cable core and metallic sheath, the propagation-related parameters show the following characteristics when in a high-frequency region [2]:

1. Impedance matrix

   In a high-frequency region, the following relation is satisfied in Equation 3.1:

   \[ Z_{cs} = Z_{ss} = Z_s \]
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or

\[
[Z_i] = \begin{bmatrix}
Z_c & Z_s \\
Z_s & Z_c
\end{bmatrix}, \quad Z_c = Z_{cc}
\]  

(3.73)

2. Voltage transformation matrix

\[
(v) = [A]^{-1}(V), \quad [A]^{-1} = \begin{bmatrix}
0 & 1 \\
1 & -1
\end{bmatrix}
\]  

(3.74)

where

- \((v)\) is the modal voltage
- \((V)\) is the actual phase voltage

3. Modal propagation constant

The modal propagation constant \(\gamma\) is given in the following equation from the actual propagation constant matrix \([\Gamma]\) explained in Chapter 1:

\[
[\gamma] = [A]^{-1}[\Gamma][A]
\]  

(3.75)

where \([\Gamma]^2 = [Z][Y]\).

Considering Equation 3.74 with Equations 3.71 and 3.73:

\[
[\gamma] = \begin{bmatrix}
\gamma_e & 0 \\
0 & \gamma_c
\end{bmatrix}
\]  

(3.76)

where

\(\gamma_e = Z_s(Y_s - Y_c)\) is the earth-return mode (mode 1)
\(\gamma_c = Y_c(Z_s - Z_c)\) is the coaxial mode (mode 2)

4. Characteristic impedance

The modal characteristic impedance \([z_0]\) is given as

\[
[z_0] = \begin{bmatrix}
z_{0e} & 0 \\
0 & z_{0c}
\end{bmatrix}
\]  

(3.77)

where

- \(z_{0e} = Z_{0s}\) is the earth-return mode (mode 1)
- \(z_{0c} = Z_{0c} - Z_{0s}\) is the coaxial mode (mode 2)
The actual characteristic impedance $[Z_0]$ is obtained from these equations in the following form:

$$[Z_0] = [A][Z_0][B]^{-1} = \begin{bmatrix} Z_{0c} & Z_{0s} \\ Z_{0s} & Z_{0s} \end{bmatrix}$$ (3.78)

where $[B]^{-1} = [A]$, is the current transformation matrix.

From these equations, it should be clear that the coaxial mode current flows through the core conductor and returns through the metallic sheath in a high-frequency region. In fact, a communication signal cable and a measuring cable intentionally use coaxial mode propagation for signal transmission because the propagation characteristic is entirely dependent on the insulator between the core’s outer surface and the sheath’s inner surface. In such a case, propagation velocity $c$, and characteristic impedance $Z_{0c}$ of the coaxial mode are evaluated approximately by:

$$c = \frac{c_0}{\sqrt{\varepsilon_{i1}}}, \quad Z_{0c} = \frac{60}{\sqrt{\varepsilon_{i1}}} \ln \frac{R_3}{R_2}$$ (3.79)

### 3.3.1.2 Example of Transient Analysis

Figure 3.12 illustrates a circuit diagram of a single-phase coaxial cable. In the figure, the characteristic impedance of each section is defined as follows:

$$[Z_1] = \begin{bmatrix} R_0 & 0 \\ 0 & R_s \end{bmatrix}, \quad [Z_2] = \begin{bmatrix} Z_{0c} & Z_{0s} \\ Z_{0s} & Z_{0s} \end{bmatrix}, \quad [Z_3] = \begin{bmatrix} R_c & 0 \\ 0 & R_s \end{bmatrix}$$ (3.80)

![Circuit diagram of a single-phase coaxial cable.](image-url)
where
\( R_0 \) is the source impedance
\( R_s \) is the sheath-grounding resistance
\( R_c \) is the core-terminating resistance

The sheath-grounding resistance \( R_s \) generally ranges from 0.1 to 20 Ω depending on the earth resistivity and the grounding method. The source impedance is either the bus impedance or the transformer impedance. If the cable is connected to an overhead line, \( R_0 \) and \( R_c \) are the surge impedances of the overhead line. If the cable is extended beyond node 2, \( R_c \) is the core self-characteristic impedance \( Z_{0c} \) or the coaxial mode characteristic impedance \( z_{0c} \).

As explained in Chapter 1, the refraction coefficient matrices at nodes 1 and 2 are given in the following forms:

\[
[\lambda_{1f}] = \frac{2}{\Delta_1} \begin{bmatrix}
Z_{0c}(Z_{0s} + R_c) - Z_{0s}^2 & R_0Z_{0s} \\
R_cZ_{0s} & Z_{0s}(Z_{0c} + R_0)
\end{bmatrix}
\]

\[
[\lambda_{2f}] = \frac{2}{\Delta_2} \begin{bmatrix}
R_c(Z_{0s} + R_s) & -R_sZ_{0s} \\
-R_sZ_{0s} & R_s(Z_{0c} + R_c)
\end{bmatrix}
\]

where
\[\Delta_1 = (Z_{0c} + R_0)(Z_{0s} + R_c) - Z_{0s}^2\]
\[\Delta_2 = (Z_{0c} + R_c)(Z_{0s} + R_s) - Z_{0s}^2\]

Now, we consider the transient response of a coaxial cable to connect a PG to a circuit (that is, a current lead wire). Then, the following condition is given assuming that the receiving end of the core is open-circuited and the sheath is perfectly grounded:

\[ R_0 = z_{0c}, \quad R_s = 0, \quad R_c = \infty \]  

(3.82)

Going by this result, Equation 3.89 becomes

\[
[\lambda_{1f}] = \begin{bmatrix}
1 & 1 \\
0 & 2
\end{bmatrix}, \quad [\lambda_{2f}] = \begin{bmatrix}
2 & -2 \\
0 & 0
\end{bmatrix}
\]

(3.83)

Since the traveling wave voltage at node 1 is \( E/2 \) in Figure 3.12 from Thevenin’s theorem (as explained in Chapter 1), the node 1 voltage \( (v_1) \) at \( t = 0 \) is calculated as
\[ v_1 = [\lambda_{1f}] \begin{pmatrix} E \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} E \\ 2 \\ 0 \end{pmatrix} = (E_{12}) \quad \text{at } t = 0 \] \[ (3.84) \]

Traveling wave \((E_{12})\) from node 1 to node 2 is transformed into modal wave \((e_{12})\) as follows:

\[ (e_{12}) = [A]^{-1}(E_{12}) = \begin{pmatrix} 0 \\ E \\ 2 \end{pmatrix} \] \[ (3.85) \]

Equation 3.85 indicates that a coaxial mode wave carries \(E/2\) to the receiving end; that is, the cable works as a coaxial mode signal transfer system.

The coaxial mode wave arrives at node 2 at \(t = t_a\). The wave is then transformed into an actual phase-domain wave \((E_{2f})\) as follows:

\[ (E_{2f}) = [A](e_{12}) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ E \\ 2 \end{pmatrix} \] \[ (3.86) \]

\[ (3.84) \]

\[ (v_2) = [\lambda_{2f}](E_{2f}) = \begin{pmatrix} E \\ 0 \end{pmatrix} \quad \text{at } t = t_a \]

The voltage \(E\) from the PG, therefore, appears at the open end of the coaxial cable at \(t = t_a\).

3.3.2 Wave Propagation Characteristics

We will now discuss the wave propagation characteristics of a three-phase single-core cable. Figure 3.13 and Table 3.2 show a cross section and the parameters of a tunnel-installed cable.

Assuming that the tunnel is a pipe conductor, the propagation parameters are evaluated by using a pipe-type (PT) cable option of the EMTP cable constants.

Table 3.3a shows the calculated results of the impedance, the admittance, the modal attenuation constant, and the propagation velocity on the solidly bonded case; Table 3.3b shows the results on the cross-bonded case with the homogeneous model at frequency \(f = 100 \text{ kHz}\).

3.3.2.1 Impedance: \(R, L\)

In Table 3.3a-1, the top \(3 \times 3\) matrix expresses a three-phase core impedance \([Z_{cc}]\), the top right and bottom left matrices are three-phase core-to-sheath \([Z_{cs}]\) and sheath-to-core \([Z_{sc}] = [Z_{cs}]^T\) impedances and the bottom \(3 \times 3\) matrix expresses three-phase sheath impedances \([Z_{ss}]\). The upper line is
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the resistance (Ω/km) and the lower line is the inductance (mH/km). The
\([Z_{cc}]\) matrix of the solidly bonded cable in Table 3.3a-1 is the same as that of
the cross-bonded cable in Table 3.3b-1. The impedance in the last column,
\(Z_{ii}''\) (= \(Z_{ii}''_4\)), in Table 3.3b is given as the average of \([Z_{cs}]\) in Table 3.3a-1, as
shown in Equation 3.41.

**TABLE 3.2**

<table>
<thead>
<tr>
<th>Cable Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_1)</td>
<td>0</td>
</tr>
<tr>
<td>(r_2)</td>
<td>30.45 mm</td>
</tr>
<tr>
<td>(r_3)</td>
<td>71.15 mm</td>
</tr>
<tr>
<td>(r_4)</td>
<td>74.80 mm</td>
</tr>
<tr>
<td>(r_5)</td>
<td>81.61 mm</td>
</tr>
<tr>
<td>(\rho_e)</td>
<td>100 Ω m</td>
</tr>
</tbody>
</table>
### TABLE 3.3
Parameters of a Tunnel-Installed Cable at 100 kHz

(a) Solidly Bonded Cable

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Core-1</th>
<th>Core-2</th>
<th>Core-3</th>
<th>Sheath-1</th>
<th>Sheath-2</th>
<th>Sheath-3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>R</strong></td>
<td>83.150</td>
<td>81.660</td>
<td>82.120</td>
<td>82.200</td>
<td>81.660</td>
<td>82.120</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>1.172</td>
<td>0.801</td>
<td>0.800</td>
<td>0.989</td>
<td>0.801</td>
<td>0.800</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>81.660</td>
<td>82.690</td>
<td>81.890</td>
<td>81.660</td>
<td>81.740</td>
<td>81.890</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.801</td>
<td>1.172</td>
<td>0.800</td>
<td>0.801</td>
<td>0.990</td>
<td>0.800</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>82.120</td>
<td>81.890</td>
<td>83.620</td>
<td>82.120</td>
<td>81.890</td>
<td>82.670</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.800</td>
<td>0.800</td>
<td>1.171</td>
<td>0.800</td>
<td>0.801</td>
<td>0.989</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>82.200</td>
<td>81.660</td>
<td>82.120</td>
<td>82.200</td>
<td>81.660</td>
<td>82.120</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.989</td>
<td>0.801</td>
<td>0.800</td>
<td>0.989</td>
<td>0.801</td>
<td>0.800</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>81.660</td>
<td>81.740</td>
<td>81.890</td>
<td>81.660</td>
<td>81.740</td>
<td>81.890</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.801</td>
<td>0.990</td>
<td>0.800</td>
<td>0.801</td>
<td>0.990</td>
<td>0.800</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>82.120</td>
<td>81.890</td>
<td>82.670</td>
<td>82.120</td>
<td>81.890</td>
<td>82.670</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.800</td>
<td>0.800</td>
<td>0.989</td>
<td>0.800</td>
<td>0.800</td>
<td>0.989</td>
</tr>
</tbody>
</table>

**a-2. Capacitance [C], in (nF/km)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Core-1</th>
<th>Core-2</th>
<th>Core-3</th>
<th>Sheath-1</th>
<th>Sheath-2</th>
<th>Sheath-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core-1</td>
<td>191.0</td>
<td>0</td>
<td>0</td>
<td>–191.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Core-2</td>
<td>0</td>
<td>191.0</td>
<td>0</td>
<td>0</td>
<td>–191.0</td>
<td>0</td>
</tr>
<tr>
<td>Core-3</td>
<td>0</td>
<td>0</td>
<td>191.0</td>
<td>0</td>
<td>0</td>
<td>–191.0</td>
</tr>
<tr>
<td>Sheath-1</td>
<td>–191.0</td>
<td>0</td>
<td>0</td>
<td>236.2</td>
<td>–19.0</td>
<td>–18.3</td>
</tr>
<tr>
<td>Sheath-2</td>
<td>0</td>
<td>–191.0</td>
<td>0</td>
<td>–19.0</td>
<td>235.4</td>
<td>–18.7</td>
</tr>
<tr>
<td>Sheath-3</td>
<td>0</td>
<td>0</td>
<td>–191.0</td>
<td>–18.3</td>
<td>–18.7</td>
<td>237.1</td>
</tr>
</tbody>
</table>

**Mode**

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

**a-3. Current Transformation Matrix [T]**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Core-1</th>
<th>Core-2</th>
<th>Core-3</th>
<th>Sheath-1</th>
<th>Sheath-2</th>
<th>Sheath-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core-1</td>
<td>0</td>
<td>1.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Core-2</td>
<td>1.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Core-3</td>
<td>0</td>
<td>0</td>
<td>1.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sheath-1</td>
<td>0</td>
<td>–1.000</td>
<td>0</td>
<td>0.336</td>
<td>0.653</td>
<td>–0.229</td>
</tr>
<tr>
<td>Sheath-2</td>
<td>–1.000</td>
<td>0</td>
<td>0</td>
<td>0.227</td>
<td>–0.140</td>
<td>0.768</td>
</tr>
<tr>
<td>Sheath-3</td>
<td>0</td>
<td>0</td>
<td>–1.000</td>
<td>0.456</td>
<td>–0.503</td>
<td>–0.509</td>
</tr>
</tbody>
</table>

**a-4. Modal Propagation Constant**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Core-1</th>
<th>Core-2</th>
<th>Core-3</th>
<th>Sheath-1</th>
<th>Sheath-2</th>
<th>Sheath-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attenuation (dB/km)</td>
<td>0.134</td>
<td>0.134</td>
<td>0.134</td>
<td>1.854</td>
<td>0.026</td>
<td>0.030</td>
</tr>
<tr>
<td>Velocity (m/µs)</td>
<td>169.6</td>
<td>169.6</td>
<td>169.6</td>
<td>220.3</td>
<td>287.7</td>
<td>288.1</td>
</tr>
</tbody>
</table>

(Continued)
The above results correspond to the fact that the cross-bonding acts as a transposition of an overhead transmission line, and the three sheath conductors are reduced to one equivalent conductor as explained in Section 3.2.3.2.

3.3.2.2 Capacitance: C

The capacitance matrix looks similar to the impedance matrix in Table 3.3a-2. The capacitance between the core and sheath $C_{ck}$ of the homogeneous model is identical to that of the solidly bonded cable. The equivalent capacitance $C'_{44}$ of the cross-bonded cable in Table 3.3b-2 is given as the sum of the elements as shown in Equations 3.44 and 3.45.
3.3.2.3 Transformation Matrix

The role of the transformation matrix \([T_i]\) in Table 3.3a-3 and b-3, is to transform modal current \((i)\) to phasor current \((l)\), that is

\[
(l) = [T_i](i)
\]  

(3.87)

In the solidly bonded cable, the first three modes (columns) shown in Table 3.3a-3 express coaxial-propagation modes (that is, the “core-to-sheath” mode [2]). The other modes (columns) correspond to one of the transformation matrices of an untransposed three-phase overhead line [2]. In this case, mode 4 expresses an earth-return mode and modes 5 and 6 correspond to aerial modes.

The reduced transformation matrix of a cross-bonded cable is shown in Table 3.3b-3. The composition of the top left \(3 \times 3\) matrix (the first three modes) is similar to that of an overhead transmission line. The current of the third mode returns from the equivalent sheath instead of the earth. The fourth mode expresses the equivalent earth-return mode of the cross-bonded cable system.

3.3.2.4 Attenuation Constant and Propagation Velocity

Modes 1–3 in the solidly bonded cable shown in Table 3.3a-4 are coaxial modes and are the same as mode 3 in the homogeneous cross-bonded cable model. Although the attenuations of the inter-core modes (modes 1 and 2) shown in Table 3.3b-4 are almost identical to that of the coaxial mode of the solidly bonded cable, the velocities are lower. The velocity of the coaxial mode \(v_c\) is determined by the permittivity of the main insulator \(\varepsilon_1 = 2.3\) shown in Table 3.2:

\[
v_c = \frac{c_0}{\sqrt{\varepsilon_1}} = \frac{300}{\sqrt{2.3}} \approx 198 \text{ m/µs}
\]  

(3.88)

where \(c_0\) is the speed of light.

The velocity converges to this value as the frequency increases.

The attenuation and velocity of the earth-return mode (mode 4) in both cable models are identical.

Because the cable is installed in a tunnel (that is, the cable is in air), the attenuation constant and the propagation velocity of mode 4 (earth return) and modes 5 and 6 (the first and second inter-sheath) in the solidly bonded cable show similar characteristics to those of an overhead line [1]. The attenuation constants of modes 5 and 6 are much smaller and the propagation velocity is much greater in the solidly bonded cable than those in the other modes.

3.3.3 Transient Voltage

Figures 3.14 and 3.15 show transient voltage waveforms at the sending-end core voltages and the first cross-bonding point of a cross-bonded
FIGURE 3.14
Calculated transient core voltages on a tunnel-installed cable: (a) cross-bonded, (b) mixed.
(Continued)
FIGURE 3.14 (Continued)
Calculated transient core voltages on a tunnel-installed cable: (c) homogeneous, and (d) solidly bonded.
Calculated transient sheath voltages on a tunnel-installed cable: (a) cross-bonded and (b) mixed.
cable system with five major sections \( (l_1 = l_2 = l_3 = 400 \text{ m} \) with total length \( l = 5 \times 3 \times 0.4 = 6 \text{ km} \) when a step voltage of 1 pu is applied to a sending-end core (phase \( a \)) through a resistor of \( R_s = 200 \Omega \). This resistor models a backward surge impedance. The physical parameters of the cable are shown in Figure 3.6 and Table 3.2. Each receiving-end core is grounded through a resistor of \( R_e = 200 \Omega \). The sheaths are short-circuited and grounded by a resistor of \( R_s = 0.1 \Omega \) at both ends of each major section. The cable is represented by a constant parameter, Dommel’s model.

The calculated results for minor sections exactly modeled by multiphase-distributed parameter lines are shown in Figures 3.14a and 3.15a. The induced voltages on the cores are observed in Figure 3.14a. The voltage is generated by the reflections at the cross-bonded points. After 70 \( \mu \text{s} \), the voltage on the applied phase is gradually increased. The time is determined based on the round-trip time of the coaxial traveling wave:

\[
T = \frac{2l}{v_c} = \frac{2 \times 6000}{169.6} = 70.8 \mu \text{s}
\]

(3.89)

where \( l \) and \( v_c \) denote the total cable length and the traveling velocity, respectively, of the coaxial mode shown in Table 3.3a-4 or b-4.

The time constant \( \tau_1 \) of the voltage increase is determined by the sending-end resistance \( R_s \), the terminating resistance \( R_e \), and the total cable capacitance \( C_{cl} \). The capacitance is obtained from Table 3.3a-2 or b-2.

\[
\tau_1 = (R_e // R_s)C_{cl} = 100 \times 0.191 \times 6 = 110 \mu \text{s}
\]

(3.90)

The maximum sheath voltage at the first cross-bonded joint shown in Figure 3.15a becomes 0.05 pu, which is about 40% of the core voltage at the time. This voltage is generated by a reflection at the cross-bonded joint. This is an inherent characteristic of the cross-bonded cable. The high voltage (HV) is a key factor in the insulation design of a cross-bonded cable system.

Although the exact model of the cross-bonded cable is useful for the simulation of a simple grid, the simulation of a large-scale cable system with a cross-bonded cable is too complicated and difficult. The homogeneous model explained in Section 3.2.3 is comparatively simple and useful. Figures 3.14 and 3.15b illustrate the transient voltages when the first major section is expressed accurately and the other major sections are expressed by pi-equivalent circuits whose parameters are determined by the homogeneous model. By comparing the results from the exact model (a), a simplification is possible, providing sufficient information for an insulation design of the cable system. Figure 3.14c illustrates the result of a case in which all major sections are expressed by homogeneous pi-equivalent circuits. It is clear from the figure that the calculated result has enough accuracy for the simulation of the switching surge, although the sheath voltages at the cross-bonded joints cannot be obtained.
Figure 3.14d shows the transient response of the core voltage in a solidly bonded cable. It shows a stair-like waveform with a length of 70 µs. This length is determined by the round-trip time shown in Equation 3.89. Sheath voltages of the solidly bonded cable are much smaller than those of the cross-bonded cable. The results indicate that not all cross-bonded cables can be simplified by a solidly bonded cable from the viewpoint of not only the sheath voltages but also the core voltages.

3.3.4 Limitations of the Sheath Voltage

As mentioned in Section 3.2.2, the limitations of the sheath voltage are key for making decisions regarding sheath bonding and other cable system designs related to the sheath. There are two types of limitations in the sheath voltage: (1) continuous voltage limitation and (2) short-term voltage limitation.

Continuous voltage limitation is the limitation of the sheath voltage induced by the normal load flow in phase conductors without any faults. It is enforced by government or district regulations in many countries and differs in each area based on said regulations. This limitation was enforced for the safety of the maintenance crews who may come into contact with the sheath circuit. Even if this limitation is not enforced, utilities follow their own standards for continuous voltage limitation.

Since SVLs are designed not to be operated by continuous sheath voltages, continuous voltage limitation is maintained mainly by cable layouts, cross-bonding designs, and grounding resistances. The cable span length (minor length) is more often limited by restrictions in transportation, but can also be limited by a continuous voltage limitation.

A short-term voltage limitation is specified in IEC 62067 Annex G (informative) as impulse levels [13]. Considering short-term voltage limitation, the following phenomena are studied:

- SLG faults (external to the targeted major section)
- Three-phase faults (external to the targeted major section)
- Switching surges
- Lightning surges

When only power-frequency components are considered, SLG faults and three-phase faults are studied using theoretical formulas. Some utilities study SLG faults and three-phase faults using EMTP in order to consider transient components of the sheath voltage. Switching surges rarely become an issue for the sheath overvoltage.

Lightning surges have to be studied for a mixed overhead line/underground cable as shown in Figure 3.16. A lightning strike on the GW can propagate into the sheath circuit, since the transmission tower and cable sheath often share the grounding mesh or electrode at the transition site. The level
of the sheath overvoltage is highly dependent on the grounding resistance at the transition site. The space of the transition site is sometimes limited; it is necessary to achieve a low grounding resistance in order to lower the sheath overvoltage.

A BFO can occur when lightning strikes the GW. In addition, a lightning strike can directly hit the phase conductor due to shielding failure. In these cases, the lightning surge in the phase conductor can directly propagate into the cable core leading to sheath overvoltage. Since the lightning surge is not highly attenuated in this case, the voltage across the sheath interrupts at the first SSJ needs to be studied in addition to the sheath-to-earth voltage at the transition site.

Lightning surges are also studied when a limited number of feeders are connected to a substation together with a cable. In Figure 3.17, the substation has only two lines and one transformer considering the maintenance outage. The lightning surge on the overhead line can propagate into the cable core without significant attenuation depending on the substation layout.

3.3.5 Installation of SVLs

SVLs are installed at SSJs in order to suppress short-term sheath overvoltages. Figure 3.18 shows the connection of SVLs when the sheath circuit is cross-bonded in a link box. SVLs are often arranged in a star formation with their neutral points earthed. If study results show that the sheath overvoltage exceeds the TOV rating of SVLs, the ECC can be installed as shown in Figure 3.18. Other countermeasures include changing the neutral point from
Transients on Cable Systems

solidly earthed to unearthed and changing the SVL connection from a star formation to a delta formation.

When the link box is not installed, SVLs are located immediately next to sheath-sectionalizing joints as shown in Figure 3.19. In this connection, SVLs are arranged in a delta formation. This formation has an advantage in

FIGURE 3.17
Example of a substation with a limited number of feeders.

FIGURE 3.18
Connection of SVLs in a link box.
suppressing short-term sheath overvoltage, as bonding leads to SVLs can be much shorter than when using the link box.

3.4 Studies on Recent and Planned EHV AC Cable Projects

This section introduces recent and planned EHV AC cable projects and cable system transients studied for the projects. In order to compensate for the large charging capacity of EHV AC cables, shunt reactors are often installed together with these cables. The large charging capacity and large shunt reactors lower the natural frequency of the network which, at times, make it necessary to conduct resonance overvoltage studies. Load-shedding overvoltages and the zero-missing phenomena are the other power system transients specifically related to cable systems.

Similar to overhead line projects, switching transients such as cable energization, ground fault, and fault clearing are also studied for EHV AC cable projects as standard work. However, severe overvoltages related to these switching transients on cable systems have not been reported in the literature.

This section focuses on well-known long cable projects, which normally require shunt compensation, since the TOVs discussed can only be observed with these cables. Therefore, this section includes only cross-bonded land cables and submarine cables. It does not include short cables, typically installed inside power stations and substations, since power system transients specifically related to cable systems are normally not included in the study of short cables.

3.4.1 Recent Cable Projects

Table 3.4 lists long 500-/400-kV cables that are currently in operation. The number of long 400-kV cable projects is larger than the number of 500-kV cable projects [14–22]. The system voltage of 400 kV is adopted mainly based on the geographical area such as in Europe and the Middle East.
## TABLE 3.4

<table>
<thead>
<tr>
<th>Location</th>
<th>Route Length (km)</th>
<th>Number of Circuits</th>
<th>Insulation</th>
<th>Commissioned in</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>500-kV Cables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vancouver, Canada</td>
<td>30</td>
<td>2</td>
<td>SCFF(^{b})</td>
<td>1984</td>
</tr>
<tr>
<td>Vancouver–Texada Island</td>
<td>8</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mainland Japan–Shikoku</td>
<td>22.2</td>
<td>1</td>
<td>SCFF</td>
<td>1994</td>
</tr>
<tr>
<td>Island, Japan</td>
<td>22.2</td>
<td>1</td>
<td></td>
<td>2000</td>
</tr>
<tr>
<td>Tokyo, Japan</td>
<td>39.8</td>
<td>2</td>
<td>XLPE</td>
<td>2000</td>
</tr>
<tr>
<td>Shin-Toyosu Line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shanghai, China [14]</td>
<td>17</td>
<td>2</td>
<td>XLPE</td>
<td>2010</td>
</tr>
<tr>
<td>Moscow, Russia [15]</td>
<td>10.5</td>
<td>2</td>
<td>XLPE</td>
<td>2012</td>
</tr>
<tr>
<td><strong>400-kV Cables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copenhagen, Denmark [16]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Southern route</td>
<td>22</td>
<td>1</td>
<td>XLPE</td>
<td>1997</td>
</tr>
<tr>
<td>Northern route</td>
<td>12</td>
<td>1</td>
<td></td>
<td>1999</td>
</tr>
<tr>
<td>Spain–Morocco [17]</td>
<td>28</td>
<td>1</td>
<td>SCFF</td>
<td>1997</td>
</tr>
<tr>
<td>Madrid, Spain [19]</td>
<td>31</td>
<td>1</td>
<td></td>
<td>2006</td>
</tr>
<tr>
<td>San Sebastian de los Reyes–Loeches–Morata Line</td>
<td>12.8</td>
<td>2</td>
<td>XLPE</td>
<td>2004</td>
</tr>
<tr>
<td>Jutland, Denmark [20]</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trige–Nordjyllandsværket (4.5 + 2.5 + 7.0)</td>
<td>2</td>
<td>XLPE</td>
<td>2004</td>
<td></td>
</tr>
<tr>
<td>London Ring [21]</td>
<td>20</td>
<td>1</td>
<td>XLPE</td>
<td>2005</td>
</tr>
<tr>
<td>Qatar</td>
<td>15</td>
<td>2</td>
<td>XLPE</td>
<td>2007</td>
</tr>
<tr>
<td>Istanbul, Turkey</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>London–West Ham</td>
<td>7.2</td>
<td>1</td>
<td>XLPE</td>
<td></td>
</tr>
<tr>
<td>Rotterdam, Netherlands</td>
<td>12.6</td>
<td>1</td>
<td>XLPE</td>
<td>2008</td>
</tr>
<tr>
<td>Enecogen</td>
<td>12.5</td>
<td>1</td>
<td>XLPE</td>
<td>2010</td>
</tr>
<tr>
<td>Dubai and Abu Dhabi, UAE</td>
<td>11.5</td>
<td>1</td>
<td>XLPE</td>
<td>2011</td>
</tr>
<tr>
<td>Mushrif–Al Mamzar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abu Dhabi, UAE [22]</td>
<td>13</td>
<td>2</td>
<td>XLPE</td>
<td>2011–2012</td>
</tr>
<tr>
<td>Sadiyat–ADST</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doha, Qatar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Umm Al Amad Super–Lusail Development Super 2</td>
<td>22</td>
<td>3</td>
<td>XLPE</td>
<td>2012–2013</td>
</tr>
</tbody>
</table>

\(^{a}\) Include 500-/525-/550-kV cables and 380-/400-/420-kV cables.

\(^{b}\) Self-contained fluid filled.
The world’s first long 500-/400-kV cable was installed in Canada by BC Hydro in 1984 [23–25]. This 500-kV AC submarine cable is a double-circuit line that connects Vancouver Island to mainland Canada through Texada Island. The distance between Vancouver Island and Texada Island is approximately 30 km; the distance between Texada Island and mainland Canada is approximately 8 km. In between, the line has an overhead section on Texada Island. Shunt reactors totaling 1080 MVar were installed to compensate for the large charging capacity.

The longest 500-kV cable in the world, the Shin-Toyosu line, was installed in Japan by the Tokyo Electric Power Company in 2000. This double-circuit line has four 300-MVar shunt reactors for the compensation of the large charging capacity. This is the first 500-kV cable on which extensive power system transient studies are available in the literature [26,27]. In addition to ordinal switching transients, the overvoltage caused by system islanding, series resonance overvoltage, leading current interruption, and zero-missing phenomenon was also studied.

Here, we introduce the overvoltage caused by system islanding, studied on the Shin-Toyosu line. When one end of a long cable is open, a part of the network can be separated from the main grid and connected with the long cable, which can lead to severe overvoltage. Figure 3.20 illustrates the equivalent circuit where one end of the long cable is open due to a bus fault. A cable fault will not lead to overvoltage since it results in the removal of the long cable from the equivalent circuit.

From this equivalent circuit, the overvoltage caused by system islanding can be expressed using the following equations:

\[ v(t) = V_m \sin \omega t - \frac{\omega}{\omega_0} V_m \sin \omega_0 t \]  \hspace{1cm} (3.91)

\[ V_m = \frac{E_m L}{L_0(1 - \omega^2 CL) + L'} \quad \omega_0 = \sqrt{\frac{1}{CL_0} + \frac{1}{CL}} \]  \hspace{1cm} (3.92)

\[ \text{Fault clearance} \quad \text{Underground cable with shunt reactors} \quad \text{Equivalent source} \]  \hspace{1cm} (Figure 3.20)

Equivalent circuit of the overvoltage caused by system islanding.
where

\[ L_0 \] is the source impedance of the islanded system
\[ E_m \] is the source voltage behind \( L_0 \)

The charging capacity of the long cable and the inductance of the shunt reactors directly connected to the cable are expressed as \( C \) and \( L \), respectively.

Equation 3.91 shows that the overvoltage contains two frequency components: the nominal frequency \( \omega \) and the resonance frequency \( \omega_0 \). Since the overvoltage is caused by the superposition of two frequency components, the resulting overvoltage is oscillatory and its level is often difficult to estimate before the simulation. The result of a simulation performed on the Shin-Toyosu line is shown in Figure 3.21.

Most of the 500-/400-kV cables shown in Table 3.4 are installed in highly populated areas, hence the route lengths are limited to 10–20 km. These cables are equipped with shunt reactors for the compensation of the charging capacity, but their unit size and the total capacity are not as large due to the shorter route lengths. For these cables, only studies such as the reactive power compensation, the design of the cable itself, and the laying method

**FIGURE 3.21**
Example of an overvoltage caused by system islanding.
are discussed in the literature. Transient studies on these cables are not available.

### 3.4.2 Planned Cable Projects

Table 3.5 lists planned lengthy 400-kV cable projects that will be operational within a couple of years. There is no mention of planned 500-kV cable projects in any publicly available sources.

Various transient studies have been performed on the 400-kV cable that will connect Sicily to mainland Italy [28,29]. In addition to switching transients, these studies include the harmonic overvoltage caused by line energization, leading current interruption, and zero-missing phenomena. The studies identified the resonant condition at the second harmonic when the cable is energized from Sicily’s side under a particular condition. The harmonic overvoltage caused by the resonant condition is avoided by the operational constraint.

### 3.5 Cable System Design and Equipment Selection

#### 3.5.1 Study Flow

This section discusses the cable system design, except for overvoltage analysis and insulation coordination. The cable system design includes the selection and specification of the cable itself and the related equipment such as CBs and VTs. The cable system design related to the sheath is discussed in Section 3.2.

During the planning stage, transmission capacity and reactive power compensation are normally studied. These studies mainly determine the cable

#### TABLE 3.5

Planned Long 400-kV Cables

<table>
<thead>
<tr>
<th>Location</th>
<th>Route Length (km)</th>
<th>Number of ckt</th>
<th>Insulation</th>
<th>Planned Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sicily–Mainland Italy</td>
<td>38</td>
<td>2</td>
<td>SCOF</td>
<td>2013</td>
</tr>
<tr>
<td>Abu Dhabi Island, UAE [22]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sadiyat–ADST</td>
<td>13</td>
<td>1</td>
<td></td>
<td>2014</td>
</tr>
<tr>
<td>Bahia–Sadiyat</td>
<td>25</td>
<td>3</td>
<td>XLPE</td>
<td>2013</td>
</tr>
<tr>
<td>SAS Al Nakheel–Mahawi</td>
<td>21</td>
<td>2</td>
<td></td>
<td>2013</td>
</tr>
<tr>
<td>Mahawi–Mussafah</td>
<td>13</td>
<td>3</td>
<td></td>
<td>2013</td>
</tr>
<tr>
<td>Strait of Messina, Italy</td>
<td>42</td>
<td>2</td>
<td>XLPE</td>
<td>2015</td>
</tr>
<tr>
<td>Sorgente–Rizziconi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*a Include 380-/400-/420-kV cables.
route, the voltage level, the conductor size, and the amount and locations of shunt reactors.

When the transmission development plan is designed for the cable, the cable route will be studied further. One characteristic of cables, compared to overhead lines, is that the laying of the cables and soil conditions of the location affect planning studies in addition to the land availability. These factors affect the burial depth, soil thermal resistivity, and cable separation between phases, which may necessitate changes to the conductor size and the amount and locations of shunt reactors initially determined in the planning studies.

Figure 3.22 illustrates the study flow and the relationship of studies on the cable system design. In the figure, the boxes show study items, and the following bullets list the items that are mainly evaluated in these studies. The figure explains how the transmission capacity and the reactive power compensation studied during the planning stage affect the other studies conducted for the cables.

The amount of shunt reactors (that is, the compensation rate of a cable) is a key figure that has a major impact on the following studies. A compensation rate close to 100% is often preferred since it can eliminate the reactive power surplus created by the introduction of the cable. It also offers a preferable condition for the TOV but causes a severe condition for the zero-missing phenomenon. The negative effect on the zero-missing phenomenon is not a primary concern as there are countermeasures established for tackling this effect.

![Study flow and the relationship of studies.](image-url)
Usually, shunt reactors for 500-/400-kV cables are directly connected to the cables in order to mitigate the TOV when one end of the cable is opened. Shunt reactors for other cables are often connected to buses as the area compensation is applied at these voltage levels. When shunt reactors are connected to buses, the zero-missing phenomenon does not occur. In this case, however, the inductive VT connected to the cable needs to have enough discharge capability, and the line breaker needs to have sufficient leading current interruption capability.

### 3.5.2 Zero-Missing Phenomenon

A DC offset current (zero-missing current) appears when an EHV underground cable is energized with its shunt reactors [30–35]. In this case, an AC component of a charging current has an opposite phase angle to the AC component of a current flowing into the shunt reactors. If the compensation rate of the cable is 100%, the sum of these AC components becomes zero and only the DC component remains. Since the DC component decays gradually with time, it can take more than 1 s, depending on the compensation rate, before a current that flows through the line breaker crosses the zero point.

Figure 3.23 shows an example of current waveforms when an EHV cable is energized with its shunt reactors. In this example, the AC component of the energization current is very small, since the compensation rate is close to 100%. The simulation was run for 0.2 s, but the energization current did not cross the zero point during this duration. Since the zero-missing phenomenon is caused by a DC component of an energization current, it reaches its

![FIGURE 3.23](image)

Zero-missing current in underground cable energization.
peak when the cable is energized and the maximum DC component is contained in the current. In order to realize this condition in phase \( a \), the cable was energized at the zero-voltage point of phase \( a \).

When a line breaker is used to interrupt this current without zero crossing, the arc current between the contacts cannot be extinguished within a couple of cycles and may continue for an extended duration. This extended duration may lead to the line breaker’s failure depending on the amount of arc energy generated. The duration is mainly determined by the magnitude of the DC component and the relationship between the arc resistance and arc current inside the line breaker. Typical durations for EHV cables can be several hundreds of milliseconds in severe conditions.

The zero-missing phenomenon can theoretically be avoided by limiting the compensation rate to lower than 50%, but this is not the most common way of addressing the problem. Normally, a compensation rate near 100% is adopted (especially for 500-/400-kV cables) and the countermeasures listed in Table 3.6 are applied to the cables. All these countermeasures (except for Countermeasure (4)) have already been applied to the cable line in operation. Countermeasure (1) in particular has a number of application records to long EHV cable lines and is discussed in detail later in this section.

Countermeasure (3) will be applied to the 400-kV Sicily–mainland Italy cable [29]. This countermeasure can be implemented rather easily as long as a cable line is installed together with single-phase CBs and current differential relays. For this reason, Countermeasure (3) is more suited for EHV cable lines than HV cable lines.

In this countermeasure, the faulted phase is opened instantly and healthy phases are opened about 10 s later when the DC component has decayed

| TABLE 3.6 |
| Countermeasures of Zero-Missing Phenomenon |

<table>
<thead>
<tr>
<th>Countermeasures</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 1. Sequential switching | • Requires higher leading current interruption capability  
                          • Requires single-phase CB and current differential relay |
| 2. Point-on-wave switching  
   (synchronized switching) | • May cause higher-switching overvoltage  
                              • Requires single-phase CB |
| 3. Delayed opening of healthy phases | • Requires single-phase CB and current differential relay |
| 4. Breaker with preinsertion resistor | • May not be possible to apply near generators  
                                      • May be necessary to develop a new CB (expensive) |
| 5. Additional series resistance in the shunt reactor for energization | • Requires special control to bypass the series resistance after energization |
| 6. Energize the shunt reactor after the cable | • Causes higher steady-state overvoltage and voltage step |
When this countermeasure is applied near a generator (especially when the cable line offers a radial path to the generator), it is necessary to evaluate the negative sequence current capability of the generator as Countermeasure (3) causes an unbalanced operation for a prolonged duration.

In Countermeasure (5), a resistance is connected in a series to shunt reactors when a cable line is energized. The resistance needs to be sized sufficiently for the DC component to decay fast enough. After the cable line is energized, the resistance is bypassed in order to reduce the losses. Considering the additional cost for the resistance, this countermeasure is more suited for HV cable lines than EHV cable lines.

Countermeasure (6) cannot always be applied; it is especially difficult to apply to long EHV cables due to their steady-state overvoltage.

3.5.2.1 Sequential Switching

Figure 3.24 shows a zero-missing current with an SLG fault in phase b. This assumes that a cable failure exists in phase b before energization, but it is not known to system operators until the cable is energized.

The zero-missing current is observed only in a healthy phase (phase a). The current in the faulted phase (phase b) crosses the zero point as it contains a large AC component due to the fault current. Hence, the line breaker of the faulted phase can interrupt the fault current.

FIGURE 3.24
Zero-missing phenomenon with an SLG fault.
Figure 3.25 shows the time sequence of sequential switching when the cable line is energized from Substation A. In Step 1, the line breaker of phase $b$ is opened at 60 ms after the fault and the fault is cleared by tripping this CB. Since the fault is already cleared by the opening of the phase $b$ line breaker, some time can be allowed before opening the line breakers of other healthy phases.

**FIGURE 3.25**
Time sequence of sequential switching.
In Step 2, shunt reactors are tripped before the line breakers of healthy phases. It is necessary to trip the shunt reactors of only healthy phases. At this time, it is not necessary or recommended to trip shunt reactors of the faulted phase since the current through shunt reactor breakers of the faulted phase does not cross the zero point.

It is recommended to trip at least half of the shunt reactors of healthy phases as shown in Figure 3.25 as the tripping will normally lower the compensation rate below 50%. The remaining shunt reactors will be useful in maintaining the charging current within the leading current interruption capability of the line breakers.

In Step 3, it is possible to open the line breakers of the healthy phases. Figure 3.26 shows that the current in the healthy phases contains the AC component and crosses the zero point after tripping the shunt reactors.

### 3.5.3 Leading Current Interruption

When the leading current is interrupted at current zero, it occurs at a voltage peak assuming that the current waveform is leading the voltage waveform by 90°. After the interruption, the voltage on the source side of the CB changes according to the system voltage; the voltage on the other side is fixed at peak voltage $E$ as shown in Figure 3.27. The most severe overvoltage occurs during a restrike after half cycle when the voltage on the source side becomes $-E$. As the voltage difference between the source side and the other
side is $2E$, the overvoltage can go as high as $-3E$. The restrike can be repeated to cause a very severe overvoltage.

The leading current interruption capability of CBs is specified in IEC 62271-100 (see Table 3.7) considering the severe overvoltage that it can cause.

When the charging capacity of a long EHV cable line is not compensated by shunt reactors that are directly connected to the cable, the leading current interruption capability requires careful attention [30]. Considering the typical capacitance of $0.2 \mu F/km$, the maximum line length for a 400-kV cable line is limited approximately below 26 km without shunt reactors directly connected to the cable. Here, it is assumed that the leading current is interrupted at one end, and the other end is opened before the interruption.

Usually, long EHV cable lines are compensated by shunt reactors that are directly connected to the cable. When the compensation rate is high enough, the leading current interruption capability is not a concern. If sequential switching is applied to a cable line as a countermeasure to the zero-missing phenomenon, however, the tripping of shunt reactors will lower the compensation rate. This is the only situation that requires careful attention.
3.5.4 Cable Discharge

If a shunt reactor is directly connected to a cable, the cable line is discharged through the shunt reactor when it is disconnected from the network. In this case, the time constant of the discharge process is determined by the quality factor (Q factor) of the shunt reactor. Since the Q factor is around 500 for EHV shunt reactors, the time constant of the discharge process is around 8 min.

If the cable is disconnected from the network and energized again within a couple of minutes, a residual charge remains in the cable line, which can be highly dependent on the time separation between the disconnection and the re-energization. Under this condition, the re-energization overvoltage can exceed the switching impulse withstand voltage (SIWV) when the residual voltage has an opposite sign to the source voltage at the time of re-energization. This is usually an issue for overhead lines since auto-reclose is applied to them. For cables, it is uncommon to apply auto-reclose as they may experience higher overvoltages because of their higher residual voltage. System operators should be aware that they need to wait for about 10 min (perhaps more to be conservative) before re-energizing a cable, though it is not common to re-energize a cable after a failure.

If a shunt reactor is not directly connected to the cable line, the cable line is discharged through inductive VTs. In this case, the discharge process will be completed within several milliseconds. The inductive VTs need to have enough discharge capability for a cable line to be operated (a) without a shunt reactor or (b) if all the shunt reactors are tripped by sequential switching. It takes several hours for the inductive VTs to dissipate heat after dissipating the cable charge. If the inductive VTs are required to dissipate the cable charge twice within several hours, the required discharge capability will be doubled.

3.6 EMTP Simulation Test Cases

PROBLEM 1

3.1. Assume that the sample cable in Section 3.2.4 is buried as a single-phase cable. Find the impedance and admittance matrices for the single-phase example cable using EMTP. Use the Bergeron model and calculate the impedance and admittance matrices at 1 kHz.

3.2. From the impedance and admittance matrices found in (1), find the phase constants for the earth-return mode and the coaxial mode using the voltage transformation matrix \[ A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \].

3.3. Find the propagation velocity for the coaxial mode and calculate the propagation time when the cable length is 12 km.
3.4. Using the cable data created in (1), find the propagation time for a 12-km cable with EMTP and compare it with the propagation time theoretically found in (3); assume that the sheath circuit is solidly grounded with zero-grounding resistance at both the ends.

Solution 1

3.1. \[ Z = \begin{bmatrix} 0.0010 + j0.0111 & 0.0010 + j0.0102 \\ 0.0010 + j0.0102 & 0.0010 + j0.0102 \end{bmatrix} \text{(}\Omega/\text{m}) \]

\[ Y = \begin{bmatrix} j1.5068 \times 10^{-6} & -j1.5068 \times 10^{-6} \\ -j1.5068 \times 10^{-6} & j1.3095 \times 10^{-5} \end{bmatrix} \text{ (mho/m)} \]

3.2. \( \beta_e = 3.4446 \times 10^{-4} \text{ (Np/m)} \): earth-return mode

\( \beta_c = 3.5733 \times 10^{-5} \text{ (Np/m)} \): coaxial mode

3.3. \( c_c = 1.7584 \times 10^8 \text{ (m/s)} \), \( t = 0.0682 \text{ (ms)} \)

3.4. A step voltage of 1.0 kV is applied at one end of the 12-km cable at 0 s. The coaxial mode arrives at the other end at 0.0682 ms. As shown in Figure 3.28, the propagation time found in EMTP exactly matches the time found in (3).

PROBLEM 2

Calculate zero- and positive-sequence currents using EMTP for the sample cable in Section 3.2.4. For this calculation, assume that the lengths of a minor section and a major section are 400 and 1200 m, respectively. As the total length of the cable is 12 km, the cable will have 10 major sections. Grounding resistance at earthing joints should be set to 10 \( \Omega \).
Solution 2

(a) Cross-Bonded Cable

<table>
<thead>
<tr>
<th></th>
<th>Zero-Sequence</th>
<th>Positive-Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude (A)</td>
<td>Angle (deg.)</td>
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<tr>
<td>EMTP simulation</td>
<td>133.8</td>
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<td>Proposed formulas,</td>
<td>124.1</td>
<td>−21.23</td>
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<td>Equations 3.8/3.14</td>
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<td></td>
<td>356.4</td>
<td>−86.35</td>
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<tr>
<td></td>
<td>356.7</td>
<td>−86.79</td>
</tr>
</tbody>
</table>

(b) Solidly Bonded Cable

<table>
<thead>
<tr>
<th></th>
<th>Zero-Sequence</th>
<th>Positive-Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude (A)</td>
<td>Angle (deg.)</td>
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<td>Proposed formulas,</td>
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<td>Equations 3.19,</td>
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<td>3.20/3.26</td>
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</tr>
<tr>
<td></td>
<td>722.7</td>
<td>−49.08</td>
</tr>
</tbody>
</table>

References


13. IEC 62067 ed. 2.0. 2011. Power cables with extruded insulation and their accessories for rated voltages above 150 kV ($U_{\text{m}} = 170$ kV) up to 500 kV ($U_{\text{m}} = 550$ kV)—Test methods and requirements.


