48
Free-Electron Lasers and Synchrotron Light Sources

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48.1 Introduction

For many years, it has been a goal of optical researchers to produce coherent radiation at wavelengths where existing sources are weak. Free-electron lasers (FELs) are rapidly filling voids in the electromagnetic spectrum left by more conventional sources. FELs are attractive because of their potential to produce both high peak and high average power and because of their wavelength flexibility. When compared to conventional lasers, FELs are neither simple nor inexpensive. FELs can, however, produce intense coherent light at wavelengths where other sources are either weak or non-existent. This makes them the ideal choice when one moves away from the visible region of the spectrum to the far IR, or to the vacuum UV, X-ray and beyond. At present FELs operate over a broad spectrum from millimetre waves to the vacuum ultraviolet and at average power levels from watts to several kilowatts. No single device spans this entire range of wavelength or power. In spite of having the same underlying physical principles, the technology behind mm-wave FELs is quite different from that of UV devices.

The development of synchrotron radiation (SR) light sources has, until recently, been independent of the development of FELs. Therefore, for the most part, FELs and SR sources are treated independently in this chapter. Heretofore, FELs and synchrotron sources have not been in direct competition because they have occupied different regions of the electromagnetic spectrum. X-ray FELs currently proposed will offer direct competition to synchrotron sources (see Sections 48.5 and 48.6). Such X-ray FELs are now often referred to as ‘fourth-generation synchrotron light sources’. Therefore, it is likely that the distinction between FELs and SR sources will become increasingly blurred, particularly at X-ray wavelengths.

48.1.1 History and Basic Principles of FELs

FELs are a marriage of accelerator and laser technology. The essential technical elements are an electron accelerator, a magnetic undulator (or wiggler)1 and photon optics. FEL operation is based on a fundamental radiative mechanism—that of an oscillating electron in a sinusoidal magnetic field of an undulator magnet—which we call spontaneous emission.

1 The terms wiggler and undulator are often used interchangeably in the FEL community; however, they have quite specific meaning in the SR community, see Figure 48.7.
FIGURE 48.1 The FEL radiative process: As the electron beam moves in a magnetic undulator, it radiates light (full line with long undulations). Initially, the electron beam is unstructured on a scale comparable to the wavelength of emission. The electron emitters are randomly phased with respect to the optical wave and so incoherent or spontaneous emission results. As the electrons progress along the wiggler, their interaction with the optical field in the presence of the undulator field results in microbunching of the electrons (broken line). At saturation, the electrons become tightly bunched in clumps spaced one optical wavelength (λ) apart. Once the electrons are bunched, the emission becomes coherent and laser-like. After passage through the undulator, the electrons are separated from the light by a bending magnet (not shown).

(see Figure 48.1). This radiation is enhanced or amplified by the modulation of the electron beam density. In low-gain systems, an optical cavity provides the feedback and the optical mode selection necessary for proper performance; in high-gain FELs, an optical cavity is often unnecessary. Typical undulators in use today are 1–5 m long and have less than 100 magnetic periods, each several centimetres long. Future undulators for X-ray FELs may be as long as 100 m.

The history of FELs can be traced back to microwave tubes, which were the first sources of coherent radiation from free electrons in vacuum. Such early devices relied on slow-wave structures with closely coupled boundary conditions and were limited to long wavelengths. The possibility of producing coherent radiation from electrons bending in a magnetic field without the necessity of boundary structures was first postulated by Schwinger in relation to SR [1]. In the 1950s, Motz [2] performed a series of experiments with a high-energy electron beam passing through an undulator-like device. These experiments produced incoherent emission in the visible. Between 1957 and 1964, Phillips demonstrated an FEL-like microwave tube called the ubitron (for ‘undulating beam interaction’) in which a bunched electron beam in an undulator produced 150 kW of coherent radiation at a wavelength of 5 mm [3]. In 1971, Madey [4] proposed an optical wavelength device, subsequently referred to as an FEL, which included a conventional optical resonator. Madey’s proposal led directly to a series of seminal experiments that demonstrated FEL amplification [5] and oscillation [6] at near-IR wavelengths.

Is the FEL a laser? This question was originally addressed by Motz [7]. There is no doubt that the FEL mechanism is quite different from that of a conventional laser. FEL light output is, however, narrow band, transversely coherent and has well-defined spatial and temporal mode structures. In the optical region of the spectrum, FEL light looks very similar to conventional laser light. Madey originally coined the term free-electron laser to describe a device operating in the optical region with an optical resonator [8]. Today, the use of the term has been extended to include almost any device that produces coherent radiation from free electrons at any wavelength and which does not use closely coupled resonant structures.

48.2 Physics of FELs

In an FEL, electron energy transitions occur in a continuum. Consequently, the discrete level transitions that exist in conventional lasers are not a factor. The decoupling of the FEL mechanism from the constraints of atomic and molecular media allows for a light source that is continuously tunable and potentially capable of both high peak and high average power. Note that in a conventional laser the thermal energy associated with heating of the lasing medium is carried away at acoustic velocities (=10⁷ m s⁻¹), whereas in an FEL the heat is carried away at the speed of the electrons (>10⁸ m s⁻¹). Therefore, if sufficient electron beam power is available, FELs should be capable of extremely high average power operation.

The key step in an FEL is the generation of ‘spontaneous emission’. In an FEL, the ‘lasing medium’ is the electron bunch (10⁸–10¹⁰ electrons typically). Consider a relativistic electron travelling in the z direction encountering a region with a transverse sinusoidal magnetic field \( \mathbf{B} = B_0 \sin (k \cdot x) \hat{z} \), as shown in Figure 48.1. The undulator period (\( \lambda_u = 2\pi/k_y \)) is typically a few cm, and \( B_0 \) is usually in the range of 0.1–0.5 T. In this chapter, we consider only planar undulators, as shown in Figure 48.1. Planar undulators made with permanent magnets or electromagnets are by far the most common variety. The electrons wiggle in the y direction as they move through the field region. In the electron rest-frame, the undulator field is transformed into an intense virtual photon field with orthogonal electric and magnetic fields.

These virtual photons become real photons by Compton scattering off the relativistic electrons and are Doppler shifted...
to short wavelength when viewed in the laboratory frame. In the laboratory frame the real photons have wavelength:

\[ \lambda = \frac{\lambda_0}{2h\gamma} (1 + a_u^2) \]  

(48.1)

where \( h = 1, 2, 3, \ldots \) is the harmonic number, \( a_u = (eB_0/\sqrt{2\pi}mc) \) is known as the rms normalized vector potential of the undulator (\( a_u \) is usually of order unity) and \( \gamma \) is the electron energy divided by the rest energy of the electron. For a 50 MeV electron, \( \gamma \approx 100 \); the light wavelength is a factor of \( 10^4 \) smaller than the undulator period. Most FELs operate at harmonic number \( h = 1 \); however, a number of FELs have operated successfully at the third (\( h = 3 \)) and fifth (\( h = 5 \)) harmonic. A second-harmonic FEL has been demonstrated, but because the spontaneous emission and gain functions are zero on the electron beam axis, lasing was quite difficult to achieve [9].

### 48.2.1 Spontaneous Emission

The spontaneous emission is generally not very intense. A 50 MeV electron beam with a peak current of 100 A in a pulse several picoseconds long will produce a peak spontaneous emission power of the order of 1 W in a typical undulator. The intensity of the spontaneous radiation on axis near the fundamental (\( h = 1 \)) resonant wavelength is given by [10]

\[ \frac{d^2I}{d\omega d\Omega} = \frac{N_e^2 e^2 \gamma^2}{2\pi e c} \frac{a_u^2}{\gamma(1 + a_u^2)} \left( \frac{\sin(\pi N_u \Delta \lambda/\lambda)}{\pi N_u \Delta \lambda/\lambda} \right)^2 F^2[1, \xi]. \]  

(48.2)

where \( F[1, \xi] = J_0(\xi) - J_1(\xi) \), \( J_1 \) is a Bessel function of order \( x \) and \( \xi = a_u^2/2(1 + a_u^2) \).

The bandwidth of the emission (\( \Delta \lambda/\lambda \)) is approximately \( 1/2N_u \). The effective angular aperture is \( \theta = \sqrt{\lambda_0/\lambda N_u} \), and the source radius is \( \omega = \sqrt{\lambda_0 N_u}/4\pi \). The number of photons per electron, per pass through the undulator within the angular aperture and bandwidth is given by

\[ N_p = \frac{\pi}{2} \frac{a_u^2}{(1 + a_u^2)} F^2[1, \xi] \]  

(48.3)

where \( \alpha \) is the fine structure constant. For the case of \( a_u = 0.5 \), \( N_p = 2 \times 10^{-3} \) photons are emitted per electron per pass.

For a bunch of electrons, whose bunch length is much longer than \( \lambda \), the total intensity is proportional to the number of electrons \( N_e \) in a bunch; typically, \( N_e = 10^9 \). If the electrons were grouped into bunches approximately \( \lambda \) apart in the longitudinal direction, the number of photons emitted by the bunch would be proportional to \( N_e^2 \). Such a process results in ‘coherent emission’ of photons.

### 48.2.2 FEL Lasing

The essence of the FEL mechanism is the conversion of spontaneous emission into narrow-band coherent emission through means of a feedback mechanism involving the interaction of the spontaneous photons and the electrons in the presence of the undulator magnetic field. The electrons either gain or lose energy depending on their phase with respect to the optical field. This results in a longitudinal clumping of the electrons. In the typical case where the electron pulse length (\( l \)) is much greater than the optical wavelength, the electron pulse will become modulated into \( l/\lambda \) micropulses spaced approximately \( \lambda \) apart. This modulation results in a narrowing of the spectrum and an increase in the output power, i.e. lasing. In low-gain systems, the optical feedback is provided by a resonator cavity, just as in conventional laser oscillators. In high-gain FELs, self-amplified spontaneous emission (SASE) occurs without the necessity of an optical cavity.

In an FEL the gain in the optical field depends on the peak current and the average optical power output depends on the average current. The quality of the electron beam is critical to FEL performance. For optimum gain, the electron beam must overlap well with the optical mode. Poor electron beam quality will result in poor overlap and reduced FEL performance.

In order to go from spontaneous emission to saturation, a total gain of the order of \( 10^6 \) is often required. Typically, low-gain oscillators can take up to several tens of passes to saturate, while high-gain amplifiers can achieve saturation in a single pass. In all cases, there is an incentive to have a large electron beam brightness, i.e. high peak current and small emittance (see Section 48.3), so as to optimize gain.

The efficiency of FELs depends on the details of the undulator used. For a low-gain FEL with a simple undulator with constant period and constant magnetic field amplitude and \( N_u \) magnetic periods, \( \eta \), the efficiency of conversion of electron energy into light, is approximately \( \eta = 1/(4N_u) \), a number comparable to the fractional width of the spontaneous emission spectrum. Typically, for simple FEL systems, \( \eta = 1\%–2\% \). If the undulator period or magnetic field amplitude is tapered to compensate for the reduction in electron-beam energy as power is extracted from the electrons, then much higher efficiencies can, in principle, be obtained. The present record for FEL efficiency at optical wavelengths is 5\% at a wavelength of 10 \( \mu \)m [11]. In principle, efficiencies up to 20\% appear feasible. The effective system efficiency can be enhanced by energy recovery of the electron beam in storage rings or in recirculating linac systems.

### 48.2.3 FEL Gain and Saturation

We present here a set of formulae that will allow the reader to estimate FEL performance in two particular cases: (i) low-gain oscillators and (ii) high-gain SASE or amplifiers. In both cases, we deal only with the Compton regime where plasma oscillations of the beam are not important. This regime is representative of almost all short-wavelength FELs. When plasma oscillations are important, the term Raman regime is used. The Raman regime is important for long-wavelength FELs driven by low-energy electron beams.

Figure 48.2 shows idealized FEL spontaneous emission and gain functions. The independent variable is \( \Omega = N_e \pi (\omega - h\omega_\nu)/\omega_\nu \), where \( \omega = 2\pi c/\lambda \). The spontaneous emission peaks at the FEL resonant wavelength \( \lambda_0 \) and the width of the spectrum to the first zero is \( 2(hN_e^2) \). The gain function is given by the derivative of the spontaneous emission function. The peak optical
The negative side of the gain function corresponds to a situation where electrons gain energy from the optical field. An FEL operating on the negative side of the gain function is called an inverse FEL (IFEL), and IFELs have been proposed as electron accelerators.

48.2.3.1 Low-gain Oscillators

In a low-gain oscillator, optical power \( P \) builds up through many passes of the electron beam through the undulator. The gain length \( L_G \) for the growth of the optical power is such that \( L_G \gg N_u \lambda_u \) (the undulator length). The small signal power gain per pass is denoted by \( \delta \), with \( \delta \ll 1 \). In the small signal regime where \( P \ll P_{sat} \), where \( P_{sat} \) is the saturation power, we have

\[
P = P_0 (1 + \delta - \Gamma)^s, \tag{48.5}
\]

where \( P_0 \) is the spontaneous emission power and \( \Gamma \) represents the fractional loss of power per pass resulting from coupling and internal cavity losses.

The peak value of \( \delta = \delta_0 \) is given by

\[
\delta_0 = 1.3 \times 10^{-3} \frac{h^2 a_u^2 f}{\gamma (1 + d_u)} I_s N_u^2 F^2[h, \xi] \tag{48.6}
\]

where \( I_s \) is the peak electron current and \( f \) is a unitless quantity called the ‘filling factor’ that varies between 0 and 1 and depends on the degree of overlap between the optical mode and electron beam (with \( f = 1 \) corresponding to perfect overlap); and

\[
F[h, \xi] = J_0(\xi) - J_1(\xi) \quad \text{for } h = 1
\]

\[
= \frac{1}{2} 
\]

\[
J_{(h+1)}(h \xi) - J_{(h+1)}(h \xi) \quad \text{for } h = 3, 5, 7, \ldots \tag{48.7}
\]

The spectral width of the gain function is shown in Figure 48.2.

The expression of \( \delta_0 \) is valid in the case of a perfect electron beam with zero emittance and no energy spread. In practice, however, imperfections in the electron beam, such as finite emittance and energy spread, reduce the gain so that the small-signal gain is given approximately by

\[
\delta_s = \frac{8 \sigma_\gamma}{(1 + 4 N_u a u \epsilon_0 \gamma^2 \lambda_u)} (1 + 27 h^2 N_u^2 \sigma_\gamma^2) \tag{48.8}
\]

where \( \epsilon_0 \) is the rms normalized emittance and \( \sigma_\gamma \) is the rms fractional energy spread of the electron beam. Lasing on the higher harmonics is difficult because of the increased sensitivity of the terms in the denominator of the expression for \( \delta_s \) to \( \epsilon_0 \) and \( \sigma_\gamma \) as the harmonic number \( h \) increases.

In the low-gain regime, the optical mode structure is largely governed by the details of the optical cavity design. Many FELs use linear near-concentric cavities, with the Rayleigh range \( (c_p) \) being chosen to match the electron and optical beam sizes at the centre of the undulator. To maximize the output power, the optimum outcoupling is usually chosen to be approximately one-third of \( \sigma_\gamma \). The maximum saturated power output is given by

\[
P_{sat} = P_{beam} N_u a_u.
\]

48.2.3.2 High-gain FELs

The behaviour of high-gain FELs is quite different from that of low-gain devices. In contrast to the low-gain regime, the gain may be sufficiently high to saturate the FEL in a single pass
through the undulator [13]. The high-gain regime is of particular interest in regions of the spectrum where high reflectivity mirrors do not exist, such as in the X-ray region. The optical power grows exponentially over most of the undulator distance \( z \), except near the beginning and close to saturation. For an ideal beam, in the exponential gain regime at the fundamental wavelength,

\[
P(z) = P_0 \exp(z/L_G) \tag{48.9}
\]

where \( P_0 = \rho^2 mc^3 (\gamma \lambda_e) \) and \( L_G = 0.0462 \omega_e / \rho \) and the parameter

\[
\rho = 5.7 \times 10^{-5} \left( \frac{a_F \lambda_e}{\gamma} \right)^2 \left( \frac{I_e}{\gamma R_e^2} \right)^{1/2} \tag{48.10}
\]

is referred to as the ‘Pierce (or FEL) parameter’, with \( R_e \) the electron beam rms radius. The saturated power is given by \( P_{\text{sat}} = \rho P_{\text{beam}} \). Typically, saturation is achieved after a length approximately equal to 20 \( L_G \). For the FEL to perform as described by the simple 1D equations above the following conditions must be met:

i. \( L_G / z_R \ll 1 \) (little diffraction over a gain length)
ii. \( 4\pi \rho \lambda_e \gamma \ll 1 \) (small electron beam emittance)
iii. \( \sigma / \rho \ll 1 \) (small energy spread).

When these conditions are not met, the gain is reduced and a more complicated set of equations is required [14] to calculate the performance. For short wavelengths, e.g. UV and X-ray, the requirement on the electron-beam emittance is the most difficult to meet (see later).

### 48.3 Technology for FELs

The type of accelerator used to drive an FEL depends very much on the lasing wavelength. Because the wavelength scales inversely as the square of the electron energy, short-wavelength devices tend to be driven by large and sophisticated accelerators, whereas long-wavelength FELs are driven by smaller and simpler accelerators. It is interesting to note that this size scaling is opposite to that of conventional microwave and rf generators, i.e. in an FEL the wavelength of emission scales inversely with the square of the electron energy; therefore, short-wavelength FELs are generally bigger than longer-wavelength devices. Note that the gain of an FEL scales approximately inversely with electron energy.

The type of FEL configuration, e.g. oscillator or amplifier, also scales with wavelength. Table 48.1 gives a summary of accelerator technology in relation to FELs.

At the longest wavelengths, the FEL gain is usually so high that the devices can operate from spontaneous emission to saturation in the amplifier mode (self-amplified spontaneous emission or SASE). At intermediate wavelengths (near-IR, visible and near-UV), the gain is lower; therefore, oscillators with optical cavities dominate. At present, storage-ring-driven FEL oscillators are leading the way to short UV wavelengths. At the shorter wavelengths (<100 nm), mirrors become impractical and so one has to consider linear-accelerator-driven SASE amplifiers with very long undulators designed to achieve saturation in a single pass. Linear accelerators are better suited as drivers for very-short-wavelength FELs rather than storage rings. This is because in a high-energy linear accelerator, higher electron beam peak current and lower beam emittance can be achieved than in a ring of equivalent energy.

At hard X-ray and \( \gamma \)-ray wavelengths, even SASE systems are likely to become impractical. Such photons can be produced by Compton backscattering of FEL-generated photons against the electron beam [15,16].

A measure of electron beam quality is the rms normalized emittance

\[
\epsilon_a = \gamma R_e \theta_e \tag{48.11}
\]

where \( R_e \) is the rms electron beam radius and \( \theta_e \) is the rms electron beam divergence angle. High electron beam brightness \( (B = I_e / \epsilon_a^2) \) corresponds to high peak current and low emittance. One of the goals of accelerator development for FELs is to have \( I_e \) as high as possible while keeping \( \epsilon_a \) as low as possible. In order to work well, the electron and optical beams must overlap effectively. Note that the product of the optical beam radius \( w \) and the emission angle \( \theta_e \) is given by \( w \theta_e = \lambda_e / 4\pi \). Effective overlap of the beams requires \( R_e \theta_e < w \theta_e \), or in terms of the emittance,

\[
\frac{\epsilon_a}{\gamma} < \frac{\lambda_e}{4\pi}. \tag{48.12}
\]

The current state of the art corresponds to \( I_e = 100 \text{ A} \) with \( \epsilon_a = 2 \mu \text{m} \). The requirements of X-ray FELs are such that this low emittance value will have to be maintained at currents of several kA. The generation and transport of an electron

<table>
<thead>
<tr>
<th>Wavelength Range</th>
<th>Accelerator Type</th>
<th>FEL Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Far-IR (mm)</td>
<td>Induction linac, van de Graaff, Pulse-line accelerator</td>
<td>SASE amplifier, oscillator</td>
</tr>
<tr>
<td>Near-IR</td>
<td>rf linac, microtron</td>
<td>Oscillator</td>
</tr>
<tr>
<td>Visible</td>
<td>rf linac, storage ring</td>
<td>Oscillator</td>
</tr>
<tr>
<td>UV &gt; 100 nm</td>
<td>Storage ring, rf linac</td>
<td>Oscillator</td>
</tr>
<tr>
<td>XUV</td>
<td>rf linac, storage ring</td>
<td>Amplifier, harmonic generation</td>
</tr>
<tr>
<td>1 keV–MeV</td>
<td>rf linac, storage ring</td>
<td>SASE (keV), Compton backscatter (keV–MeV)</td>
</tr>
</tbody>
</table>
beam from its source to the undulator involves a considerable amount of manipulation of the electrons. Maintaining the quality of the electron beam (i.e. keeping the emittance low and current high) is the subject of extensive research in the FEL community.

At optical wavelengths, considerable effort has been put into developing compact cost-efficient FELs, and also toward improving the quality of electron beams, particularly in regard to electron sources for rf linacs [17]. The technology of choice, which both enhances the performance and reduces the size of the FEL, is the rf electron gun, operated either in the thermionic mode or in the laser switched mode. Typically FELs require peak currents ($I_e$) in the range of 100 A for low-gain oscillators to several kA for high-gain amplifiers. In the case of low average power devices, increasing the electron beam brightness has the effect of making the accelerator more compact by allowing short-wavelength lasing at lower electron energy than would otherwise be possible, e.g. 370 nm using a 45 MeV electron beam [18].

### 48.4 FEL Applications

Because of their cost and complexity, FELs are most competitive at wavelengths where other sources are weak. In order for FELs to be of continued interest to users, they must deliver a performance that far surpasses conventional sources in brightness, power and wavelength range. At present, most applications-orientated FELs operate in the near- to mid-IR and near-UV regions of the spectrum.

In the following description of some FEL applications, we highlight applications that have great practical utility but which still require work to reach maturity. Any application requiring average optical power in excess of a few tens of watts should be considered speculative at this time. Table 48.2 gives a list of FELs that are operated as user facilities. Further details of these and other operating and proposed FELs can be found elsewhere [19,20].

Medicine is a very rich field for FEL applications [21]. Medical applications generally require rather modest average power levels of the order of several watts.

In the area of laser surgery, there is considerable interest in finding wavelengths at which tissue can be cut cleanly with little collateral damage. Ablation of tissue with high-peak power laser pulses in the IR and UV is of particular interest. There is some preference for the use of IR wavelengths because of the potential reduced risk of photochemical effects (with possible persistent collateral damage or genetic modifications) compared to UV wavelengths.

Studies at the Vanderbilt University FEL have shown that wavelengths near 6.45 $\mu$m, corresponding to the amide II absorption band of proteins, are particularly well suited for

<table>
<thead>
<tr>
<th>Country</th>
<th>Institution</th>
<th>Device Name</th>
<th>Wavelength ($\mu$m)</th>
<th>Micropulse Length (ps)</th>
<th>Peak Power (MW)</th>
<th>Average Power (W)</th>
</tr>
</thead>
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<tr>
<td>China</td>
<td>Institute for High Energy Physics</td>
<td>-</td>
<td>10–16</td>
<td>4</td>
<td>20</td>
<td>2</td>
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<td>France</td>
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<td>SuperACO</td>
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<td>12</td>
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<td></td>
<td>CLIO</td>
<td>3–50</td>
<td>1.5–6</td>
<td>10</td>
<td>9</td>
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<td>Germany</td>
<td>DESY</td>
<td>TESLA-FEL*</td>
<td>(0.0001–0.001)</td>
<td>(0.2)</td>
<td>(37 000)</td>
<td>(210)</td>
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<td>The Netherlands</td>
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<td>FELIX-2</td>
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<td>iFEL</td>
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<td>5–22</td>
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<td>FEL I 2</td>
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<td></td>
<td></td>
<td>FEL I 5</td>
<td>50–100</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>USA</td>
<td>Duke University</td>
<td>OK-4</td>
<td>0.19–0.4</td>
<td>10</td>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mark III</td>
<td>3–10</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
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<td></td>
<td>Jefferson Laboratory</td>
<td>IR Demo</td>
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<td>1–2</td>
<td>10</td>
<td>2000</td>
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<tr>
<td></td>
<td></td>
<td>IR-Upgrade*</td>
<td>(1–15)</td>
<td>(15)</td>
<td>(10 000)</td>
<td>(3000)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>UV-FEL*</td>
<td>(0.3–1)</td>
<td>(4)</td>
<td>(4)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>University of Maryland*</td>
<td>MIRFEL</td>
<td>(10–150)</td>
<td>(4)</td>
<td>(4)</td>
<td>(0.5)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>Stanford University</td>
<td>FIREFLY</td>
<td>19–65</td>
<td>1–5</td>
<td>0.3</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>SCA</td>
<td>3–10</td>
<td>0.7</td>
<td>10</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>University of California, Santa Barbara</td>
<td></td>
<td>150–2000</td>
<td>10*</td>
<td>0.004</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Vanderbilt University</td>
<td>MKIII</td>
<td>2–10</td>
<td>2</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>SLAC*</td>
<td>LCLS-2*</td>
<td>(0.00015–0.0015)</td>
<td>(0.3)</td>
<td>(20 000)</td>
<td>(1.5)</td>
<td></td>
</tr>
<tr>
<td>BNL*</td>
<td>DUVFEL</td>
<td>0.266</td>
<td>2</td>
<td>50</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

The asterisk indicates facilities that are proposed or under construction; numbers in parentheses indicate design goals.

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**Handbook of Laser Technology and Applications**

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soft tissue surgery [22]. Remarkable results have been achieved in ocular and neural tissue, where extremely clear cuts were made with little collateral damage. In some cases, collateral damage extended only to a depth of 10 µm into the tissue from the cut boundary. Human studies are now underway.

The FEL is particularly suited for bulk materials processing because of its tunability for optimum wavelength, potential for scaling to high average power and low cost per unit of light energy. Because of the large investment of capital equipment and operating personnel, FELs are not cost effective for industrial processing at low average power levels. As the average power level increases, however, the cost effectiveness of the FEL as a photon source improves dramatically. Studies have shown that at average power levels in the 100–200 kW range, the cost of UV photons delivered should be less than $0.0002kJ⁻¹ [23].

A general category of material processing involves surface modification. An example is the alteration of the surface characteristics of various fibres to make fabric less shiny, more hydroscopic, softer or more accepting of dyes [24]. Such an application would require approximately 100 kW of 193 nm light per min per 1000 m² of fabric. Other examples include improving the adhesion properties of material to reduce the requirement for glues and enhancing the antimicrobial properties of nylon by converting the amide groups to amines.

In the gas phase, infrared photochemistry by multiple-photon dissociation has been the subject of promising FEL research [25]. The IR excitation process that leads to molecular dissociation occurs by the successive absorption of a few tens of IR photons near a strong IR resonance. The particular features of the FEL make it a very appropriate device as a driver for bond-selective photochemistry. The FEL offers easy tunability to match molecular absorption resonances and the high-peak power necessary for multi-photon processes.

In general, the wavelength range covered by existing FELs is not far removed from that of conventional lasers. In many cases, FEL applications have been extensions of work already begun with other lasers. New opportunities exist at very short wavelengths (i.e. very high photon energies), where the only existing sources are incoherent.

The development of tunable, hard X-ray FELs such as the linac coherent light source (LCLS) at the Stanford Synchrotron Radiation Laboratory and the TESLA FEL facility at the Deutsches Elektronen Synchrotron (DESY) in Germany are commonly being referred to as ‘fourth-generation light sources’ [26]. This contrasts with the previous three generations of incoherent synchrotron light sources. The major advantage of FELs is that they produce a coherent beam with a peak brightness many orders of magnitude higher than that possible from synchrotrons (see Figure 48.12). Consequently, applications of light sources in the X-ray region will undergo an explosion similar to that experienced in the visible region brought on by the invention of the laser.

The proposed X-ray FELS will have the spatial and temporal resolution to observe features on the angstrom length scales of molecular bonds and the sub-picosecond time scales of molecular vibrations. This would allow chemists to directly observe reaction processes such as the photo-dissociation of isolated gas-phase molecules, photochemically induced bond breakage and recombination and structural transformations in photosynthetic processes. Studies of nanoscale dynamics involve the overlap of different time and length scales. Techniques that will be possible include X-ray coherent spectroscopy; X-ray transient grating spectroscopy; and the study of the dynamics of entangled polymers, glassy dynamics and collective mode dynamics in liquids and gases.

Very high-energy (≫10 keV) photons can be generated by Compton backscattering of FEL photons against high-energy electrons. The head-on collision results in a pencil-like beam of hard photons whose energy is closely correlated with scattering angle. The highest energy photons are on the scattering axis, with energy falling monotonically as the scattering angle increases. Such a source is quite different in character from a conventional bremsstrahlung source in which photon energy is only loosely correlated with angle. Though the photon beam is incoherent, the correlation of photon energy with angle allows the collimation of the photon beam into a narrow beam with a small energy spread. Experiments have shown that a quasi-monochromatic beam of multi-MeV γ-rays could be produced by scattering near-UV FEL photons against 500 MeV electrons circulating in a storage ring [16]. Such beams will have many applications [27] in nuclear physics studies [28] and radiographic imaging [29].

### 48.5 Future Directions for FELs

Developments in both high average power and short-wavelength have fuelled a tremendous increase in interest in FELs.

The demonstration of 2 kW average optical power in the near-IR by the Jefferson laboratory group using a superconducting energy recovery linac (Figure 48.3) has opened the door for high average power applications in materials processing. Continued development of high-power FEL technology in the UV should produce considerable interest from industry.

Ultrashort-wavelength FELs will be an important component in the next generation of light sources that will produce coherent, short-pulse radiation at soft and hard X-ray wavelengths. The technical challenges are particularly great in this region where high-reflectivity optics are not available. Developments in linear accelerator and storage ring technology and SR research, however, have made the new frontiers both alluring and reachable [26,30]. An example of a proposed device is the linac coherent light source [26].

![Figure 48.3](https://example.com/figure48.3.png)
is designed to operate from 0.15 to 2.5 nm and will require a 15 GeV electron beam with a peak current of \( I_e > 3000 \) A and a normalized emittance of 1–2 µm feeding a 100-m long undulator. The LCLS will produce an X-ray beam with a peak brightness of \( 10^{33} \) photons \( (s \text{ mm}^2 \text{ mrad}^2 0.1\% \text{ bandwidth})^{-1} \) in sub-picosecond pulses (see Figure 48.12). Such a device would have a peak spectral brightness several orders of magnitude greater than existing synchrotron sources. Another X-ray FEL designed to operate at longer wavelengths than the LCLS is under construction at DESY in Germany [26]. Figure 48.12 shows a comparison of the performance of these proposed FELs and existing synchrotron light sources.

Another spectral region that is ripe for FEL development is the far-IR at wavelengths of several tens to several hundred micrometres. In this region lie many molecular resonances of biological interest that have not yet been investigated. Unlike the X-ray FELs, far-IR devices can be very compact and can be driven by electrons that are a few MeV in energy. It is anticipated that far-IR devices will fit on an optical bench, and will be suitable for individual investigator use. For further details on FELs, the reader is referred to [31–36].

48.6 Synchrotron Light Sources

48.6.1 Introduction

A synchrotron light source typically consists of an electron storage ring which produces both SR in the bending magnets of the ring and wiggler or undulator radiation in the insertion devices installed in the straight sections of the ring. A detailed discussion of the properties of synchrotron, wiggler and undulator radiation will be given in a later section. As an example of a synchrotron light source, the schematic layout of part of the National Synchrotron Light Source at Brookhaven National Laboratory is shown in Figure 48.4.

The key elements of a synchrotron light source are the electron storage ring and the electron injection system:

The ‘electron storage ring’ stores \( 10^{12} \) electrons for 5 h and has the following components:

- dipole magnets which bend electrons in a circular orbit and produce SR;
- quadrupole magnets which provide focusing of the electron beam;
- rf cavity which replenishes the electron energy lost to SR;
- a vacuum system which reduces ring pressure to \( \sim 10^{-7} \) Pa;
- straight sections which are typically \( \sim 5 \) m of empty space to accommodate insertion devices; and
- beamlines providing an optical transport line (\( \sim 5–30 \) m) to take the SR from the ring to the experimental end stations.

The ‘electron injection system’ has the following components:

- electron gun which is the electron source;
- linac which pre-accelerates the electrons to 50–100 MeV; and
- a booster ring which accelerates the electrons to the energy of the storage ring (500–8000 MeV).

48.6.2 Brief History of Synchrotron Light Sources

In the literature one often reads that light sources are classified in terms of ‘generations’ as follows:

- First Generation: SR is obtained parasitically from bending magnets in a high-energy physics ring (circa 1970s);
- Second Generation: dedicated storage rings producing SR from bending magnets with a only a few insertion devices of moderate brightness (circa 1980s);

FIGURE 48.4 Schematic layout of a generic synchrotron light source.
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- Third Generation: dedicated storage rings optimized for many insertion devices yielding very high brightness (circa 1990s); and
- Fourth Generation: an open research topic and possibly not a storage ring at all but an FEL on a linear accelerator (circa 2000s!)

At present, most of the first-generation light sources have been replaced by dedicated storage rings. There are more than 50 second- and third-generation rings in operation around the world with new rings being brought on line each year (see Table 48.3).

The rings in Table 48.3 are grouped into three ranges according to the energy of the electron beams. Using the formulae in the next section for synchrotron and undulator radiation, it can be shown that the rings of a particular energy produce photon beams as follows:

- 0–1.5 GeV rings: infrared, ultraviolet photon beams (less than 2 keV);
- 1.5–3 GeV rings: soft X-ray photon beams (1–10 keV); and
- 6–8 GeV rings: hard X-ray photon beams (10 keV and beyond).

### 48.6.3 Operational Characteristics of Synchrotron Light Sources

Typical stored currents in an electron storage ring are 750 mA for low-energy rings and 200 mA for high-energy rings; this corresponds to the order of $10^{12}$ electrons circulating in the ring. It should be noted that the electrons are not distributed uniformly around the ring; they are formed into circulating bunches with free space in between. The length of the electron bunch depends on the arrangement of the magnets, the energy of the ring and the voltage and frequency of the rf system. Bunch lengths typically range from 1 to 5 cm (rms). The number of electron bunches in a particular ring can range from one to several hundreds depending on the design of the ring and desires of the experimenters.

The operation of a generic ring proceeds as follows. All the hardware (magnets, rf, etc) is powered to the appropriate setpoints. An electron beam is emitted from a thermionic cathode and accelerated in the linear accelerator to low energy (~50–100 MeV). This beam is injected into a booster ring that cycles at a rate of 1–30 Hz and the energy of the electron is raised to the level of that in the storage ring (~500–8000 MeV). The electron beam is ejected from the booster ring into a transport line and injected into the storage ring. The circulating electron beam generates radiation when it is bent in the dipole magnets of the ring or in one of the insertion devices in a straight section of the ring. The electron beam is stored for roughly 5–12 h. The electrons are lost from the ring due to Coulomb collisions among electrons in a bunch and also due to collisions with residual gas molecules in the ring. To prolong the storage time, the vacuum in the storage ring should be very good (~$10^{-7}$ Pa). This operation cycle is simply repeated throughout the life of the ring.

One of the key figures of merit of a synchrotron light source is the emittance of the electron beam. The Holy Grail is to produce a light beam without any measurable effects on the electron beam. Since a zero emittance electron beam is not possible, the desire is to reduce the emittance, such that $\varepsilon_\gamma/\lambda < \lambda/\pi$, the so-called ‘diffraction limited emittance’. Figure 48.5 displays the horizontal emittance of the electron beam versus energy of the electron beam for many existing and planned synchrotron light sources.

The main factors determining the emittance of a light source are the energy of the ring and the layout of the quadrupole and dipole magnets that make up the ring, the so-called ‘lattice’. The lattice is usually constructed from basic cells as displayed in Figure 48.6; the number of cells in the full ring is termed the superperiodicity, $N_s$. The emittance, $\varepsilon_n$, of a ring depends on the energy of the ring and the number of superperiods: $\varepsilon_n \propto E^2/N_s^{3/2}$ [36].

Thus, for a ring of particular energy, one varies the number of cells to obtain the desired emittance. Of course the circumference of the ring increases as the superperiodicity is increased. A ring with an energy of 1 GeV will have a circumference of roughly 75 m, a 3 GeV ring ranges from 170 to 350 m, while a 6–8 GeV ring grows to 1–2 km.

### 48.6.4 Electromagnetic Radiation from Charged Particles

When an electron circulates in an electron storage ring designed as a synchrotron light source, there are three types of radiation that are of interest to the experimentalist who makes use of this light:

- synchrotron or bending magnetic radiation;
- wiggle magnet radiation; or
- undulator magnet radiation.

In Figure 48.7, we give a schematic diagram of the three radiation processes showing the electric field as a function of time and the intensity as a function of frequency for radiation from a single electron. In general, the radiation from the electrons is not coherent so the total intensity is simply given by the sum of the intensity for each electron. In the succeeding sections, we shall discuss the properties of each type of radiation and provide some formulas in ‘practical units’ to facilitate rapid calculations.

#### TABLE 48.3

Approximate Number of Synchrotron Light Source Rings Worldwide as a Function of Ring Energy

<table>
<thead>
<tr>
<th>Region</th>
<th>0–1.5 GeV</th>
<th>1.5–3 GeV</th>
<th>6–8 GeV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asia</td>
<td>13</td>
<td>5</td>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>Europe</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>India</td>
<td>1</td>
<td>1</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Middle East</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>North America</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Russia</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>South America</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
48.6.4.1 Synchrotron or Bending Magnetic Radiation

When an electron moves in a transverse magnetic field, the accelerated electron emits electromagnetic radiation known as synchrotron radiation (SR), as shown schematically in Figure 48.8. SR has a very broad spectrum for low photon energies and falls off exponentially above a critical photon energy, $E_c = \frac{\hbar \omega_c}{3} = \frac{3\hbar c}{\gamma^{3/2}\rho}$, where $\gamma = \frac{E}{m_e c^2} = E [\text{MeV}]/0.511 [37–40]$. $\rho$ is the bending radius of the electron in the dipole magnetic field and is related to the electron energy and the dipole magnetic field, $\rho [\text{m}] = \frac{E [\text{GeV}]}{0.511 B [\text{T}]}$. SR was first experimentally observed at the General Electric Corporation in 1947.

$E_c = \frac{\hbar \omega_c}{3} = \frac{3\hbar c}{\gamma^{3/2}\rho}$, where $\gamma = \frac{E}{m_e c^2} = E [\text{MeV}]/0.511 [37–40]$. $\rho$ is the bending radius of the electron in the dipole magnetic field and is related to the electron energy and the dipole magnetic field, $\rho [\text{m}] = \frac{E [\text{GeV}]}{0.511 B [\text{T}]}$. SR was first experimentally observed at the General Electric Corporation in 1947.

<table>
<thead>
<tr>
<th>Radiation Type</th>
<th>Electric Field(t)</th>
<th>Intensity(ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchrotron</td>
<td>$\tau \propto \frac{1}{\omega}$</td>
<td>$1/\omega$</td>
</tr>
<tr>
<td>Wiggler</td>
<td>$\tau \propto \frac{1+K^2/2}{\omega^2}$</td>
<td>$1/\omega^2$</td>
</tr>
<tr>
<td>Undulator</td>
<td>$\tau \propto \frac{1}{\omega^2}$</td>
<td>$1/\omega^2$</td>
</tr>
</tbody>
</table>

FIGURE 48.5 Horizontal electron beam emittance versus electron energy for several existing and planned synchrotron light sources worldwide.

FIGURE 48.6 Basic building block of a light source lattice: the Chasman–Green double-bend achromat cell.

FIGURE 48.7 Schematic diagram of the three radiation processes of interest in a synchrotron light source. The second column displays the electric field as a function of time, and the third column displays the intensity as a function of frequency.

FIGURE 48.8 Schematic diagram for the emission of synchrotron radiation by a relativistic electron on a curved trajectory.
In practical units, the critical photon energy or wavelength can be written in terms of the electron energy and the dipole magnetic field as

\[ E_c [\text{keV}] = 0.665 B \cdot T \cdot E^2 [\text{GeV}] \]  
(48.13)

\[ \lambda_c \approx 18.64 \frac{B[T]E^2[\text{GeV}]}{[\text{[T]E^2[GeV]}]} \]  
(48.14)

Half of the total SR power is radiated above the critical photon energy and half below. Due to the relativistic motion of the electron, the SR is highly collimated in the forward direction with a characteristic vertical opening angle [37],

\[ \psi(\omega) = \begin{cases} 
\frac{1}{\gamma} \left( \frac{\omega - \omega_c}{\omega_c} \right) & \omega < \omega_c \\
1 & \omega = \omega_c \\
\frac{1}{\gamma} \left( \frac{\omega - \omega_c}{\omega_c} \right)^2 & \omega > \omega_c 
\end{cases} \]  
(48.15)

The spectral and angular distribution of the number of photons (flux) is given by [37–40]

\[ \frac{dN}{d\Omega} = \frac{3\alpha \gamma^6}{4\pi^2} \left( \frac{\omega_c}{\omega} \right)^3 \left( \frac{1}{\gamma^2} + \psi^2 \right)^2 \left[ K^2_{\text{2/3}}(x) + \frac{\psi^2}{1/\gamma^2 + \psi^2} K^2_{\text{1/3}}(x) \right] \frac{I}{e} \frac{\Delta \omega}{\omega} \]  
(48.16)

with units of photons s\(^{-1}\) mrad \(\theta\) \(^{-1}\) and where \(\gamma \equiv \alpha(1 + \gamma^2\psi^2)^{1/2}/2\omega_c\). The first term in the square brackets corresponds to radiation with the electric field vector polarized in the plane of the orbit and the second term to radiation polarized perpendicular to the plane.

In the forward direction (\(\psi = 0\)), the previous equation can be written in practical units as

\[ \frac{dN}{d\Omega} = 1.325 \times 10^{16} E^2 [\text{GeV}] I[A] \frac{I}{e} \frac{\Delta \omega}{\omega} \]  
(48.17)

Figure 48.9 is a plot of the functions \(H_2(y, 0)\) and \(G_1(y)\). Figure 48.10 is a plot of \(dP/d\theta\) as a function of photon energy for radiation from a representative range of storage rings.

The spectral power per horizontal angle but integrated over all vertical angles is given in practical units by

\[ \frac{dP}{d\theta} \frac{\text{mW}}{\text{mrad}} = 8.73 \times 10^4 \frac{E^4 [\text{GeV}]}{[\rho \text{[m]}]} I[A] G_2(y) \frac{I}{e} \frac{\Delta \omega}{\omega} \]  
(48.21)

Finally, the total power radiated by a circulating current \(I\) is given in SI units by

\[ P_r = \frac{4\pi r e \gamma^4 I}{3e} \]  
(48.22)

or in practical units by

\[ P_r [\text{kW}] = \frac{88.5 E^4 [\text{GeV}]}{[\rho \text{[m]}]} I[A]. \]  
(48.23)

In practical units, this can be written as

\[ \frac{dN}{d\theta} = 2.457 \times 10^{16} E [\text{GeV}] I[A] G_1(y) \frac{I}{e} \frac{\Delta \omega}{\omega} \]  
(48.19)

FIGURE 48.9 Plot of \(G_1(y)\) and \(H_2(y, 0)\) versus \(y\).

FIGURE 48.10 SR flux integrated over all vertical angles and for 5 mrad of horizontal angle versus photon energy for several synchrotron light sources of different energies.
48.6.4.2 Insertion Devices

To enhance the performance of synchrotron light sources beyond what is available from the dipole magnet radiation, so-called insertion devices are used in the straight sections of the electron storage ring. As mentioned in Section 48.2, an insertion device is characterized by a periodic magnetic field \( B[x] = B_0 \sin[2\pi x/\lambda_0] \), and it is constructed from alternating polarity permanent magnets as shown in Figure 48.11. Although FELs do exist in a few storage rings, the radiation from insertion devices in synchrotron light sources is simply spontaneous emission.

The figure of merit of an insertion device is the ‘K parameter’ given by [40],

\[
K = \frac{eB}{mc_0} = 0.934B[T]\lambda_0[cm]. \quad (48.24)
\]

Note that the K parameter is simply related to the rms normalized vector potential introduced in Section 48.2 as \( a_u = K/\sqrt{2} \).

In the literature on synchrotron light sources, ‘K’ is the mostly widely used notation; however, we shall use ‘\( a_u \)’ to make the notation consistent throughout this chapter.

For \( a_u \leq \sqrt{2} \), the insertion device is termed an undulator, and for \( a_u > \sqrt{2} \), the device is termed a wiggler. Note that \( a_u = 2 \) is not a threshold: there is a continuous transition from an undulator to a wiggler. Schematically, the difference between an undulator and wiggler is shown in Figure 48.7, where the electric field as a function of time and the intensity as a function of frequency are given.

Wiggler Radiation: For a wiggler, the electric field as a function of time is simply a periodic train of pulses similar to those of ordinary SR. In the frequency domain, the intensity is then given by a large number of lines with an overall envelope given by ordinary SR but increased by twice the number of periods in the insertion device. This is the key advantage of a wiggler. By wiggling, the particle back and forth many times one can greatly enhance the amount of flux compared to that available from, say, 5 mrad of arc in a dipole magnet. Another use of a wiggler is to shift the critical photon energy to higher values from, say, 5 mrad of arc in a dipole magnet. One additional feature of an undulator is that the total power radiated in a wiggler also applies for an undulator. Thus, the number of lines in the frequency domain increases but for \( a_u \leq \sqrt{2} \) the spectrum is still characterized by a few distinct lines. The narrow line structure arises from the interference between light emitted in successive periods. The very high intensity in a narrow spectral bandwidth is the signature feature of an undulator source. The formula given for the total power radiated in a wiggler also applies for an undulator. Thus, one additional feature of an undulator is that the total power incident on the beamline optical elements is greatly reduced as compared to a wiggler since \( P \propto a_u^2 \).

Radiation in the forward direction occurs at odd harmonics of the frequency \( \omega \) of Equation (48.27),

\[
\omega_h = \frac{2hc\kappa \gamma^2}{(1 + a_u^2)} \quad (48.28)
\]

with a linewidth of

\[
\frac{\Delta \omega_h}{\omega_h} = \frac{1}{hN_u}. \quad (48.29)
\]

In contrast to most FELs, the higher harmonics of undulator and wiggler radiation are widely used in synchrotron light sources. For completeness, we generalize the expression for the spectral angular intensity in the forward direction given in Section 48.2.1 to include higher ‘odd’ harmonics and an electron beam with a current \( I \). The intensity in units of photons s\(^{-1}\) sr\(^{-1}\) is then given by

\[
\left. \frac{dN_h}{d\Omega_{h,o}} \right|_{\omega=\omega_h} = \alpha N_u^2 \frac{I}{e} \frac{\Delta \omega}{\omega} \frac{2h^2 a_u^2}{\left(1 + a_u^2\right)^2} F^2 \left[ h, \xi \right] \sin^2 \left( x_h/2 \right) \quad (48.30)
\]

with \( I \) in amperes, \( h = 1, 3, 5, \ldots, x_h = 2\pi hN_u (\omega - \omega_h/\omega) \) and \( F[h, \xi] \) was given in Section 48.2.3.1.

Again generalizing the results in Section 48.2.3, the most useful part of the radiation is confined to a central cone about the forward direction within an angular divergence given by [40],

\[
\sigma' = \frac{1}{\gamma} \left[ \frac{(1 + a_u^2)^{3/2}}{2hN_u} \right] = \left[ \frac{\lambda_h}{L} \right]^{3/2} \quad (48.31)
\]

Using this definition of the radiation opening angle, the spectral and angular distribution in the forward direction at \( \omega = \omega_h \) can be rewritten as [40],

\[
\left. \frac{dN_h}{d\Omega_{h,o=\omega_h}} \right|_{2\pi a_u^2} = \frac{N_h}{2\pi \sigma'^2} \quad (48.32)
\]
where

\[ N_h = \pi \alpha N_e \frac{L}{e} \frac{\Delta \omega}{\omega} \left( \frac{2 \hbar}{\omega} \right) \left( 1 + \frac{\hbar^2}{a_e^2} \right)^2 F^2(h, \xi) \]  

(48.33)

is the intensity of photons in the central cone. It is interesting to note that this quantity does not depend explicitly on the energy of the electron!

A critical figure of merit for an undulator source is the on-axis peak brightness of the undulator radiation at \( \omega = \omega_h \) defined as \[ B_h = \frac{N_h}{4\pi^2 \Sigma \Sigma'} h \]  

(48.34)

where \( \Sigma_{\alpha,\beta} = \sqrt{\epsilon_{\alpha,\beta} + \lambda L/(4\pi)^2} \) is the effective rms beam size of the combined electron beam \( \sigma_{\alpha,\beta} = \sqrt{\epsilon_{\alpha,\beta}, \beta_{\alpha,\beta}} \) and photon beam \( \sigma_{\alpha,\beta}^h = \sqrt{\lambda L/(4\pi)^2} \) and, similarly, for the rms angular divergence \( \sigma_{\alpha,\beta}' = \sqrt{\epsilon_{\alpha,\beta} + \sigma_{\alpha,\beta}^2} \).

Figure 48.12 displays the average and peak brightness of several synchrotron light sources and FELs.

### 48.7 Future Directions for Synchrotron Light Sources

With more than 50 rings in existence and more under construction and in the planning stages, storage-ring-based synchrotron light sources will continue to serve the largest user base for decades to come. For high average brightness, from the infrared portion of the spectrum to the hard X-ray regime, with pulse-lengths in the range of 50–500 ps, electron storage rings are the source of choice at present. If higher peak brightness and/or sub picosecond pulses are required then the source of choice is an FEL.

It must be noted that the underlying physics of electron storage rings means it will be difficult, if not impossible, to continue increasing the brightness significantly as the rings are simply becoming too large [41,42]. One should note a paper entitled ‘Towards the Ultimate Storage Ring-Based Light Source’ [43]. New technologies will be needed to continue the trend of ever increasing brightness.

A candidate for a future light source that is beginning to appear on the drawing boards of machine designers is the so-called energy recovery linac (ERL), shown schematically in Figure 48.3 [44–49]. This device uses a superconducting linear accelerator and a pair of half arcs to return the accelerated beam back through the linac with a 180° phase shift to recover the energy in a bunch before the spent beam is dumped at an energy of the order of 10 MeV. At present, the ERL at the Thomas Jefferson National Accelerator Facility is the only machine in operation. It has an energy of 50 MeV, a circulating current of 5 mA, and it also contains a high average power infrared FEL [50]. Machines of higher energy (0.6–6 GeV) and average currents similar to that in ring (100 mA) are being explored for feasibility and should be possible in the next decade.

Potential advantages of an ERL-based light source are as follows [46]:

- diffraction limited electrons beams for 10 Å and below;

![FIGURE 48.12 Average and peak brightness versus photon energy for several synchrotron light sources and FELs. (Courtesy of J Galayda/SLAC.)](image-url)
• average brightness exceeding that of existing storage rings;
• sub-picosecond electron bunches;
• round electron beams;
• virtual ‘top off’ operation; and
• SC linac can serve as an FEL driver.

REFERENCES