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Chapter 3

The numerical Discontinuous Deformation Analysis (DDA) method: Benchmark tests

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Abstract: This chapter reviews benchmarking studies of 2D and 3D-DDA performed by the rock mechanics group at Ben-Gurion University of the Negev over the past decade. DDA fundamentals are briefly reviewed first, followed by a comprehensive review of verification and validation studies. We conclude by presenting some limitations of 3D-DDA in its current formulation.

1 INTRODUCTION

Discontinuous Deformation Analysis (DDA) has been in use by the professional rock mechanics community for three decades, and has been successful in modeling discontinuous rock masses and masonry structures. DDA is an implicit, discrete element, numerical method proposed in the late 1980s (Shi, 1988, 1993; Shi & Goodman, 1985, 1989), that provides a useful tool for investigating the dynamics of blocky rock masses and systems composed of multiple blocks, such as masonry structures.

In the 1980s Shi (Shi, 1988) proposed the two-dimensional DDA (2D-DDA) as a PhD thesis at UC Berkeley, and later on, the 3D-DDA was published (Shi, 2001). A review of the essentials of DDA is provided by Jing (1998). Reviews of DDA within the scope of other numerical methods used today to solve problems in rock mechanics and rock engineering are provided by Jing (2003), Jing and Hudson (2002) and Jing and Stephansson (2007). MacLaughlin and Doolin (2006) published a comprehensive review of 2D-DDA validations. The 3D-DDA, being a more recent development, has not been verified extensively; some new and useful validations of 3D-DDA are presented in this chapter.

Many research groups have made modifications to the original code developed by Dr. Shi (Shi, 1988, 1993; Shi & Goodman, 1985) in an attempt to better address some of the fundamental issues in DDA.

The contact algorithm has been the subject of many research groups. Lin et al. (1996) modified the original contact model of DDA, which is based on the penalty method, by adopting the Lagrange type approach. Ning et al. (2010) modified the contact algorithm of DDA by adopting the Augmented Lagrangian method. Later on Bao et al. (2014b) introduced another version of the augmented Lagrangian method, which makes use of advantages of both the Lagrangian multiplier method and the penalty method so as not to alter the structure of the system of equations. Bao and Zhao (2010, 2012) have made some enhancements to the vertex to vertex contact. In the 3D-DDA...
front, Beyabanaki et al. (2009) verified the 3D-DDA with the analytical solution for the case of a block on an incline, and modified the edge-to-edge contact constraints by using the augmented Lagrangian method instead of the penalty method. By doing so, they claim to retain the simplicity of the penalty method, and reduce its disadvantages. Ahn and Song (2011) proposed a new contact-definition algorithm for 3D-DDA using a virtually inscribed sphere installed in every contacting vertex, and reported an increase in the efficiency and stability of the code, using their suggested algorithm. Mikola and Sitar (2013) developed a 3D-DDA formulation using an explicit time integration procedure, and a different contact detection algorithm. Wu et al. (2014) addressed the issue of contact-detection algorithm, claimed to be the most computation-consuming stage in 3D-DDA simulations. They use a novel multi-shell cover system, and claim that this method greatly reduces the contact detection volume and iterations. They report improved efficiency when their algorithm is implemented in the 3D-DDA code.

In an attempt to overcome the DDA simply deformable blocks assumption and therefore uniform distribution of stresses within blocks, Shi (1997) developed the Numerical Manifold Method, using superposition of a mathematical cover over the physical mesh of the blocks. Bao and Zhao (2013) have integrated the advantages of both DDA and the finite element methods (FEM), and developed the hybrid nodal based DDA (NDDA), thus improving the accuracy of stress distribution and allowing for crack propagation within blocks. Jiao et al. (2012) developed a two-dimensional contact constitutive model to simulate the fragmentation of jointed rock. Several other research groups have developed higher order DDA codes to address this issue (e.g. Grayeli & Mortazavi, 2006).

Considering friction and cohesion of block interfaces, Zhang et al. (2014a) added an additional contact type determination for the edge-to-edge contact, in order to better address the issue of discontinuities with both cohesion and friction. They verified and validated their improved code, and found it to be accurate. Zhang et al. (2015) replaced the Mohr-Coulomb joint failure criterion in the original 3D-DDA code, which has a constant friction coefficient, with rate-and-state friction laws. They tested their code by modeling slide-hold-slide and velocity stepping tests, and reported that their numerical results compared well with experimental results. Wang et al. (2013), in an attempt to overcome the cohesion treatment in the original DDA code, developed displacement-dependent interface shear strength. They report that landslides, simulated using the modified algorithm, exhibit a two-stage failure pattern: a relatively slow, downward progressive failure stage followed by a rapid, massive run-out failure stage.

Jiao et al. (2007) developed a viscous boundary for DDA based on the standard viscous boundary condition provided in the original DDA formulation, in order to deal with dynamic wave propagation problems. Later on Bao et al. (2012) implemented new viscous boundary conditions to the 2D-DDA, in order to improve the absorbing efficiency. Zhang et al. (2014b) made two modifications to the original DDA code, in order to investigate the seismic response of underground houses. They incorporated viscous boundary conditions, and applied the seismic input as stress input from the bottom of the model. They report that the numerical solution of the modified DDA is close to the theoretical solution. Ning and Zhao (2012b) coupled the superposition boundary algorithm, to obtain a non-reflecting boundary, with the DDA, for wave propagation modeling in infinite media.
Jiang et al. (2012) introduced the softening block approach for the simulation of stage-wise sequential excavations in jointed rock masses. Their proposed modification to the original code eliminates the need to remove the excavated blocks from the calculation model. They present validations for their method, and report of accurate results. Tal et al. (2014) improved the original numerical manifold method capability to model stability of underground openings embedded in discontinuous rock masses by implementing an algorithm which models the excavation sequence during simulations, starting with a domain with no opening at all and progressively adding openings according to the planned construction phases. Later on this modification was implemented in the original DDA code and was applied in stability analysis of deep tunnels excavated in the columnar jointed basalts of the Baihetan hydroelectric project in South West China (Hatzor et al., 2015).

Kim et al. (1999) and Jing et al. (2001) have made a modification where they compute water pressure and seepage through rock mass, this way coupling fluid flow in fractures. More recent implementations of hydro-mechanical coupling in DDA was developed by Chen et al. (2013) and Ben et al. (2011). Koyama et al. (2011) combined the DDA and the finite element method for fluid flow simulation to model the interaction between solid particles movement and fluid flow. Jiao et al. (2015) have made modifications to the original code in order to simulate the hydraulic fracturing process. In their proposed approach, they first perform the calculation of fluid mechanics to obtain seepage pressure near the tips of existing cracks, and then treat the fluid pressure as linearly distributed loads on corresponding block boundaries, by adding these components to the force matrix of the global equilibrium equation. They verified their new approach and found that it effectively simulates hydraulic rock fracturing. Morgan and Aral (2015) have also approached the subject of hydro-mechanical coupling for hydraulic fracturing. They combined the DDA with finite volume fracture network model for simulation of compressible fluid flow in fractures. The authors claim to improve the fluid, contact and coupling components to increase the accuracy of the solution, and report successful verification of their coupled H-M model against analytical and semi-analytical solutions.

Due to the first order displacement function used in DDA, the error, when computing block rotations, may become excessive if the blocks are undergoing large rotations (Ohnishi et al., 1995). Wu et al. (2005) developed a post-contact adjustment method to overcome rotation errors when addressing rock fall problems in the original code. Liu et al. (2012) coupled the 3D-DDA with tetrahedron finite elements to overcome the problem of block expansion under rigid body rotation. They validated their method with a physical model of a wedge, and got a very good agreement.

A model for cable bolt-rock mass interaction was integrated with DDA by Moosavi and Grayeli (2006). Other useful developments and applications of DDA are summarized in a series of ICADD proceedings (International Conference on Analysis of Discontinuous Deformation) published biannually since 1995.

In this chapter illustrative benchmark tests performed by members of the rock mechanics group at Ben-Gurion University of the Negev (BGU) during the past decade are reviewed. These tests could be performed in every development of the DDA code, for verification purposes. We compare all benchmark tests reported here with analytical solutions, some developed at BGU and some adopted from existing publications. The authors do acknowledge other verification studies performed by other research
groups, such as the extensive study of sliding blocks performed at Nanyang Technological University in Singapore (Ning & Zhao, 2012a), slope stability kinematics (MacLaughlin et al., 2001) and analysis of three-hinged beams (Yeung, 1996) performed at U.C. Berkeley, and more, but here only verifications resulting from BGU research are discussed, for brevity.

2 DDA FUNDAMENTALS: A BRIEF REVIEW

DDA considers both statics and dynamics using a time-step marching scheme and an implicit algorithm formulation. The static analysis assumes the velocity of the different block elements is zero at the beginning of each time step, while the dynamic analysis assumes the velocity at the beginning of a time step is inherited from the previous one. The criterion for convergence in DDA is that there will be neither tension nor penetration between the blocks at the end of a time step in the entire block system. These two constraints are applied using a penalty method, where stiff springs are attached to the contacts. Extension or compression of the springs are energy consuming, therefore the minimum energy solution utilized in DDA assures no penetration or tension between the blocks.

In the original DDA code a damping submatrix was not included in the equilibrium equations. There are two ways to introduce damping in the original code: the time step marching scheme introduces algorithmic damping (Doolin & Sitar, 2004), that is determined by the time step size used, and will be briefly discussed later, and kinetic damping. The latter can be applied by assigning a number lower than 1 for the dynamic control parameter: a value of zero means the analysis is static and the velocity is zeroed at the beginning of each time step, a value of unity means the analysis is fully dynamic and the velocity at the beginning of a time step is inherited from the previous one, and any value between zero and one corresponds to the percentage of the velocity that is inherited from one time step to the next. For example, a value of 0.98 corresponds to 2% kinetic damping.

For the sake of brevity, the DDA basic equations will not be reviewed here. The interested reader is encouraged to refer to basic DDA references (Shi, 1988, 1993; Shi & Goodman, 1985).

3 DDA VERIFICATIONS

3.1 Sliding block on an inclined plane

A block sliding on an inclined plane is a classic problem in rock mechanics, as it is a simple and intuitive model for some cases of rock slopes, and has a straightforward analytical solution. The DDA has been verified by several researchers with the analytical solution for a block on an inclined plane. Tsesarsky et al. (2005) and Kamai and Hatzor (2008) have verified the 2D-DDA with the analytical solution for static and dynamic input, where in the latter a horizontal acceleration was added. Ning and Zhao (2012a) have verified the 2D-DDA for a block sliding on an inclined plane under both horizontal and vertical accelerations, and under different mechanisms of loading. In this subsection the analytical solution for a block sliding on an incline is reviewed, and
the results of former verification studies of the 2D-DDA, as well as new verifications of 3D-DDA are presented.

3.1.1 Single face sliding

The model of a block on an inclined plane is presented in Figure 1. The inclination angle of the slope is \( \alpha \), and the friction angle of the sliding interface is \( \phi \). The forces acting on the block are its self-weight \( mg \), the normal from the plane \( N \) and the frictional force at the interface between the block and the plane, \( f \). In the most generalized case, where an external force is applied on the block in the form of a harmonic function (Figure 1), the downslope displacements of the block can be calculated by:

\[
d(t) = \frac{1}{2} g (\sin \alpha - \cos \alpha \tan \phi) t^2 - \frac{A}{\omega^2} \sin(\omega t) (\cos \alpha + \sin \alpha \tan \phi) + \dot{d}_0 t + d_0
\]

where \( g \) is the acceleration of gravity, \( \tan \phi \) is the friction coefficient of the sliding interface, \( A \) and \( \omega \) are the amplitude and angular frequency of the harmonic input acceleration, respectively, \( \dot{d}_0 \) is the initial velocity of the sliding block and \( d_0 \) is its initial displacement.

3.1.1.1 2D-DDA

Kamai (2006) performed a 2D-DDA verification study of a block sliding on an inclined plane subjected to gravity \( (A = 0) \), starting at rest \( (\dot{d}_0 = 0, d_0 = 0) \), for an inclination angle of \( \alpha = 28^\circ \), and different values of friction angle. She obtained a good agreement between the two solutions (Figure 2), demonstrated by the relative numerical error, \( E_N \), defined as:

\[
E_N = \left| \frac{d_A - d_N}{d_A} \right| \cdot 100\%
\]

where \( d_A \) and \( d_N \) are the analytical and numerical displacements, respectively. She found that the relative numerical error increases with increasing friction angle, and was lower than 8\% for \( \phi = 25^\circ \), and at the order of 0.05\% for \( \phi = 5^\circ \).

Kamai and Hatzor (2008) then proceeded to loading the block with a sinusoidal horizontal acceleration, as in Figure 1. In this case, downslope sliding will initiate only when the yield acceleration, \( a_y = g \tan (\phi - \alpha) \), is exceeded, at time
\[ \theta = \sin^{-1}(\tan(\phi - a)g/A) \ast (1/\omega), \]

a model proposed by Newmark (1965) and Goodman and Seed (1966), and today largely referred to as ‘Newmark type’ analysis. In this case, Eq. 1 becomes (assuming that \( d(\theta) = \dot{d}(\theta) = 0 \)):

\[
d(t) = g \left[ (\sin a - \cos a \tan \phi) \left( \frac{1}{2} t^2 - \theta t + \frac{1}{2} \theta^2 \right) \right]
+ \frac{A}{\omega^2} \left[ (\cos a + \sin a \tan \phi) (\omega \cos(\omega t)(t - \theta) - \sin(\omega t) + \sin(\omega t)) \right]
\]

The downslope displacements, \( d(t) \), are calculated while \( a_y \) is exceeded for the first time at \( \theta_1 \), or the block’s velocity is positive. If neither condition is fulfilled, the block is at rest, and will initiate sliding only once \( a_y \) is exceeded again, at \( \theta_2 \), and so on.

Kamai and Hatzor (2008) have used this solution to verify the 2D-DDA. The inclination angle they used for the slope was \( \alpha = 20^\circ \), with different friction angles (for the entire set of results, please refer to their original paper). They obtained an excellent agreement between the analytical and numerical solutions, with relative numerical error lower than 1% for most of the simulation time (Figure 3), with a friction angle of 30°.

3.1.1.2 3D-DDA

(1) One direction of motion

The 3D-DDA mesh for the model of the block on an incline is similar to the one in Figure 1, and is constructed of a triangular prism base block, fixed in space, serving as the incline, and a box as the sliding block.

This verification study of the block on an incline is performed in three steps: first the response of the block when subjected to gravity, starting at rest, is examined, then the block is given initial horizontal velocity, and finally the block is subjected to one-dimensional horizontal sinusoidal acceleration. When subjecting the 3D block to initial velocity and acceleration in one direction only, the problem is basically reduced to a 2D problem.
(a) Block starting at rest
In this step the block is subjected to gravity alone ($A = 0$), starting at rest ($\dot{d}_0 = 0$, $d_0 = 0$). In this case, Eq. 1 becomes:

$$d(t) = \frac{1}{2} g (\sin \alpha - \cos \alpha \tan \phi) t^2$$

The numerical and physical parameters used can be found in (Yagoda-Biran & Hatzor, 2016). The downslope displacement history is compared for friction angle of 20° (remembering the inclination angle of the slope is 45°). Note the excellent agreement between the analytical and numerical solutions (Figure 4a), further demonstrated by the low relative error in Figure 4b: after 0.2 s the numerical error drops to below 1%.

(b) Block starting with initial velocity
The next step of the verification study is applying initial velocity $\dot{d}_0$ to the sliding block. In this case, where no external forces are applied on the block, Eq. 1 becomes:

$$d(t) = \frac{1}{2} g (\sin \alpha - \cos \alpha \tan \phi) t^2 + \dot{d}_0 t$$

An initial velocity of 1 m/s is applied horizontally in the dip direction. The agreement between the analytical and the numerical solution is good (Figure 5a), as demonstrated by the relative numerical error plotted in Figure 5b: less than 1% after 0.5 s of the analysis.
The third step of the verification study is subjecting the block to one-dimensional horizontal sinusoidal acceleration, as in Figure 1. The amplitude and frequency used for the input acceleration are $2 \text{ m/s}^2$ and $1 \text{ Hz}$, respectively.

Here the 'Newmark' type analysis is used as the analytical solution, as in the 2D-DDA verification explained earlier. The friction angle of the interface between the slope and the sliding block is set to $\phi = 50^\circ$, higher than the inclination angle $\alpha = 45^\circ$, so block sliding will initiate only when the yield acceleration is exceeded. In Figure 6a the downslope displacement histories calculated by the Newmark analysis and the

![Graphs showing displacement and relative error over time.](embed)
3D-DDA code are presented. The agreement between the two is good, and can again be expressed in terms of relative error, presented in Figure 6b, which during most of the analysis remains below 3%.

(2) Block sliding on an incline – loading in two directions
Bakun-Mazor et al. (2009, 2012) have derived a semi-analytical 3D-formulation for solving dynamic three dimensional displacements of single and double plane sliding. In their paper, Bakun-Mazor et al. (2009) presented an analytical formulation which they called Vector Analysis (VA), based on the limiting equilibrium equations of vector forces acting on a block on an inclined plane. The dynamic equations of motion of their analytical solution have a discrete nature, therefore the solution is considered semi-analytical. The formulation is explained in details in (Bakun-Mazor et al., 2009, 2012). They compared the 3D-DDA numerical results to the results they obtained with their 3D solution, when subjecting the block to sinusoidal accelerations in the horizontal dip and strike directions, with different amplitudes and frequencies, and obtained a good agreement, where the numerical error in the final position of the block after 6 seconds was approximately 8% (Figure 7).

3.1.2 Double face sliding with 3D-DDA
Double face sliding refers to the classic problem in rock mechanics – the wedge failure. Bakun-Mazor et al. (2009) verified the 3D-DDA with their VA solution for a wedge failure (Figure 8a). The input acceleration to the wedge was a sine function acting parallel to the line of intersection between the boundary planes along which the wedge...
slides. The plunge of the line of intersection was 30° below the horizon, and the friction of the sliding interfaces was set to 20°. They obtained a good agreement between the 3D-DDA and their VA formulation where the relative numerical error remained below 7% for the entire simulation (Figure 8b).

Figure 8 Dynamic sliding of a wedge: comparison between 3D-DDA and VA solutions. (a) The wedge model in the 3D-DDA. (b) Wedge response to one component of horizontal sinusoidal input motion and self-weight. Lower panel presents the relative error. After Bakun-Mazor et al. (2009).
3.2 Failure mode mapping for a block on an inclined plane

A block on an incline, of which sliding failure was reviewed in the previous section, has actually four possible modes: it can stay at rest, it can slide, it can topple, or it can slide and topple simultaneously. The actual failure mode is controlled by three factors, when the block is subjected to gravity: the inclination of the slope $\alpha$, the slenderness of the block $\delta$, and the friction angle of the interface, $\phi$ (see Figure 9a). The boundaries between the modes were modified over the years (Ashby, 1971; Bray & Goodman, 1981; Hoek & Bray, 1981; Yeung, 1991) (see Figure 9b), and recently mapping the failure mode when the block is subjected to an external pseudo-static earthquake inertia force $F$ (see Figure 9a), has been demonstrated (Yagoda-Biran & Hatzor, 2013). Yagoda-Biran and Hatzor (2013) have found that when adding a horizontal pseudo-static force, with its resultant with the weight vector forming an angle $\beta$ with the vertical direction (see Figure 9a), the mode of the block is now controlled by the slenderness of the block $\delta$, the friction angle of the interface $\phi$, and a new angle $\psi = \alpha + \beta$, rather than $\alpha$.

3.2.1 2D-DDA

The first comparison between 2D-DDA and analytically derived failure mode chart for toppling and sliding was performed by Yeung (1991) at U.C. Berkeley, and the results are discussed extensively in his PhD dissertation. Inconsistencies found in his comparison actually led him to develop a new equation for the boundary between toppling and sliding and toppling, turning it from static to dynamic in nature.

In this section, the agreement between the DDA and the mode chart can’t be discussed in terms of relative error, since in this case only the first motion of the block is of interest, so the degree of agreement can be thought of as binary – either the solutions agree, or they don’t.

![Figure 9](image_url)  
*Figure 9  a) Schematics of the block on an incline and the angles controlling its mode. b) An example for the failure mode chart for gravitational loading, at $\phi = 30^\circ$. After Yagoda-Biran & Hatzor (2013).*
3.2.1.1 Comparison between 2D-DDA and modified failure mode chart under pseudo-static loading

Yagoda-Biran and Hatzor (2013) used the 2D-DDA to verify their newly developed failure mode chart when a pseudo-static inertia force is considered. They put forth a set of rules to determine the first motion of the block as obtained with DDA, and compared it to the prediction of their modified mode chart. The slope angle $\alpha$ used was 10° for most of the simulations, and they used a wide range of slenderness and friction. The change in the angle $\psi$ was controlled by changing the magnitude of the applied pseudo-static force. An excellent agreement was obtained between the two solutions, with the DDA returning the modes as predicted by the analytical mode chart for 106 out of the 110 simulations.

3.2.2 3D-DDA

In the case of 3D-DDA, the numerical solution was compared for both static and pseudo-static cases.

3.2.2.1 Gravitational loading

Since the 3D-DDA has not been verified many times in the past, Yagoda-Biran and Hatzor (2013) first verified the 3D-DDA with the previously published failure mode chart as derived by (Ashby, 1971; Bray & Goodman, 1981; Hoek & Bray, 1981; Yeung, 1991), using the same criteria for determining the first motion of the block as used for the 2D comparison. The range for $\alpha$ was between 14 and 50°, $\phi$ was 20° in most of the simulations, and $\delta$ was between 11 and 50°. An excellent agreement was obtained between the two solutions; 49 out of the 51 DDA simulations produced the failure mode predicted by the analytical mode chart.

3.2.2.2 Pseudo-static loading

After verifying the 3D-DDA with the original failure mode chart for gravitational loading, Yagoda-Biran and Hatzor (2013) proceeded with comparing the 3D-DDA and their newly developed mode chart incorporating a pseudo-static force, using the same criteria for determining the first motion of the block as used for the 2D comparison. The slope angle $\alpha$ used was 10° for most of the simulations. The block slenderness range $\delta$ was between 6 and 70°, and the range of input friction angle $\phi$ between 6 and 80°. The change in the angle $\psi$ was controlled by changing the magnitude of the applied pseudo-static force. All 89 simulations performed with the 3D-DDA returned failure modes as predicted by the analytical mode chart.

3.3 Block rotation under dynamic excitation

Makris & Roussos (2000) studied the problem of the dynamic rocking of a free-standing column subjected to a sinusoidal input acceleration and their analytical solution can be reviewed in (Makris & Roussos, 2000; Yagoda-Biran & Hatzor, 2010). The free body diagram for the problem is shown in Figure 10a. The analytical solution assumes that no sliding occurs at the base of the rocking block.
Yagoda-Biran and Hatzor (2010) compared between results from 2D-DDA simulations and the Makris and Roussos (2000) solution. They selected a geometry of a block where $b = 0.2$ and $h = 0.6$ m, and used an acceleration input function of the form $a(t) = a_p \sin(\omega_p t + \psi)$, with changing amplitude $a_p$ and $\omega = 2\pi$ from $t = 0$ to $t = 0.5$ s. For this specific geometry and frequency of motion, the analytical solution shows that the block will not topple with $a_p = 5.43 m/s^2$, but will topple with $a_p = 5.44 m/s^2$.

Yagoda-Biran and Hatzor (2010) found the value of contact spring stiffness (i.e. the optimal penalty parameter) that will give the same results, in terms of stability-failure, in the 2D-DDA, and then compared the rotation time histories of the column calculated by the analytical and DDA solutions. They obtained an excellent agreement between the two solutions, where the relative error drops below 10% after about 0.3 s of simulation, and below 1% after about 0.5 s (Figure 10b). Yagoda-Biran and Hatzor (2010) found that the error grows larger and the DDA deviates from the analytical solution as soon as the first impact between the rocking column and the fixed base occurs. They explained it by the way damping is implemented in the two solutions. While in the analytical solution the motion during impact is energetically damped due to conservation of angular momentum following the constant value of the coefficient of restitution (Makris & Roussos, 2000), in DDA oscillations at contact points are restrained due to inherent algorithmic damping (Doolin & Sitar, 2004; Ohnishi et al., 2005b).

### 3.4 Block response to shaking foundation

In the preceding sections the dynamic response of the blocks was a result of direct dynamic input to the blocks centroid. In this sub-section dynamic loadings will be induced by displacement input to a foundation block, a state that resembles the true loading mechanism during an earthquake.

![Figure 10](image_url)
3.4.1 2D-DDA

Kamai and Hatzor (2008) verified the 2D-DDA with a semi-analytical solution for a block responding to induced displacements at its foundation. The model comprises of three blocks as follows (Figure 11): a stationary base block (0), an intermediate block (1) to which the input displacements are applied, and an overlying very flat block (2) which responds to the induced displacements in Block 1.

Block 1 is subjected to a horizontal displacement input function in the form of a cosine, starting from zero:

\[ d(t) = D \left( 1 - \cos(2\pi ft) \right) \]  

The only force acting on Block 2, other than its weight and the normal from Block 1, is the frictional force, which determines the acceleration of block 2. For full derivation of equations please refer to Kamai and Hatzor (2008). Kamai and Hatzor (2008) defined a set of inequalities and boundary conditions that determine the magnitude and direction of the acceleration of Block 2, as a function of the acceleration of block 1 and the relative velocity between the blocks. Kamai and Hatzor (2008) compared the displacements computed with 2D-DDA and the analytical solution for changing values of friction angle of the interface between blocks 1 and 2, and for changing amplitudes of input displacement. They found that generally the relative numeric error stayed below 5%, and was more sensitive to friction coefficient than amplitude changes (Kamai & Hatzor, 2008). Example of the response of block 2 is presented in Figure 12.

3.4.2 3D-DDA

The verification of the case of a responding block to moving foundation in three dimensions is based on the one-dimensional verification described by Kamai and Hatzor (2008), with the exception that here the displacements, velocities and accelerations are vectors.

3.4.2.1 The semi-analytical solution

The model used for the verification study is presented in Figure 11. Each of the two moving blocks, blocks 1 and 2, has time dependent displacements \( d(t) \), velocities \( \dot{d}(t) \) and accelerations \( \ddot{d}(t) \). The displacement induced to block 1, \( d_1 \), is in the form of a cosine function:

Figure 11 The model used in the DDA verification of block response to induced displacements.
\[ d_1(t) = A \left( 1 - \cos(2\pi f t) \right) \]  

where \( A \) and \( f \) are the amplitude and frequency of motion, respectively. The forces acting on block 2 are its weight, \( m_2g \), the normal from block 1, \( N = m_2g \), and the frictional force between the two blocks, \( \mu m_2g \), where \( \mu \) is the friction coefficient. Newton’s second law of motion yields that the acceleration of block 2 is \( \ddot{d}_2 = \mu g \). Following Kamai and Hatzor (2008), the direction of the frictional force, and therefore of \( \ddot{d}_2 \), is determined by the direction of the relative velocity between the two blocks, \( \dot{d}^* = \dot{d}_1 - \dot{d}_2 \), defined by the unit vector of the relative velocity, \( \hat{d}^* \). When \( |\dot{d}^*| = 0 \), the acceleration of block 2 (\( \ddot{d}_2 \)) is determined by the acceleration of block 1 (\( \ddot{d}_1 \)). When the acceleration of block 1 exceeds the yield acceleration \( \mu^*g \), over which block 2 no longer moves in harmony with block 1, the frictional force direction is determined by the direction of \( \dot{d}^* \), but the magnitude of \( \ddot{d}_2 \) is equal to \( \mu^*g \). This rationale can be formulated as follows:

\[
\text{If } |\dot{d}^*| = 0 \ldots \text{ and } |\ddot{d}_2| \leq \mu^*g \ldots \text{ then } \ddot{d}_2 = \ddot{d}_1
\]

\[
\text{and } |\ddot{d}_2| > \mu^*g \ldots \text{ then } \ddot{d}_2 = (\mu^*g) \cdot \hat{d}_1
\]  

If \( |\dot{d}^*| \neq 0 \ldots \) then \( \ddot{d}_2 = (\mu^*g) \cdot \hat{d}^* \)

This set of conditions and inequalities was applied with a time step of 0.0001 s. Since the analytical solution is calculated numerically, it is actually a semi-analytical solution.

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**Figure 12** Response of Block 2 to displacement input of \( f=1 \) Hz, and amplitude of 0.5 m. Curve – analytical, symbols – DDA. After Kamai & Hatzor (2008).
3.4.2.2 The numerical model

The actual model used for the 3D-DDA is shown in Figure 11. Block 2 was designed to be very flat, so as to avoid rotations during motion. The physical and numerical control parameters used in the verification analyses can be found in (Yagoda-Biran & Hatzor, 2016).

3.4.2.3 One direction of motion

The first step was inducing displacements to block 1 in the $x$-direction (see Figure 11), similar to the work reported by Kamai and Hatzor (2008). This was done with changing amplitudes, frequencies and friction angles, which can be viewed in (Yagoda-Biran & Hatzor, 2016). One example can be seen in Figure 13, where the input motion is of amplitude of 0.5 m, frequency of 2 Hz and friction coefficient of 0.6. The two solutions compare very well (Figure 13a), and the relative numerical error stays below 1% after about 0.3 s into the simulation.

3.4.2.4 Two directions of motion

In the second step of the verification study, displacements were induced to block 1 in the $x$ and $y$ directions (Figure 11), each with different amplitude and frequency. Some results are presented in Figure 14, where the resultant horizontal displacement vs. time is presented in a). Figure 14c is a 3D plot of the $x$ and $y$ displacements vs. time, presented as the vertical axis. Again, the agreement between the 3D-DDA and the analytical solution is good, as expressed by the numerical error in Figure 14b, which remains below 10% for the entire simulation. A deviation between the numerical and analytical solutions is observed with increasing simulation time.
3.2.4.5 Three directions of motion

The third verification step was subjecting block 1 to sinusoidal displacements in all three directions: \( x \), \( y \) and \( z \). Adding input in the vertical direction affects the response of block 2 as it changes the normal force between the two blocks, and therefore the frictional force between them. This in turn changes the acceleration.
of block 2, $d_2$. Applying time-dependent displacements in the $z$ direction is actually equivalent to time-dependent changes in $g$: when block 1 has positive $z$ acceleration ($\ddot{d}_1\hat{k} > 0$), it is added to $g$. When $\ddot{d}_1\hat{k}$ is negative, it is subtracted from $g$. The analytical solution in this case assumes no other effect of the vertical displacement of block 1 on the horizontal displacement of block 2. The induced displacement function is now:

$$d_1(t) = 0.1 \left(1 - \cos(2\pi 2t)\right) \cdot i + 0.1 \left(1 - \cos(2\pi 4t)\right) \cdot j + 0.1 \left(1 - \cos(2\pi t)\right) \cdot \hat{k}$$

Equation (9)

In Figure 15 results of the verification study with three components of induced displacements are presented. Figure 15a presents the resultant horizontal ($x$-$y$ plane) displacement vs. time, for different values of the $k$-normal contact spring stiffness. The black curve is the analytical solution, while the other curves are the 3D-DDA numerical solutions for different values of $k$. The range of contact spring stiffness that best fits the analytical solution is between $1*10^7$ and $1*10^9$ N/m, with stiffness of $k = 1*10^7$ N/m, or 0.0003 $E*L$, being the optimal selection, where $E$ is the Young’s modulus of the block and $L$ is the length of the line across which the contact springs are attached. When considering 3D-DDA, it might be more relevant to compare $k$ to $E*A$, where $A$ is the area across which the contact springs are attached. In this case, $k = 1*10^7$ is $\approx0.0001 E*A$, not much different from $E*L$. Figure 15b demonstrates this with the relative numerical error. Note that for the results obtained with $k = 1*10^7$ N/m, the relative error stays below 3% for the entire analysis, and the error is well below 10% for $k = 1*10^8$ and $1*10^9$ N/m as well.

![Figure 15](image-url)

**Figure 15** a) Comparison between analytical and 3D-DDA solutions. The best fit is obtained with contact spring stiffness of $k = 1*10^7$ N/m, but the overall trend of the analytical solution is maintained for all values of stiffness. b) Relative numerical error for the different solutions presented in a).
3.5 Shear wave propagation

Bao et al. (2014a) tested the ability of the 2D-DDA to correctly simulate wave propagation. They modeled a set of stacked horizontal layers (Figure 16a), generated a shear wave by inducing horizontal displacements at the base, and compared the waveforms at the measurement point in the model (Figure 16a). Because the DDA blocks are linear elastic and un-damped, the analytical amplitude in the tests is the same amplitude as that of the incident wave at the rigid base. After performing sensitivity analysis to the block size and time step size, they managed to successfully preserve the wave form in the DDA model (Figure 16b).

Bao et al. (2014a) also validated the DDA with SHAKE program for site response applications. SHAKE (Lysmer et al., 1972; Schnabel et al., 1972) is a program that analyzes the 1-dimensional response of a stack of linear-elastic layers in the frequency domain. The stack can be composed of several layers with varying properties, and it is subjected to seismic motion through its base. Although in this chapter we review verification studies, we choose to present this validation part, because the SHAKE program has been verified many times, its accuracy is well established for the underlying assumptions and boundary conditions, and validating the DDA with SHAKE for wave propagation applications seems to be an important step. Bao et al. (2014a) generated a stack of 15 layers in 2D-DDA, each with different mechanical properties (Figure 17a), and applied a real earthquake time history at its base. They modeled the same sequence of layers in SHAKE, and compared the spectral amplifications obtained with the two approaches at the top of the layer stack, with respect to its base. A very good agreement between the two methods was obtained (Figure 17b), both for homogeneous and inhomogeneous media, both for frequency and amplitude. Their results suggest the DDA can be used to model wave propagation through discontinuous media, provided that the numerical control parameters are well conditioned.

![Figure 16](image_url) Wave propagation through a stack of layers with DDA. a) DDA model. b) One-cycle sinusoidal incident wave time history used to induce vertical shear wave propagation in the DDA, and wave forms as obtained at the measurement point. After Bao et al. (2014a).
4 DISCUSSION

4.1 Numerical control parameters

The user-defined numerical control parameters have a significant effect on the results of DDA simulations. In this section a discussion of these parameters and suggestions as to optimal selection of them will be made. It is however important to stress that although some general guidance for optimal selection of the parameters is presented here, given the effect they have on the results of the simulation, their calibration should be performed whenever possible, and routine sensitivity analyzes are highly recommended.

4.1.1 Normal contact spring stiffness

The normal contact spring stiffness, \( k \), is the stiffness of the virtual springs assigned at the dynamically formed contacts. In many sensitivity analyzes it was found that the value of \( k \) significantly affects the results of the simulation.

Shi (1996), in his user manual, recommended that as a rule of thumb, \( k = E \times L \), where \( E \) is the Young’s modulus of intact block material and \( L \) is the average block diameter. However, it is sometimes reported that the optimal value does not follow this rule. In their study of wave propagation with DDA, Bao et al. (2014a) found that a \( k \) value lower by 1.5 orders of magnitude than Shi’s rule of thumb is optimal. They suggested that the condition of the interface between the blocks might have an effect on the value selected: a weathered interface might effectively lower the Elastic modulus, therefore lower the optimal stiffness value. This observation is supported by Yagoda-Biran and Hatzor (2010) who modeled a physical problem somewhat similar to the one modeled by Bao et al. (2014a) and concluded that a \( k \) value of about 2 orders of magnitude lower than Shi’s rule of thumb (Shi, 1996) would be optimal. In other cases however, it seems
that the optimal stiffness value even further deviates from Shi’s rule of thumb, such as the case presented in section 3.4.2. In this case, the optimal stiffness that results in the smallest numerical error is 2 to 4 orders of magnitude lower than Shi’s recommendation. In this case however the simulation is in 3D-DDA, while Shi’s recommendations were given as a guide for 2D-DDA, therefore generalization might not be appropriate in this case.

4.1.2 Time step interval

Selecting an appropriate time step interval is an important issue with DDA simulations, and should be a balance between accuracy and computational efficiency. As reviewed in Bao et al. (2014a), various authors proposed different rules of thumb for optimal time step size in relation to the wave period: less than 1% of the primary wave period in finite element simulations of nonlinear sound wave propagation (Kagawa et al., 1992), 5% of the shortest period of incident waves in Newmark time integration scheme (Moser et al., 1999), or smaller than $2/\pi$ of the un-damped period of vibration of the system, in order to avoid bifurcation in the DDA solution (Doolin & Sitar, 2004).

In order to ensure the stability of the numerical solution, the time step interval should be smaller than the fraction of the period that is equal to the ratio between the side length of the element along the direction of wave propagation path, and the wavelength, according to the Courant–Friedrichs–Levy condition (Courant et al., 1967). This condition however does not ensure accuracy. In the next section we discuss how the choice of time step interval is reflected in the systems’ damping.

4.1.3 Damping

In the original DDA code a damping submatrix was not incorporated in the equilibrium equations. Therefore, if the original code is to be used without modifications, damping can be introduced artificially by means of either kinetic damping or algorithmic damping. Kinetic damping is applied when the transferred velocity to the consecutive time step is reduced by some measure. Any value between 0 and 1 of the user defined dynamic control parameter would correspond to the percentage of velocity transferred from one time step to the following, that is, a dynamic parameter of 0.97 corresponds to 3% damping. When studying dynamic deformation of jointed rock slopes, Hatzor et al. (2004) reported that a 2% kinetic damping is required to obtain stable solution with the 2D-DDA version they used at the time. But if true and accurate displacements are required, then no kinetic damping should be introduced at all. This can be done provided that all other numerical control parameters are properly conditioned.

Algorithmic damping (Doolin & Sitar, 2004) is associated with the time integration scheme used for integrating second order systems of equations over time. Numerical damping stabilizes the numerical integration scheme by damping out the unwanted high frequency modes. For the Newmark time integration scheme used in DDA (with $\beta = 0.5$ and $\gamma = 1.0$, see Ohnishi et al., 2005a), it also affects the lower modes and reduces the accuracy of integration scheme to first order. In DDA, the numerical damping that is associated with the time integration scheme increases with increasing time step size. If the time step is small enough, the numerical damping phenomenon is insignificant. Bao et al. (2014a) suggested a way to utilize this time step size dependence of algorithmic damping,
and obtained an equivalent damping ratio by seeking the time step size that will result in exactly the same damping ratio that would have been assumed otherwise in the structural analysis. They inspected the damped oscillations of the free end of a cantilever beam modeled with DDA with different time step intervals, and obtained an equivalent damping ratio, using the algorithmic damping in DDA as a function of time step interval. Then they modeled a stack of horizontal layers in DDA, subjected to earthquake displacements at the foundation, with a time step size of 0.001s, which corresponded in that case to 2.3% damping. They compared the amplification and resonance frequency obtained with the DDA model, to those of an equivalent SHAKE (Lysmer et al., 1972; Schnabel et al., 1972) model with an input of 2.3% damping, and obtained an extremely good agreement between the two methods (see section 3.5).

4.2 Limitations of 3D-DDA

4.2.1 Constructing a 3D-DDA mesh

Modeling three dimensional multi-block structures in 3D-DDA is an elaborate and challenging task. The block cutting code in 3D-DDA does not have a user friendly graphic interface, and does not accept three-dimensional blocks as input, but rather two-dimensional triangles, of which the blocks are constructed. For example, in order to build a rectangular face of a box, two triangles will be required, with three vertices each. Therefore, to form a simple box, one needs to input the vertices for 12 triangles. The accuracy of the input coordinates is of great importance as well: if the coordinates of two adjacent triangles do not exactly coincide, the code will not be able to cut a block. This requires sophisticated pre-processing with suitable software. When modeling problems involving only a few blocks, the process of building the mesh might be tolerable, but when constructing meshes which consist of many blocks (see section 4.2.2) the task becomes difficult and exhausting. It is therefore recommended to use computer aided design (CAD) software to construct a 3D-DDA mesh, and use a function of the CAD software to export the model. In the work presented here in section 4.2.2 we built the model in a CAD software, exported the nodes, and then wrote the model’s nodes to a file readable by the DDA. The scope of this chapter does not allow for a full presentation of the process, but the interested reader is welcome to contact the corresponding author for more information.

4.2.2 The L’Aquila case study

The DDA in its two dimensional formulation has been used several times as a tool to estimate historical seismic hazard (Kamai & Hatzor, 2008; Yagoda-Biran & Hatzor, 2010). While attempting to use the 3D-DDA in a similar manner, we have stumbled upon limitations that suggest the 3D-DDA, at its current formulation, is still not ready for solving reliably dynamic problems involving a large number of blocks. We use the case study of L’Aquila to illustrate this problem.

The city of L’Aquila, the capital of the Abruzzo region, Italy, suffered strong ground motions during the $M_w$ 6.3 earthquake of April 6, 2009. Over 300 people were killed, and many of the buildings in the old city were severely damaged and evacuated. Since there are several strong ground motion accelerographs in the city, this seemed as an
excellent case study to check the validity of the DDA for solving complicated dynamic problems in three dimensions.

We searched the old city of L’Aquila for small, simple buildings that can be easily modeled with the 3D-DDA. We found a small masonry structure that was damaged by the earthquake, but did not collapse (Figure 18). Naturally, the observed damage cannot be modeled correctly with 2D-DDA, and a 3D approach is required. The model of the structure in 3D-DDA was comprised of 197 blocks.

When trying to run forward analyzes with the model, subjecting it to the acceleration time series as recorded in a station located about 500 meters away from the structure, the solution converged only when kinetic damping of at least 3% was used, and a time step size no larger than $10^{-5}$ s. With less kinetic damping and longer time steps a stable solution could not be obtained, probably due to limitations of the contact algorithm in its current form, which does not allow the system to converge when a large number of contacts must be solved in every iteration. The choice of such a small time step inevitably leads to extremely long CPU time, especially when running simulations with long “real” run time of tens of seconds. The use of small time step intervals in the 3D-DDA, smaller than the ones used in similar simulations in 2D-DDA, was observed many times. For example, the 3D simulations in Yagoda-Biran and Hatzor (2013) used a time step size two orders of magnitude smaller than the time step size used in similar 2D-DDA simulations. For the case of the responding block to induced displacements, the time step size used in the 3D-DDA case in section 3.4 is one order of magnitude smaller than the one used in 2D-DDA (Kamai, 2006). For the case of a block on an incline, the time step size used in the 2D-DDA case is 0.002 s (Kamai & Hatzor, 2008), while the 3D-DDA simulation in section 3.1.1.2 used a time step size of 0.0001 s.

Furthermore, we noticed that the displacements of the blocks were several orders of magnitude smaller than expected. These results led us to investigate what effect does the coupling of kinetic damping and small time step has on the numerically obtained

Figure 18 a) The damaged masonry structure in the city of L’Aquila, Italy following the Mw 6.3 earthquake that struck the region, b) snapshot of the corresponding 3D-DDA mesh.
cumulative displacements. We conducted a numerical experiment, where the time dependent displacements of a mass driven by a constant force were computed, with kinetic damping of 3% and different time step sizes. As observed in Figure 19, when using kinetic damping, the cumulative displacement decreases with decreasing time step, an effect also observed in the same experiment performed with the 3D-DDA (see Yagoda-Biran & Hatzor, 2016). This effect was not observed when no kinetic damping was applied: the time step size had no effect on the cumulative displacements. Furthermore, decreasing the kinetic damping by even 2%, down to 1% damping, did not change the results significantly: the displacements were still highly restrained.

It is thus evident that a combination of a very small time step interval and kinetic damping of a small percentage significantly decreases the cumulative displacement during the simulation rendering the numerical results inaccurate and unrealistic. It is also evident that a small increase in kinetic damping coefficient will not make a great difference when very small time steps are used. Ideally, it would be preferable to use zero kinetic damping in dynamic DDA simulations, as the displacements per time step are reduced with increasing time step size anyhow due to the inherent algorithmic damping in DDA.

In cases such as these, where displacements are the desirable output, these limitations of the current version of the 3D-DDA code are not tolerable, and this effect makes the 3D-DDA, in its current form, ill-suited for dynamic simulations of multi-block systems. A completely new contact algorithm has recently been proposed by Shi (2013), but has not yet been implemented in executable codes which are available to us for testing. After the new contact algorithm is implemented 3D-DDA should be tested again for its applicability for multi-block systems and multiple contacts using benchmark tests similar to those presented in this chapter.

Figure 19 Cumulative displacement of a mass subjected to constant force, under 3% kinetic damping and different time steps, as calculated semi-analytically. After Yagoda-Biran & Hatzor (in review).
5 SUMMARY AND CONCLUSIONS

In this chapter the validity of dynamic analysis with 2D and 3D-DDA is verified by reviewing published verification studies. As the numerical discrete element DDA method is becoming more popular in rock mechanics and engineering geology research worldwide, it becomes supremely important to verify the accuracy and applicability of the method before it is accepted and established as a standard analytical approach in the practice. The DDA has been proven to accurately solve problems involving block translations (block on an incline, double face sliding) and rotations (rocking of a free standing column) when the loading is applied at the center of the block, as well as translations when the loading is applied at the foundation (responding block). It has been proven to accurately solve large displacements, as well as wave propagation problems, that involve small displacements, despite the simply deformable blocks assumption and the first order approximation. This accuracy however is highly dependent on an educated selection of the numerical control parameters, first and foremost the penalty parameter otherwise known as the contact spring stiffness, as well as the time step size. A wise selection of the time step size would balance between small computation times and high accuracy. A wise contact spring stiffness selection would ensure accuracy of the solution; the block size becomes an issue primarily when dealing with wave propagation problems.

Naturally, 2D-DDA cannot be used in cases where out-of-plane deformations are expected in the physical problem. In such cases using the 3D-DDA would seem more appropriate although as much as we have experimented with 3D-DDA in its current formulation we have concluded that obtaining a stable solution to dynamic problems involving a large number of blocks is a very challenging task.

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