Solitons and Dissipative Solitons for Ultrafast Lasers

Accessed on: 28 Dec 2023

Please scroll down for document

Full terms and conditions of use: https://www.routledgehandbooks.com/legal-notices/terms

This Document PDF may be used for research, teaching and private study purposes. Any substantial or systematic reproductions, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The publisher shall not be liable for an loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.
39

Solitons and Dissipative Solitons for Ultrafast Lasers

Ph GRELU

CONTENTS

39.1 Introduction ................................................................. 521
39.2 Passive Mode Locking of Fibre Lasers .......................... 522
  39.2.1 Saturable Absorber Materials ................................ 522
  39.2.2 Virtual Saturable Absorbers .................................... 523
  39.2.3 Non-linear Loop Mirrors .......................................... 523
  39.2.4 Figure-8 Lasers and Variants .................................... 523
  39.2.5 Non-linear Polarization Evolution Mode Locking ........ 524
39.3 The Soliton Laser: Advantages and Limitations .............. 525
  39.3.1 Soliton Energy .................................................... 525
  39.3.2 Dispersive Wave Radiation ..................................... 526
  39.3.3 Multi-Pulsing Instability ......................................... 527
39.4 The Concept of Dissipative Solitons ............................... 527
  39.4.1 Beyond Conventional Solitons .................................. 527
  39.4.2 The Dissipative Soliton Paradigm ............................ 527
  39.4.3 Dissipative Solitons in the Normal Propagation Regime . 528
  39.4.4 Dissipative Soliton and Frequency Chirping ............... 528
  39.4.5 Dissipative Soliton Molecules .................................. 528
39.5 Applications of Dissipative Solitons .............................. 529
  39.5.1 The Quest of High-Energy Pulses from Laser Oscillators . 529
  39.5.2 Innovative Chirped-Pulse Laser Architectures ............ 529
  39.5.3 Peculiar Laser Pulses ............................................ 530
39.6 Prospects ................................................................. 530
References .................................................................................. 531
Further Reading ........................................................................... 533

39.1 Introduction

From the user point of view, ultrafast lasers can be simply defined as lasers generating pulses whose temporal profile cannot be easily monitored in real time. Namely, when the optical pulses are shorter than the electronic response time of the direct optical detection scheme based on a fast photodiode connected to an oscilloscope. Owing to the tremendous progress in fast electronics during the past two decades, this practical definition has been shifting the ultrashort pulse duration border from, typically, below 100 to below 10 ps. But from the laser engineering point of view, ultrafast lasers are laser cavities where dispersive and non-linear effects do affect intracavity propagation significantly, owing to, respectively, the large optical bandwidth and the high peak power associated with ultrashort pulses. In this respect, there will be large quantitative differences according to the laser architecture. In bulk lasers, such as in the celebrated titanium–sapphire laser, the dispersive and non-linear effects accumulate every cavity roundtrip through, typically, a few-millimetre-long dense gain medium: they will become a significant issue to generate subpicosecond pulses. In contrast, fibre lasers are usually several metres long, so that pulses of tens of picoseconds can experience major dispersive and non-linear effects during a single cavity roundtrip.

The Soliton Laser. The concept of an optical soliton, originated in the early 70s (Zakharov and Shabat 1972, Hasegawa and Tappert 1973) provides an elegant solution to the handling of both chromatic dispersion and Kerr non-linearity effects. The optical Kerr effect produces a self-phase modulation (SPM) of the optical pulse. Alone, SPM alters the optical spectrum, generating longer (resp. shorter) wavelengths at the pulse leading (resp. trailing) edge. A precise balance between anomalous dispersion and SPM allows the fundamental optical soliton to maintain a chirp-free temporal waveform of minimal duration while it propagates. In 1980, the optical soliton propagation regime was evidenced experimentally in a passive optical fibre link (Mollenauer et al. 1980). Four years
later, the soliton pulse shaping effect was introduced inside a mode-locked laser cavity, by coupling a length of optical fibre to a bulk colour-centre laser cavity: the so-called soliton laser had been invented (Mollenauer et al. 1984). This seminal work also predicted that, when a suitable fibre gain medium would become available, the soliton laser would probably take the form of a single loop of fibre closed upon itself. The simplicity and low cost of such devices would make them most attractive. Since then, fibre laser technology has been developed at a tremendous rate, whereas the soliton laser concept has been considered as a safe guideline for ultrafast laser engineering. Nevertheless, in practice, laser cavities may be quite far from presenting the ideal conditions for the implementation of the soliton laser concept.

Implementation Issues. First, the laser may operate at wavelengths where the propagation medium features normal dispersion. As a matter of fact, normal dispersion is typical of most bulk materials in the visible and near infrared optical domains, such as silica for wavelengths shorter than 1.27 µm. At first sight, it seems that such a laser cavity would hardly support ultrashort pulses, due to the lack of balance between non-linear and dispersive effects. Indeed, the longer (resp. shorter) wavelengths at the pulse leading (resp. trailing) edge generated by SPM, together with the faster propagation of the longer wavelengths in normal dispersion, entails an accentuated spreading of the temporal wave packet. Therefore, dispersion compensation has become the standard procedure, through the engineering of adequate amount of geometric dispersion, which can take the form of gratings, prisms or waveguides that can be incorporated into the laser cavity. In bulk lasers, dispersion management has culminated in the design of chirped multilayer dielectric mirrors allowing the direct generation of few-cycle optical pulses out of the laser oscillator. Dispersion management in fibre lasers displays an even greater diversity. As it will be shown in the following sections, the cavity dispersion map determines the generation of specific ultrafast internal dynamics that unfold within the cavity roundtrip. Understanding the variety of these ultrafast dynamics requires to go well beyond the conventional soliton concept, calling in particular for the broader conceptual framework of dissipative solitons.

Second, in addition to the chromatic dispersion and the Kerr non-linearity, an ultrafast laser includes other physical effects that contribute to the pulse shaping. Besides Kerr non-linearity, passive mode locking requires a fast-saturable absorber. The gain medium features saturation, as well as wavelength dependence that contributes to an overall spectral filtering effect. Therefore, non-linear gain and loss are always acting. Their importance over shaping the pulse dynamics depends on the laser architecture, and associated cavity parameters. Dissipative non-linear effects become significant in lasers having relatively large gain and loss contributions during a roundtrip pulse propagation. In such situation, which is typical of the majority of ultrafast fibre laser architectures, it is unlikely that the pulse dynamics will merely follow the conventional soliton dynamics, defined by the simple balance between chromatic dispersion and Kerr non-linearity. Nevertheless, what can be first seen as a difficult challenge turned out to be a fantastic opportunity to test original laser cavity architectures supporting new types of solitary waves in the dissipative cavity regime, namely, a variety of dissipative optical solitons. Such exploration has been particularly prominent since the early 2000s.

The Dissipative Soliton. Dissipative solitons are localized formations of an electromagnetic field that are balanced through an energy exchange with the environment in presence of non-linearity, dispersion and/or diffraction (Grelu and Akhmediev 2012). Their growing use in the area of passively mode-locked lasers is remarkable: the concept of a dissipative soliton provides an excellent framework for understanding complex pulse dynamics and stimulates innovative cavity designs. Reciprocally, the field of mode-locked lasers serves as an ideal playground for testing the concept of dissipative solitons and revealing their unusual dynamics.

In the sections that follow, the focus will be on the ultrafast soliton and dissipative soliton dynamics of fibre lasers that employ a passive saturable absorber mechanism. Therefore, Section 39.2 provides a brief technical overview of the passive mode-locking techniques adapted to the fibre laser arena. Then, Section 39.3 discusses the soliton laser advantages and limitations. The conceptual framework of dissipative solitons is presented in Section 39.4, with important applications for ultrashort lasers developed in Section 39.5. To conclude, a few remarkable prospects are presented in Section 39.6.

39.2 Passive Mode Locking of Fibre Lasers

Regardless for the moment of the fibre cavity dispersion map and the type of ultrafast dynamics that unfolds, it is instructive to scan the popular fibre laser mode-locking techniques, which intend to follow the general philosophy of the fibre laser platform: compactness, fibre integration, low-cost and reliability. There are two major options: to resort on the fast-saturable absorption of an engineered localized material that will be inserted in the cavity, or to employ non-linear interferences based on the optical Kerr effect that accumulates along with propagation in the optical fibres.

39.2.1 Saturable Absorber Materials

There has been a considerable evolution of saturable absorber materials during the past two decades. First, the design of semiconductor saturable absorber mirrors based on a Bragg multilayered structure, or SESAM, has rapidly come to maturity (Keller 2003, Okhotnikov et al. 2004). A SESAM is engineered for a specific wavelength range and, in its fibre-integrated version, for a specific laser fluence. Its fabrication requires a relatively complex and expensive clean-room manufacturing process. The SESAM has become one component of choice for commercial mode-locked fibre lasers produced in series. As alternatives to SESAM, there has been a surge of investigation of the saturable absorber properties of one and two-dimensional (2D) materials. Indeed, several such materials demonstrated an ultrafast relaxation time, a wideband operation, and a relatively large damage threshold. Among them, single-walled carbon nanotubes (Set al. 2004) and graphene (Sun al. 2010) have become quite popular for fibre
laser mode locking. In particular, as a Dirac gapless 2D material, graphene displays an ultrawideband optical absorption, suitable to cover the major wavelengths of interest in current rare-earth-doped fibre lasers, namely the 1 µm (ytterbium), 1.5 µm (erbium) and 2 µm (thulium, holmium) emission bands. For their integration, 2D materials can be simply deposited on a fibre ferrule and sandwiched between two connectors, though the contrast of saturable absorption will be relatively low, of the order of a few per cent. A 2D material can also be integrated as a coating of a tapered optical fibre, or as a planar film deposited onto a side-polished D-shaped fibre: in these cases, a significant fraction of the evanescent optical field interacts with the saturable absorber material over a properly designed length. Other saturable absorber materials recently investigated include topological insulators (Jung et al. 2014) and black phosphorus (Sotor et al. 2015).

39.2.2 Virtual Saturable Absorbers

Until now, no single engineered material has shown to combine the large non-linear absorption contrast of a SESAM with the wideband operation, ultrashort relaxation time of graphene, together with a large damage threshold. However, the fibre laser platform naturally presents several options to build a laser cavity architecture where such efficient saturable absorption becomes readily available for mode locking. These options resort to the use of the distributed third-order Kerr non-linearity to create non-linear interferences that affect the overall cavity losses, a strategy initially called additive-pulse mode locking (APM) (Ippen et al. 1989). To circumvent the difficulty in stabilizing interferometric devices in general, two self-consistent fibre cavity architectures have become popular among the fibre laser physicists. The first one includes a non-linear asymmetric Sagnac interferometer—or one of its variants. The second one consists in a unidirectional fibre ring laser wherein non-linear polarization evolution takes place during field propagation. Based on the dispersive Kerr effect, there will not be a real saturable absorption effect within the material, this is why APM is also said to employ a virtual, or artificial saturable absorber effect, in contrast to the real saturable absorber effect of the materials introduced in the previous paragraph. The APM strategy takes advantage of the quasi-instantaneous response time of the Kerr effect in glass materials (in the femtosecond range), as well as its ultrabroadband applicability (for silica, from the visible to the near infrared) and very large intrinsic damage threshold. As a drawback, the transfer function associated with the related artificial saturable absorber effect will not be a monotonous function of the optical intensity, since resulting from interferences.

39.2.3 Non-linear Loop Mirrors

The non-linear asymmetric Sagnac interferometer is better known under the acronym of NOLM, which stands for non-linear optical loop mirror, following its initial proposition as a fibre-integrated all-optical switch (Doran and Wood 1988). Whereas the original Sagnac interferometer is symmetric so that it acts as a mirror—or as an inertial rotation sensor from its inception (Sagnac 1913), by breaking the optical intensity symmetry between the two counterpropagating waves, these two waves will accumulate different non-linear phase shifts. That phase shift difference dictates the outcome of the wave interference that takes place when the two waves recombine after propagation through the loop. Therefore, the NOLM output features a transmission that oscillates as a function of the input optical intensity, namely, a sinusoidal non-linear transfer function. To break the symmetry between the counterpropagating beams, there are several possibilities. In the original proposition (Doran and Wood 1988), the NOLM is based on an asymmetric coupler (Figure 39.1a). It is also possible to induce different non-linear phase shifts for counterpropagating beams by inserting a segment of highly-dispersive fibre asymmetrically, namely, to make a dispersion-imbalance NOLM, or DI-NOLM (Figure 39.1b) (Wong et al. 1997). Another alternative is to insert an optical amplifier within the loop, followed by a length of passive fibre: this geometrical gain asymmetry also leads to asymmetric non-linear phase shifts. Indeed, whereas one direction of propagation will first meet the amplifier, enhancing the non-linear phase shift in the subsequent passive fibre link, the counterpropagating wave will first propagate through the passive fibre at a lower optical intensity before meeting the amplifier at the end of the loop, see Figure 39.1c. Such active NOLM option has been termed Non-linear Amplifying Loop Mirror, or NALM (Fermann et al. 1990). In the NALM, the non-linear transfer function of the loop mirror can be adjusted by varying the gain, which provides one useful additional degree of freedom to control the non-linear transfer function and to adjust the mode-locking operation of the NALM laser (Nakazawa et al. 1991).

39.2.4 Figure-8 Lasers and Variants

The so-called figure-of-eight laser (F8L) represents the typical cavity architecture that incorporates a non-linear loop mirror within a fibre ring laser, see Figure 39.1d. One important issue of the F8L is that it does not easily lead to a self-starting mode-locked operation. This can be understood by noting that the non-linear transmission of the NOLM or the NALM part grows as a sinusoidal function of the optical intensity, namely with a flat initial slope that will not easily enhance the higher field fluctuations, up to some extent. Such issue can be circumvented by using standard—i.e. non-polarization-maintaining—optical fibres and incorporating polarization controllers within the loop: this introduces a tunable phase bias, which helps promoting the self-starting mode-locked operation. Nevertheless, using long lengths of standard fibres in a fibre laser cavity will make the laser operation quite sensitive to environmental perturbations (temperature drift, mechanical noise), so that a preferred option is nowadays to design polarization-maintaining (PM) F8Ls that include a flexible control of the effective saturable absorber. For instance, this can be achieved by inserting a second gain medium in the cavity (Runge et al. 2014), or by using a non-reciprocal phase bias. The latter can be obtained through a combination of Faraday rotators and retarding waveplates (Jiang et al. 2016). With a complete control of the phase bias range, it is possible to make a virtual saturable absorber from either the transmission or the reflection output port of the non-linear loop mirror. This has
led to the development of a compact, environmentally stable, reflection variant of the PM-fibre loop-mirror laser cavity, whose architecture has been dubbed *figure-9 laser* (Hänsel et al. 2017).

### 39.2.5 Non-linear Polarization Evolution Mode Locking

Alternatively, a fibre ring laser is a simpler laser architecture, as a mirrorless serial loop of optical fibres and components. Endowed with an optical isolator, it becomes a unidirectional fibre ring cavity, where the APM mechanism can be readily implemented by using the polarization degree of freedom of the optical field that propagates through standard (non-PM) single-mode optical fibres, as sketched in Figure 39.1.e. Since 1964, the non-linear coupling between orthogonal polarization components in an isotropic Kerr medium is known to entail the rotation of an elliptic polarization input (Maker et al. 1964). Such non-linear polarization rotation (NPR) effect is proportional to the difference of the polarization components intensities in the circular polarization basis and to the propagation distance. By discriminating the amount of polarization rotation with a polarizing element, a non-linear transmission function is obtained. The latter can be conveniently adjusted by orienting retarding waveplates placed before the polarizer. Therefore, the implementation of an NPR-based virtual saturable absorber within a fibre ring laser appears to be fairly simple. The intracavity waveplates constitute the practical degrees of freedom used to shape up the required non-linear transfer function enabling the self-starting mode locking of the laser cavity. The main non-linear transmission features, namely contrast, initial slope, and peak intensity can be varied widely, allowing to explore a large range of pulse dynamics within a given fibre ring laser setup. For these practical reasons, NPR has become one of the most popular mode-locking techniques among fibre laser physicists, since its experimental implementation in the early 90s (Matsas et al. 1992). NPR operates ideally within an isotropic Kerr medium, which is not exactly the case of standard optical fibres, which have a small amount of random birefringence. The associated beat length is in the few-metre range for standard SMF fibres, but may be significantly shorter for rare-earth doped fibres—to which the stress-induced birefringence brought by fibre coiling or splicing may add. Therefore, instead of NPR, a better description of the mechanism at play is *non-linear polarization evolution* (NPE), where the polarization beat length varies non-linearly with the optical intensity (Winful 1985). NPE followed by polarization discrimination makes an efficient virtual saturable absorber, provided that the non-linear length remains smaller than or comparable to the polarization beat length. This is generally the case with standard optical fibres but the above criterion obviously precludes the use of PM fibres. One

---

**FIGURE 39.1** Non-linear loop mirrors and passively mode-locked fibre lasers. (a) NOLM. (b) Dispersion-imbalanced NOLM. (c) NALM. (d) Figure-8 laser. (e) Unidirectional fibre ring laser mode-locked through non-linear polarization evolution (NPE). (f) All-normal-dispersion fibre laser including a free-space cavity section. Coll, collimator; DCF, dispersion-compensation fibre; DF, rare-earth doped fibre; DL, diode laser; PC, fibre polarization controller; PBS, polarizing beam splitter; SMF, standard single-mode fibre; WDM, wavelength-division multiplexer; YDF, ytterbium-doped fibre.
39.3 The Soliton Laser: Advantages and Limitations

In the 1990s, the development of ultrafast fibre lasers in the 1.5 µm window was accompanied by the fast expansion of high-bit-rate optical communications, benefiting from the newly available optical amplifier and efficient fibre-integrated components, whose cost has been dramatically reduced. The 1.5 µm spectral window combines the ultralow loss of silica with the anomalous dispersion of standard single-mode fibres (SMF) and the broadband gain (~25 nm) of erbium-doped silica glass. These conditions have favoured the development of passively mode-locked erbium-doped fibre lasers generating ultrashort soliton pulses.

39.3.1 Soliton Energy

Soliton pulse shaping takes place within the length of passive SMF that is part of the fibre cavity. When the SMF length is larger than the other characteristic propagation lengths, namely the non-linear and the dispersive length, the pulse may reach a fundamental soliton profile, where SPM exactly balances the anomalous chromatic dispersion. The fundamental soliton yields a bell-shaped hyperbolic-secant-square (sech²) pulse intensity profile, whose duration is Fourier-transform limited, namely, with a constant-phase profile across the pulse. The fundamental soliton energy $E$ and duration $T_0$ are constrained by the soliton area theorem:

$$E = \frac{2|\beta_2|}{\gamma T_0},$$  \hspace{1cm} (39.1)

where $\beta_2$ is the second-order chromatic dispersion of the optical waveguide, and $\gamma$ its effective non-linear coefficient. We recall that $\beta_2$ combines both material and geometrical dispersion, which can be engineered. The non-linear coefficient $\gamma$ is proportional to the Kerr non-linearity coefficient of the material $n_2$, and inversely proportional to the effective mode area $A_{\text{eff}}$ and to the optical wavelength $\lambda$, as follows: $\gamma = \frac{2\pi n_2}{A_{\text{eff}} \lambda}$. Note that the soliton duration parameter $T_0$ is related to the full-width pulse duration at half maximum (FWHM) by: $T_{\text{FWHM}} = 1.76 T_0$.

However, we can see that the soliton model does not predict the pulse energy or temporal duration, but only fixes the relationship between them. Technically, this reflects the fact that non-linear propagation, as modelled by the non-linear Schrödinger (NLS) equation (Zhabarov and Shabat 1972), admits a whole family of soliton solutions, as pictured in Figure 39.2. An additional constraint needs to be included to yield a specific soliton solution within a given fibre laser cavity. Such constraint comes from the gain and loss cavity contributions (Kutz 2006). One constraint is obtained by

![FIGURE 39.2 The soliton family. For a given optical waveguide, the balance between the anomalous dispersion $\beta_2$ and the non-linearity $\gamma$ defines a soliton family, which has the soliton duration $T_0$ as free parameter. Represented by the dotted line, the family spans from high-energy short pulses to low-energy long pulses. The temporal field profile $\psi$ of the fundamental (first order) soliton is given in the upper-right formula as a function of the local time $T$ in the pulse reference frame, and the propagation distance $Z$.](image-url)
the following reasoning. The mode-locked pulses tend to use the available gain bandwidth, which fixes the pulse duration, assuming Fourier-transform limited pulses. Given the amount of anomalous dispersion, this fixes the pulse energy. However, the main constraint can originate from the saturable absorber, which fixes the pulse peak power. This in turn determines the pulse duration and energy compatible with the soliton area theorem. That is why the adjustment of saturable absorbers such as those relying on non-linear interference (NALM, NPE) can lead to a wide range of pulse parameters within a given laser cavity setup.

Let us provide some typical orders of magnitude. With a laser gain bandwidth of 25 nm, soliton pulses can be as short as 100 fs ($T_{WWM}$). By using a standard SMF fibre at 1.55 µm, with dispersion $\beta_2 = -0.021$ ps$^2$ m$^{-1}$ and non-linearity $\gamma = 1.2 \times 10^{-3}$ W$^{-1}$ m$^{-1}$, and assuming that the saturable absorber saturation intensity is appropriate, the fundamental soliton has an energy of 0.6 nJ. The output pulse energy will be a fraction of that, say, around 0.1–0.2 nJ. This is the typical energy limitation that results from soliton pulse shaping, by using SMF fibres in the 1.5 µm spectral window. Note that the soliton energy limitation is higher by nearly an order of magnitude in thulium-doped fibre lasers operating at ~2 µm, as a result of larger anomalous dispersion, wavelength and mode area in standard SMF. Actually, the most efficient way to increase the fundamental soliton energy is through the engineering of large mode area fibres (LMA), which can gain more than two orders of magnitude in the mode area scaling (Richardson et al. 2010).

### 39.3.2 Dispersive Wave Radiation

Nevertheless, the overall pulse propagation within a fibre laser cavity is quite far from the propagation experienced in a single passive fibre. The large saturated-gain of the rare-earth doped fibre compensates for the losses of the optical components (optical isolator, multiplexer etc.), their coupling (splices, connectors), and the major losses resulting from the saturable absorber and the output coupler. Altogether, these losses can typically amount to ~10 dB per cavity roundtrip. In addition, the rare-earth doped fibre generally has a different mode area than that of the passive fibre, so that the waveguide dispersion contributions can differ widely. Therefore, the pulse will experience major, periodic, soliton-like reshaping, which entails the emission of dispersive waves. Dispersive waves are low-amplitude radiated waves that propagate linearly. The amount of such radiation depends on the ratio between the average non-linear length and the cavity length. When this ratio is large, the soliton evolution will not be able to follow closely the propagation medium landscape, so that an average soliton propagation can prevail, leading to a moderate level of dispersive wave radiation (Hasegawa and Kodama 1990, Kelly et al. 1991). Noting that a fibre laser cavity is generally not so short, with a length of several metres, that ratio will be typically of the order of unity or smaller: in such a case, the amount of radiation will be large, leading to a spreading oscillating pulse pedestal in the temporal domain (Gordon 1992), and to a specific spectral signature featuring high-amplitude sidebands (Nørskov et al. 1992). As a consequence, the pulse temporal profile can deviate significantly from the sech$^2$ profile of a conventional soliton, while the spectral sidebands can contribute to a significant fraction of the intracavity energy.

Gordon-Kelly Sidebands. The existence of specific radiation sidebands, as opposed to a radiation continuum, is linked to the periodicity of the perturbation undergone by the circulating pulse. Such phenomenon was predicted by Gordon in the context of periodic optical transmission lines (Gordon 1992), and by Kelly for ultrafast fibre lasers (Kelly 1992). Resonant wave mixing occurs between dispersive waves and soliton spectral components when their phase matching is satisfied after each period of the propagation landscape. Such phase matching condition yields resonant sidebands that are symmetric with respect to the soliton spectrum and unevenly spaced (see Figure 39.3a), with a spectral distribution that follows:

$$\Delta \nu_N = \pm \frac{1}{2\pi} \frac{1}{L} \left( \frac{4\pi N}{L} - P \gamma \right)$$

where $L$ is the propagation period, $N$ is the sideband order (absolute value), $P$ is the path-averaged second-order chromatic dispersion (here, negative), $\gamma$ is the average non-linearity, and $P$ the average soliton peak power. These sidebands are very familiar to fibre laser physicists, who dub them Kelly sidebands, or better, Gordon-Kelly sidebands, see Figure 39.3(a). By fitting their location, it is possible to measure the

![FIGURE 39.3](image-url) Some typical spectral features observed with passively mode-locked fibre lasers. (a) Spectral sidebands superimposed on the soliton spectrum, in the anomalous-dispersion regime. (b) Rectangular spectral profile obtained in the normal-dispersion regime. (c) Modulated soliton spectrum of a stable three-soliton molecule laser output, also confirmed in the temporal domain by the optical autocorrelation (inset).
average cavity dispersion. Note that the sidebands become asymmetrically located whenever third-order dispersion is no longer negligible with respect to the second-order dispersion. In addition, as quasi-continuous wave components, the sidebands can be subjected to modulation instability at sufficiently high power, leading to a more complex sideband distribution across the pulse spectrum (Tang et al. 2001a).

### 39.3.3 Multi-Pulsing Instability

In soliton fibre lasers, the spectral sidebands tend to grow along the increase of the pumping power, until a multi-pulsing instability develops. The latter is responsible for the generation of one or several additional pulses within the cavity (Komarov et al. 2005, Liu 2010). These multiple pulses generally share the same field profile and thus carry the same energy, giving the sense of an energy quantization within the laser cavity (Grudinin et al. 1992). After one such multi-pulse transition has occurred, the energy taken by the dispersive waves is reduced (Gutty et al. 2001). Such scenario can go on repeatedly as the pumping power is increased. For a fibre laser operated in the anomalous dispersion regime under multi-Watt pumping power, several hundreds of identical pulses can circulate simultaneously round the cavity (Haboucha et al. 2008, Lecaplain and Grelu 2013). These pulses will always interact, since being regenerated one roundtrip after another, they have virtually an infinite time to express the variety of their interaction processes, even in the case of faint interaction (Sanchez et al. 2014, Weill et al. 2016). Therefore, a wide range of pattern formations can take place, according to the hierarchy of pulse interactions that in turn depends precisely on the set of cavity parameters. Common patterns range from passive harmonic mode locking (HML) (Grudinin and Gray 1997), which is dominated by repulsive interactions (Kutz et al. 1998), to pulse bunching, dominated by attractive ones (Richardson et al. 1991). In particular, HML appears as a very promising option to multiplex the pulse repetition rate within the relatively long fibre cavity, and therefore reach the multi-GHz repetition rate that is otherwise typical from the semiconductor laser platform, while benefiting from the higher optical power and shorter pulse duration delivered by the fibre laser platform (Pang et al. 2015). Among pulse bunching cases, stable bound states can form, called soliton molecules, which will be explained in the following section. Nevertheless, multiple pulsing is generally considered as a major setback that hampers the scaling of the pulse energy with the pump power, owing to the limitation imposed by the soliton area theorem.

Whereas we have seen that a laser cavity poses a number of issues to implement the soliton dynamics, one can also take advantage of the subsequent propagation after the laser output to improve the pulse features in accordance to the applicative needs. By using an optimized passive fibre link connected to the laser output, it is indeed possible to benefit from well-known solitonic effects, such as soliton compression to get shorter pulses and higher peak power (Chernikov et al. 1993, Travers et al. 2007), or soliton self-frequency shift to obtain a widely tunable pulse source (Mitschke and Mollenauer 1986, Nishizawa et al. 2000).

### 39.4 The Concept of Dissipative Solitons

#### 39.4.1 Beyond Conventional Solitons

In the multiple-pulse regime, the rich pattern-forming dynamics that is found experimentally in fibre lasers cannot be explained by the conventional soliton dynamics that applies to the passive optical fibres. First, the formation of multiple identical ultrashort pulses within the laser cavity involves the repeated action of an attracting state of dissipative nature (Grelu and Akhmediev 2012). This is opposed to the generation of a soliton family comprising pulses of various duration and energy from passive optical fibre rings (Schwache and Mitschke 1997). Second, understanding the self-formation of peculiar pulsed laser outputs such as the ubiquitous soliton molecules (Gutty et al. 2001, Tang et al. 2001b) requires going beyond the conventional soliton propagation model, which does not predict a stable equilibrium between interacting pulses (Gordon 1983).

Besides, the generation of stable ultrashort pulses in the normal-dispersion cavity regime appears counterintuitive, as it opposes the view of the chromatic dispersion balancing the accumulated self-phase modulation. Nevertheless, it dawned theoretically as early as in 1991 that a laser including dispersion management could be advantageously shifted into the net-normal-dispersion regime, to increase the pulse energy, albeit at the expense of its time-bandwidth product (Haus et al. 1991). This prediction was validated a couple of years later (Tamura et al. 1993), leading to a substantial research activity on the stretched-pulsed dynamics generated within dispersion-managed lasers, which also inspired additional innovative laser architectures (Ilday et al. 2004). Another decisive step was taken with the theoretical prediction that stable frequency-chirped pulses could be formed within an entirely normal propagation medium, i.e. without the need for dispersion compensation (Soto-Crespo et al. 1997). Such laser regime was validated experimentally a few years later (Chong et al. 2006), demonstrating an advantageous pulse energy scaling beyond 30 nJ, see the sketch of the normal-dispersion ring cavity architecture in Figure 39.1f (Kieu et al. 2009).

#### 39.4.2 The Dissipative Soliton Paradigm

The above paradoxical ultrafast laser regimes find their interpretation within the conceptual frame of dissipative solitons (Grelu and Akhmediev 2012, see also Further Reading). As represented in the sketch of Figure 39.4a, the composite balance involving non-linear gain and loss, dispersive non-linearity and diffraction/dispersion provides a fixed soliton solution, instead of the soliton family of a passive transmission line. Such soliton, whose profile is therefore defined by the laser cavity parameters, is called a dissipative soliton. It acts as an attracting state: as long as energy is pumped into the system, any initial field condition belonging to the basin of attraction will converge toward the fixed dissipative soliton profile. This provides a simple way to understand the stability of mode-locked lasers, where an ultrashort pulse is able to propagate during hours or, equivalently, through billions of kilometres. Such a feat is impossible for conventional solitons, owing to
unavoidable losses and noise perturbation along any practical passive transmission line. Here, the losses are compensated, and the noise is eaten by the dissipative processes—by the saturable absorber in particular—so that its accumulation remains bounded.

### 39.4.3 Dissipative Solitons in the Normal Propagation Regime

The dissipative soliton paradigm applies to modelocked lasers in general, but it is particularly relevant for fibre lasers and other photonic systems where pulses experience significant amounts of non-linear gain and loss (see Further Reading). The composite balance then allows for a larger freedom in handling the contributions of the physical effects involved. Besides the double-balance extension of the soliton laser concept in Figure 39.4a, it is also possible to balance the normal chromatic dispersion, as sketched on Figure 39.4b. Tilting the arrows that represent the gain and loss terms means physically that we introduce a large level of gain/loss dispersion. Obviously, by continuity, dispersion is not a prerequisite to stable pulse propagation (Figure 39.4c). To understand better how pulses can be stabilized in the normal dispersion, Figure 39.4d represents the action of a strong spectral filtering on a highly-chirped pulse: owing to the dissipation, the extended pulse wings are clipped off, which combats the temporal broadening due to normal dispersion accentuated by self-phase modulation. Such possibility was anticipated by Haus et al. (1991): For positive GVD, the chirp parameter can become quite large (...) Gain dispersion, instead of lengthening the pulse, shortens it. At first sight, this may seem paradoxical. However, if a pulse is chirped, frequency filtering can, in fact, shorten it, if it shaves off the high- and low-frequency wings. (...) The pulse is kept in balance by a lengthening due to positive GVD. Of course, the gain dispersion also narrows the pulse spectrum. This spectral narrowing is compensated for by SPM (Haus et al. 1991).

Consequently, ultrashort pulses generated from laser cavities that are operated in the normal dispersion regime always present a large amount of frequency chirping. This is precisely one efficient way to circumvent the pulse energy limitation applying to soliton lasers. Owing to a reduced peak power, a chirped pulse can propagate without suffering breakup instabilities, at a much higher pulse energy. In essence, the strategy is similar to that of chirped-pulse amplification (CPA) (Strickland and Mourou 1985), except that it is implemented inside of the laser cavity. Therefore, the energetic chirped pulses need to be compressed after the laser output (Kieu et al. 2009).

### 39.4.4 Dissipative Soliton and Frequency Chirping

A dissipative soliton pulse always presents frequency chirping. Fundamentally, recalling that energy flows through phase gradients, the non-uniform phase profile of the pulse is dictated by the necessary energy redistribution resulting from the action of non-linear gain and loss. Therefore, in addition to a linear chirp part, the pulse generally presents non-linear frequency chirping that is more difficult to compensate. Dissipative soliton pulses can therefore present temporal profiles that are very different from the sech² shape of an NLS soliton, as well as from the typical Gaussian shape of a linearly-chirped pulse. For instance, flat-top pulse profiles are common in the normal-dispersion regime. Naturally, this is also reflected in the spectral domain, where the optical spectra of modelocked pulses in normally-dispersive laser cavities present a plateau with steep edges, see Figure 39.3b. The spectrum often includes broad bounces right before these edges, which make an “M-shaped” spectral profile (Kieu et al. 2009).

### 39.4.5 Dissipative Soliton Molecules

The dissipative soliton nature reveals itself in other situations, including for lasers operating in the anomalous-dispersion regime. For the latter, the conventional soliton theory can only be considered as a first approach to describe optical pulses having a small amount of frequency chirping. The limit of the conventional soliton theory is met when soliton pulses interact to form a stable optical soliton molecule. Soliton molecule formation is a universal behaviour observed among
the whole range of ultrafast lasers operating in the multiple pulse regime (Grelu and Akhmediev 2012). The soliton-pair molecule is the simplest one: it consists of a bound state of two mutually-trapped optical pulses at a temporal separation of a few pulse widths. It results from a complex pulse attractor, which explains the self-assembly and subsequent stability of the soliton molecule. The experimental characterization of self-phase-locked soliton molecules has yielded sub-femtosecond timing jitter for the relative separation between the two ultrashort pulses (Shi et al. 2018). Molecules comprising higher number of pulse constituents are also commonly observed. In general, soliton molecules can be analysed from the spectral domain, since the interference among multiple pulses produces spectral fringes of high contrast, see Figure 39.3c. In the soliton-pair molecule case, the temporal separation between the two solitons is simply the inverse of the spectral interfringe, whereas the offset of the fringe pattern with respect to the central wavelength yields the relative phase between the solitons.

### 39.5 Applications of Dissipative Solitons

Beyond providing a consistent frame to understand the complex dynamics of ultrafast lasers, the dissipative soliton paradigm has stimulated the creative design of new mode-locked laser architectures, bypassing the former prerequisites of a low-loss cavity and dispersion compensation. Note that this conceptual margin has been possible thanks to the large gain available from the rare-earth doped fibres.

#### 39.5.1 The Quest of High-Energy Pulses from Laser Oscillators

Increasing the output pulse energy is the most recurrent objective with fibre lasers, which confronts the pulse breakup issue at higher pump powers. The overdrive of the saturable absorber, the excess of self-phase modulation, and/or other higher-order effects (Raman effect, thermal mode instability, component damage) can either lead to multiple pulsing or to the disruption of mode locking. There are two main remediation directions. The first one is to act on the spatial (transverse) domain: the larger the effective mode area, the less the non-linear impairments, so the larger the pulse energy. This is efficient and scalable, but remains highly technical, involving for instance either LMA fibres, rod-type fibres (Ortaç et al. 2007) or thin-disk lasers (Marchese et al. 2008). Note that coherent beam combining is the current step forward in this direction, leading to megawatt-class ultrafast fibre laser oscillator systems of KW average optical power (Kienel et al. 2016). The second direction is to act on the temporal domain: by stretching the pulse, thus limiting peak optical intensities, high-energy pulses can be generated. As explained above, this is precisely the chosen field of dissipative solitons, with various laser cavity architectures possibilities, as introduced below. Naturally, the two approaches can be combined, paving the way for unprecedented pulse energy levels directly out of robust fibre or waveguide laser oscillators, leading after pulse compression to MW-level peak powers (Baumgartl et al. 2010).

#### 39.5.2 Innovative Chirped-Pulse Laser Architectures

The Stretched-Pulse Fibre Laser. The original stretched-pulse fibre laser cavity developed in the early 90s is based on a concatenation of optical fibres having opposite dispersion signs, so as to present an average dispersion close to zero (Tamura et al. 1993). Such contrast between the local and the average dispersion is responsible for a large temporal breathing of the laser pulse that repeats twice per roundtrip (Turitsyn et al. 2012). Consequently, there are two locations where the pulse duration is close to the Fourier-transform limit. Elsewhere in the cavity, the pulse is stretched, with a stretching factor that can be in excess of 10. Using standard optical fibres and components, the laser can generate pulses with energies in the few-nJ range and peak power around 20kW after external pulse compression (Lenz et al. 1995). Compared with conventional soliton lasers, dispersion-managed lasers achieve a typical ten-fold increase of the pulse energy.

Self-Similar Evolution. Is it possible to improve the stretched-pulse strategy? The existence of two locations within the cavity where the pulse is compressed remains a weak point, as instabilities are prone to develop around these locations of highest peak power. It is desirable to develop a larger pulse stretching, while limiting the peak power where the pulse is at its minimum duration. Self-similar propagation is one interesting solution in that respect. In the normal dispersion regime, a pulse can acquire a monotonic chirp and evolve toward a self-similar parabolic profile that avoids pulse breakup (Anderson et al. 1993, Fermann et al. 2000). In 2004, this propagation regime was implemented in a fibre laser cavity and subsequently developed (Ilday et al. 2004, Oktem et al. 2010). Self-similar propagation allows to gain nearly another order of magnitude in the pulse energy, while parabolic pulses can be easily compressed. Implementing such a marked internal evolution within the laser cavity requires to reset the pulse after every cavity roundtrip: this is the self-consistency issue. Self-consistency is here obtained by inserting a relatively narrow spectral filter, which shrinks the chirped pulse in both temporal and spectral domains, allowing for the subsequent roundtrip self-similar evolution. Again, such a strategy opposes the conventional handling of cavity losses, but finds its full justification within the dissipative soliton concept: high losses, if deeply associated with the non-linear dynamics, can be useful to define a suitable attracting pulse state. Similarly, spectral filtering is essential for the consistency of the roundtrip propagation in the all-normal-dispersion lasers, as explained earlier.

Ultralong All-Normal Cavities. Higher frequency chirping can be provided in ultralong fibre laser cavities operating in the normal-dispersion regime (Kelleher et al. 2009). For a given allocated pump power, which limits the average laser output power, increasing the cavity length allows for an increased pulse energy. In 2008, by using a 3.8-km long fibre, researchers achieved an additional increase of the dissipative-soliton energy by two orders of magnitude, generating 3-ns highly-chirped pulses with a 3.9 μJ energy (Kobis et al. 2008). Note that long fibre cavities generally require additional stabilization, as they are more sensitive to environmental
perturbations. It is also important to mention that propagation of wideband energetic pulses in long fibre cavities favours stimulated Raman scattering. The latter may convert the excess energy out of the coherent dissipative soliton into a noisy Raman pulse, thus limiting its energy (Aguergaray et al. 2013). However, it has been shown that an appropriate two-track feedback scheme allowed for the formation of a coherent two-color Raman dissipative soliton (Babin et al. 2014).

### 39.5.3 Peculiar Laser Pulses

**Dissipative Soliton Resonance.** Normal dispersion may not be always required to generate ultralong energetic pulses. Within an ultrafast laser, a specific balance among dissipative and dispersive physical effects can lead to the formation of ultralong rectangular pulses carrying high energy. This phenomenon, which is accentuated within a limited range of cavity parameters, is called dissipative soliton resonance (DSR), and can take place in both dispersion regimes (Greul et al. 2010). It makes an interesting alternative to ultralong cavities for the generation of ns-long pulses with μJ energy level (Seeman et al. 2016). However, in practice, the temporal coherence of experimental DSR pulses needs to be investigated further, as such parameter resonance can become unstable in presence of perturbations, leading to the formation of noise-like rectangular pulses.

**Noise-Like Pulses.** Under higher pumping power, a peculiar instability can take place. Instead of either generating additional dissipative soliton pulses, or completely disrupting the short-pulse laser operation, the laser can sustain the propagation of a high-energy optical burst, or noise-like pulse (Horowitz et al. 1997). This regime looks paradoxical, as the pulse is subjected at a same time to a strong instability and to a strong temporal localization that maintains its short duration—picosecond to nanosecond, typically. Since it propagates round the cavity, and generates a broad averaged optical spectrum, it resembles to mode locking. However, shot-to-shot spectral measurements confirm the pulse incoherence (Runge et al. 2013). This regime leads to a generalization of the dissipative soliton concept to incoherent pulses, this time with the presence of a chaotic attractor. Noise-like pulses could be used in a few applications requiring low coherence and high pulse energy, such as in optical coherence tomography and laser ranging.

### 39.6 Prospects

Ultrafast fibre laser development has been continuously expanding over the past two decades, stimulated by novel soliton concepts and fuelled by major advances in materials and integrated components. Obviously, laser applications are always eager to implement one or several of the following upgrades: higher pulse energy, higher pulse-repetition rate, new spectral window, shorter pulse duration. In near future, some applications may also request more agility, versatility, and self-starting resilience from the ultrafast laser source itself, in the wake of the early development of smart lasers, i.e. lasers combining dissipative soliton dynamics, automated ultrafast measurement and machine learning (Andral et al. 2015, Woodward and Kelleher 2016, Winters et al. 2017).

Soliton energy scalability will benefit from the ongoing development of ultrafast lasers operated in the transverse multimode regime. Indeed, the discovery of novel spatiotemporal propagation regimes reenables the transverse degree of freedom for dissipative soliton waveguide and fibre lasers. This represents a wide margin to increase the transverse effective area of the guided pulse, and support the propagation of higher-energy solitons (Wright et al. 2017, Guenard et al. 2017, Qin et al. 2018). The generalization of the optical temporal soliton to the transverse spatial dimensions is also called a light bullet, which could find applications in parallel all-optical processing. Beyond mode-locked laser cavities, other oscillator architectures involving stable attracting states can also be developed. Such is the case with Mamyshev oscillators, based on Mamyshev regenerators (MR) (Mamyshev 1998). A MR combines a high amount of SPM followed by a strong spectral filtering to regenerate optical pulses at a carrier wavelength that is significantly offset from the initial central wavelength by, say, Δλ. By adding a spectrally-symmetric MR, namely an optical regenerator that regenerates optical pulses at a carrier wavelength offset by −Δλ, and operating within an optical amplifying loop, a stable roundtrip propagation can be obtained. That stable pulse propagation is attributed to a special type of dissipative soliton attractor, which tolerates a large amount of SPM and therefore circumvents efficiently the pulse breakup at high optical intensity (Regelskis et al. 2015, Fu et al. 2018). Note that Mamyshev oscillators cannot be truly considered as mode-locked lasers, as in most configurations, these are not lasing in quasi-continuous wave and therefore do not sustain cavity modes. These oscillators require an optical injection to initiate the pulse propagation, or in the language of non-linear dynamics, to enter the basin of attraction of the dissipative soliton.

Besides the quest of higher-energy dissipative solitons, there is also an intense research activity focused on special soliton patterns generated from mode-locked fibre lasers. This research encompasses the study of soliton molecules, which has been accentuated by the recent availability of real-time ultrafast characterization techniques. Among these techniques, the time-stretch dispersive Fourier-transform technique has become the most popular, as it allows to monitor single-shot optical spectra from low-power laser oscillators within a simple experimental setup (Goda and Jalali 2013). Therefore, various ultrafast dynamics of soliton molecules can be precisely monitored and interpreted (Krupa et al. 2017, Herink et al. 2017). Whereas this research remains fundamental so far—the fibre laser platform serving as a universal testbed of non-linear dynamics—some applications of pulse patterns could appear in future, such as for optical data encoding in multilevel schemes (Rohmann et al. 2012). In addition, understanding better and controlling dissipative soliton interactions in a laser cavity could lead to passively mode-locked laser sources at high harmonics (multi-GHz repetition rates) with unprecedented stability. To this goal, hybrid laser cavities combining the respective advantages of the optical fibre and those of the planar waveguide technology represent an
interesting possibility (Peccianti et al. 2012). Finally, by considering further polarization, carrier wavelength and spatial modes, possibly in combination, and their non-linear coupling, the possibilities of soliton and dissipative soliton dynamics in fibre and waveguide lasers seem limitless.

REFERENCES


Godz K and Jalali K B 2013 Dispersive Fourier transformation for fast continuous single-shot measurements Nat. Photon. 7 102.


Grelu Ph and Akhmediev N 2012 Dissipative solitons for mode-locked lasers Nat. Photonics 6 84.


Keller U 2003 Recent developments in compact ultrafast lasers Nature 424 831.


Pang M et al. 2015 Stable subpicosecond soliton fiber laser passively mode-locked by gigahertz acoustic resonance in photonic crystal fiber core Optica 2 339.


Seeman G et al. 2016 Generation of high energy square-wave pulses in all anomalous dispersion Er/Yb passive mode locked fiber ring laser Opt. Express 24 8399.


Weill R, Bekker A, Smulakovsky V, Fischer B and Gat O 2016
Noise-mediated Casimir-like pulse interaction mechanism in lasers Optica 3 189.
Wong W S, Namiki S, Margalit M, Haus H A and Ippen E P 1997
Wright L G, Christodoulides D N and Wise F W 2017
Spatiotemporal mode-locking in multimode fiber lasers Science 358 94.

FURTHER READING
2013 Nature Photonics Issue on Fiber Lasers 7(11) 841–82.
A special issue with emphasis on high-power fibre lasers.


The reference book to understand the variety of nonlinear effects relevant in fiber optics and apprehend the conditions under which they exert and can be utilized.


These two multi-authored volumes unfold the background and applications of dissipative solitons applied to various research fields, in photonics and other disciplines of nonlinear science.

A special issue covering various pulse emission regimes and wavelength.

A multi-authored review presenting many aspects of soliton dynamics from various platforms in photonics.

A multi-authored review covering high-power fibre lasers and amplifiers, and extension of wavelength coverage.
Presents a variety of advanced fibre technology and fibre laser architectures.